Special Program in Applied Mathematics and Applied Mechanics

Relaxation-type Non-local Inertial-number Rheology for Granular Materials

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Granular materials, such as sands, rocks, cereals, minerals, pills, powders… are renowned complex materials which can present both fluid and solid behaviors, rendering the unified description a challenge. In this talk, we introduce a new dynamical constitutive model that can describe many salient rheological features of granular materials. By taking an inertial number I, the ratio of a grain rearrangement time and a bulk deformation time, as an order parameter measuring the degree of local fluidization, the model is formulated as a relaxation equation for I which describes how fluidization is created due to grain avalanche and how it is dissipated due to inelastic collisions. The model contains bistability to give rise to a non-monotonic bulk friction law so that hysteresis and stick-slip near flow-no-flow transition are predicted. In inhomogeneous sheared flows, the model reduces to a stress relation which reveals two non-local mechanisms associated with long-range momentum transport due to collisions among grain structures and spreading of fluidization due to grain rearrangement inside the structures. This relation generalizes a famous Bagnold stress for uncorrelated collisional flows. Applied to an inclined plane flow, the model successfully recovers renown hysteretic phase diagram in terms of the material thickness and the inclination angle and the Pouliquen flow rule for the mean flow velocity. In addition, the model can describe a Bagnold-to-creeping flow transition observed in literature through a newly-discovered non-locality parameter characterizing the length of force chain extending from the flow bottom.


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