Special Program in Applied Mathematics and Applied Mechanics

On boundary layers of an electrostatic model: Pointwise description for concentration phenomena with curvature effects and its applications

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For the structure of the thin electrical double layer (EDL), we analyze boundary layer solutions of a nonlocal electrostatic model with small Debye screening length. The model is an elliptic type with nonlocal dependence on its unknown variable, and it is known that the limiting profiles of solutions asymptotically blow up near the boundary. In this work, a series of fine estimates that combine the Pohozaev's identity, the inverse H\"{o}lder type estimates and some technical comparison arguments is developed for establishing boundary asymptotics for solutions in arbitrary bounded domains. The concept of inverse H\"{o}lder type estimates seems novel, and rarely appears in previous related literatures. Moreover, to gain a clear picture on the curvature effect of the thin boundary layer, we concentrate on the physical domain being a ball with the simplest geometry. The current study involves three contributions. The first one is about the boundary concentration phenomena. We show that the net charge density behaves exactly as Dirac delta measures concentrated at boundary points, which presents that the extra charges are accumulated near the boundary (charged surface), and the ionic distribution approaches electroneutrality in the bulk. For the second one, the boundary asymptotic blow-up behavior is completely illustrated. An interesting outcome shows that the significant curvature effect merely occurs in the part of the boundary layer sufficiently close to the boundary, and this part is quite thinner than the whole boundary layer. The third contribution is a connection to physical applications. Using such pointwise descriptions, we calculate the corresponding EDL capacitance and provides a theoretical way to support that the EDL has higher capacitance in a

quite thin region near the charged surface, not in the whole EDL.