Special Program in Applied Mathematics and Applied Mechanics

New Aspects of Constitutive Model for Dense Granular Flows and Their Analytic Solutions to Inclined Flows

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15:00 - 18:00
308, Mathematics Research Center Building (ori. New Math. Bldg.)

There is an ever-growing needs for predictive continuum models for granular flows in geophysics and industry. However, due to the complexity of the materials which exhibit both solid and fluid behaviors, a unified constitutive model and analytical solutions that can reveal underlying mechanisms for macroscopic phenomenon remain challenges. In this talk, new aspects on analytical solution in incline flow configuration and constitutive model for granular flows are presented.

In the first scope, we will employ asymptotic method to solve for local flow velocity and bulk free surface profile in the limit of flow shallowness regarding different flow conditions and existing constitutive models. We will show that, within an intermediate range of slope angles for either continuous or finite mass incline flows, the continuum model can be reduced to a singular perturbation problem. To the leading order, the evolution of free surface at upstream is described by a kinematic-wave equation and the velocity profile is uniformly Bagnold-type, while this asymptotic behavior leads to a unrealistic shock front required to be modified via employing singular asymptotic expansion to include higher-order gradient effects. The upstream kinematic-wave dynamics accounts for the deposition process of a finite mass when the size of rigid internal structure is comparable with the flow height locally, giving analytical models for the arresting flow trajectory and the deposit length. The model predictions show satisfactory agreements with experimental data in literature.

In the second scope, we will present a new relaxation-type constitutive model able to capture many complex granular rheological behaviors including

hysteresis, non-locality and collisional momentum transport, as the first in the literature. In application to a uniform incline flow, the model is shown to reproduce known hysteretic starting and stopping heights and the Pouliquen mean flow rule and give a newly-discovered dimensionless parameter measuring non-locality to determine Bagnold-creeping flow transition.