AUSMD Fluxes for Simulations of the Shock Waves and Droplets Problems

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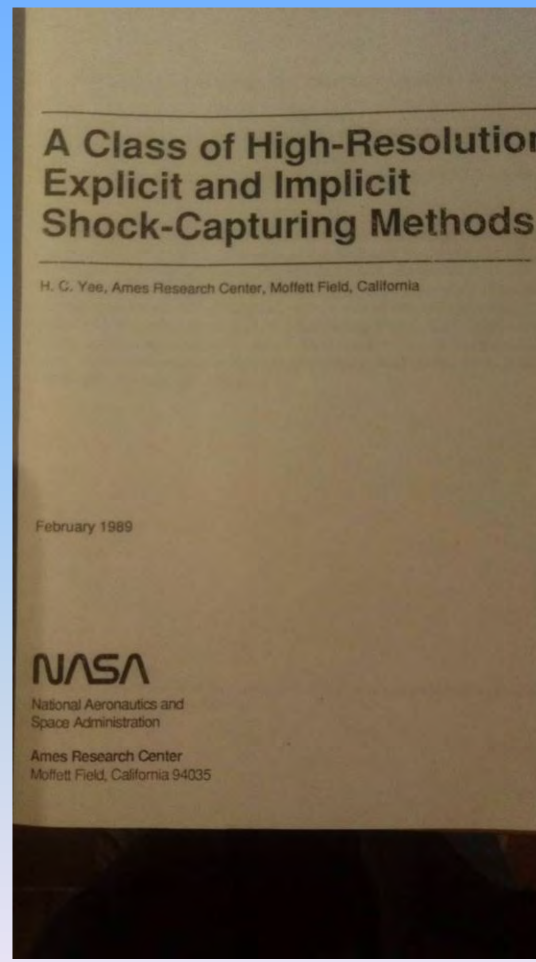

A Class of High-Resolution
Explicit and Implicit
Shock-Capturing Methods

H. C. Yee, Ames Research Center, Moffett Field, California

February 1989

NASA
National Aeronautics and
Space Administration

Ames Research Center
Moffett Field, California 94035

- A+

for My CFD course of PhD study at Ohio-State University due to short course lecture notes of TVD, ENO reorganized by H. Yee and introduced by J. Y. Yang

Flux vector Splitting and Flux Difference

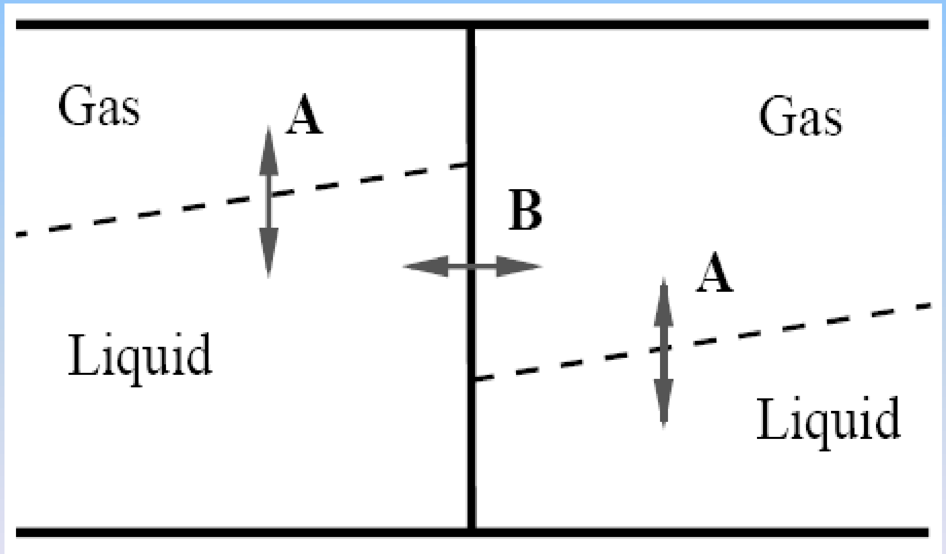
1. **Steger and Warming flux vector splitting scheme 1981** --- large disssipations
2. Van Leer Flux Splitting Scheme 1982--- large disssipations
   a. AUSM(1993) AUSM+(1996) Meng Liou
   b. AUSM+up(2006) by CH Chang and Meng Liou
   c. AUSMD , AUSMV, AUSMDV (1994-1997)
      by Wada & Liou-→ mass flux allows more choices for contact discontinuities,
      Niu(2001) for 7-eq. Two-fluid model.
3. Roe Flux Differencing: The Approximate Riemann flux 1983, HLL, HLLC, HLLE……

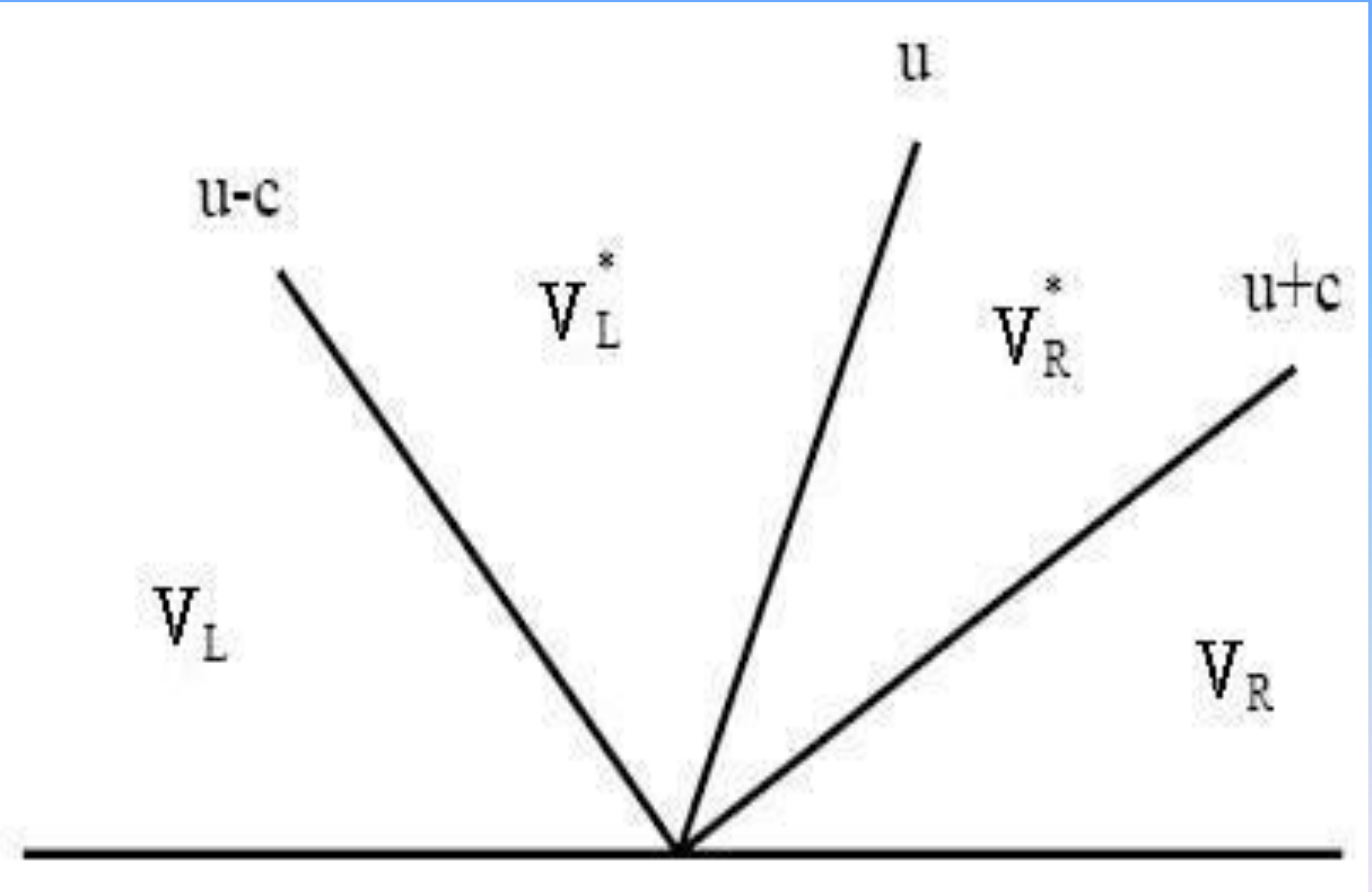
Successfully Flux Splitting for Two component flows

A. AUSM+up works well for gas-gas flow and liquid-quid flow, but failed at computing gas-liquid flows at interfaces so exact Riemann solver for liquid-gas interface is required
B. AUSMD, its mass flux allows easy implementation of evaluating the variables on interfaces of gas-liquid
C. Approximate Riemann fluxes as Roe HLLC, HLLE…… need figuring out tedious eigensystem

Numerical Flux

Exact or Approximated Riemann solvers are used to look for a robust scheme can be applied to the entire gas-gas, liquid-liquid and gas-liquid interface consistently




u
u-c
V*L
V*R
u+c
VL
VR

Characteristic equations

$dp - \rho a du = 0$ along $dx/dt = u - a$

$dp - a^2 d\rho = 0$ along $dx/dt = u$

$dp + \rho a du = 0$ along $dx/dt = u + a$

$C = \rho a$

$\bar{p} + C_L \bar{u}_L = p_L + C_L u_L$ along the characteristic of speed u + a

$\bar{p} - C_R \bar{u}_R = p_R - C_R u_R$ along the characteristic of speed u - a

The complete solution for numerical flux of the liquid-liquid interace and gas-gas interface

$$\bar{p} = \frac{1}{C_L + C_R}\left[C_R p_L + C_L p_R + C_L C_R \left(u_L - u_R\right)\right] \quad ,$$

$$\bar{u} = \frac{1}{C_L + C_R}\left[C_L u_L + C_R u_R + \left(p_L - p_R\right)\right] \quad ,$$

$$\bar{\rho}_L = \rho_L + \left(\bar{p} - \rho_L\right) / a_L^2 \quad ,$$

$$\bar{\rho}_R = \rho_R + \left(\bar{p} - \rho_R\right) / a_R^2 \quad ,$$

$$\bar{\rho} = \frac{1}{2}\left(\bar{\rho}_L + \bar{\rho}_R\right) \quad , \quad \bar{a} = \frac{1}{2}\left(a_L + a_R\right)$$

gas-gas interface become

$$p^*_{g-g} = \frac{1}{2}\left[p_L + p_R + \left(u^g_L - u^g_R \right) / \,\bar{\rho}\bar{a} \;\right] \;,$$

$$u^*_{g-g} = \frac{1}{2}\left[u^g_L + u^g_R + \left(p^g_L - p^g_R \right) / \,\bar{\rho}\bar{a} \right] \;,$$

$$\rho^*_{L_{,g-g}} = \rho^g_L + \left(p^*_{g-g} - p^g_L \right) / \,a^2_L \;,$$

$$\rho^*_{R_{,g-g}} = \rho^g_R + \left(p^*_{g-g} - p^g_R \right) / \,a^2_R \;,$$

$$\rho^*_{g-g} = \frac{1}{2}\left(\rho^*_{L_{,g-g}} + \rho^*_{R_{,g-g}} \right) .$$

For the gas-gas interface,

AUSMD flux

$$E_{g-g}=\frac{1}{2}\left[\left(\rho^{*}_{g-g}u^{*}_{g-g}\right)_{1/2}\left(\Psi_L+\Psi_R\right)-\left|\left(\rho^{*}_{g-g}u^{*}_{g-g}\right)\right|\left(\Psi_R-\Psi_l\right)\right]+P_{1/2},$$

where

$$\Psi_L=\begin{pmatrix}1\\ u_L\\ H_L\\ Y_L\end{pmatrix},\quad \Psi_R=\begin{pmatrix}1\\ u_R\\ H_R\\ Y_R\end{pmatrix}\ \text{and}\ \ \mathrm{P}=\begin{pmatrix}0\\ p^{*}_{g-g}\\ 0\\ 0\end{pmatrix}.$$

liquid-liquid interface become

$$p^*_{l-l} = \frac{1}{2}\left[p_L^l + p_R^l + \left(u_L^l - u_R^l \right) / \bar{\rho}\bar{a} \right] ,$$

$$u^*_{l-l} = \frac{1}{2}\left[u_L^l + u_R^l + \left(p_L^l - p_R^l \right) / \bar{\rho}\bar{a} \right] ,$$

$$\rho^*_{L,l-l} = \rho_L^l + \left(p^*_{l-l} - p_L^l \right) / a_L^2 ,$$

$$\rho^*_{R,l-l} = \rho_R^l + \left(p^*_{l-l} - p_R^l \right) / a_R^2 ,$$

$$\rho^*_{l-l} = \frac{1}{2}\left(\rho^*_{L,l-l} + \rho^*_{R,l-l} \right) .$$

For the liquid-liquid interface

$$E_{l-l} = \frac{1}{2}\left[\left(\rho^*_{l-l}u^*_{l-l}\right)_{1/2}\left(\Psi_L + \Psi_R\right) - \left|\left(\left(\rho^*_{l-l}u^*_{l-l}\right)\right)\right|\left(\Psi_R - \Psi_l\right)\right] + P_{1/2},$$

Where

$$\Psi_L = \begin{pmatrix} 1 \\ u_L \\ H_L \\ Y_L \end{pmatrix}, \quad \Psi_R = \begin{pmatrix} 1 \\ u_R \\ H_R \\ Y_R \end{pmatrix} \text{ and } \quad P_{1/2} = \begin{pmatrix} 0 \\ p^*_{l-l} \\ 0 \\ 0 \end{pmatrix}.$$

By choosing

$$C_L = \rho_L^g a_L^g \quad \text{and} \quad C_R = \rho_R^l a_R^l .$$

The complete solutions of gas-liquid interface can be

$$p_{g-l}^{*} = \frac{1}{C_L + C_R}\left[C_R p_L^g + C_L p_R^l + C_L C_R \left(u_L^g - u_R^l \right) \right] ,$$

$$u_{g-l}^{*} = \frac{1}{C_L + C_R}\left[C_L u_L^g + C_R u_R^l + \left(p_L^g - p_R^l \right) \right] ,(51)$$

$$\rho_{L,g-l}^{*} = \rho_L^g + \left(p_{g-l}^{*} - p_L^g \right) / a_L^2 ,$$

$$\rho_{R,g-l}^{*} = \rho_R^g + \left(p_{g-l}^{*} - p_R^l \right) / a_R^2 ,$$

$$\rho_{g-l}^{*} = \frac{1}{2}\left(\rho_{L,g-l}^{*} + \rho_{R,g-l}^{*} \right) ,$$

For the gas-liquid interface

$$E_{g-l}=\frac{1}{2}\left[\left(\rho^{*}_{g-l}u^{*}_{g-l}\right)_{1/2}\left(\Psi_{L}+\Psi_{R}\right)-\left|\left(\left(\rho^{*}_{g-l}u^{*}_{g-l}\right)\right)\right|\left(\Psi_{R}-\Psi_{l}\right)\right]+P_{1/2},$$

Where

$$\Psi_{L}=\begin{pmatrix}1\\ u_{L}\\ H_{L}\\ Y_{L}\end{pmatrix},\quad \Psi_{R}=\begin{pmatrix}1\\ u_{R}\\ H_{R}\\ Y_{R}\end{pmatrix}\ \text{and}\ \ P_{1/2}=\begin{pmatrix}0\\ p^{*}_{g-l}\\ 0\\ 0\end{pmatrix}.$$

By choosing

$$C_L = \rho_L^l a_L^l \quad \text{and} \quad C_R = \rho_R^g a_R^g$$

The complete solutions of liquid-gas interface can be

$$p_{l-g}^* = \frac{1}{C_L + C_R}\left[C_R p_L^l + C_L p_R^g + C_L C_R\left(u_L^l - u_R^g\right)\right] \ ,$$

$$u_{l-g}^* = \frac{1}{C_L + C_R}\left[C_L u_L^l + C_R u_R^g + \left(p_L^l - p_R^g\right)\right] \ ,(52)$$

$$\rho_{L,l-g}^* = \rho_L^l + \left(p_{l-g}^* - p_L\right) / a_L^2 \ ,$$

$$\rho_{R,l-g}^* = \rho_R^g + \left(p_{l-g}^* - p_R\right) / a_R^2 \ ,$$

$$\rho_{l-g}^* = \frac{1}{2}\left(\rho_{L,_{l-g}}^* + \rho_{R,_{l-g}}^*\right)$$

For the liquid-gas interface

$$E_{l-g}=\frac{1}{2}\left[\left(\rho^{*}_{l-g}u^{*}_{l-g}\right)_{1/2}\left(\Psi_{L}+\Psi_{R}\right)-\left|\left(\left(\rho^{*}_{l-g}u^{*}_{l-g}\right)\right)\right|\left(\Psi_{R}-\Psi_{l}\right)\right]+P_{1/2},$$

Where

$$\Psi_{L}=\begin{pmatrix}1\\ u_{L}\\ H_{L}\\ Y_{L}\end{pmatrix},\quad \Psi_{R}=\begin{pmatrix}1\\ u_{R}\\ H_{R}\\ Y_{R}\end{pmatrix}\text{ and }\ P_{1/2}=\begin{pmatrix}0\\ p^{*}_{l-g}\\ 0\\ 0\end{pmatrix}.$$

The general discretization form for each cell Ω_j can be organized as

$$\Omega_j \partial_t \begin{pmatrix} \alpha_k \rho_k \\ \alpha_k \rho_k u_k \\ \alpha_k \rho_k E_k \end{pmatrix}_j + \sum \left(\overline{ab} E_{g-g} + \overline{bc} E_{l-g} + \overline{bc} E_{g-l} + \overline{cd} E_{l-l} \right)_m n_m S_m$$

$$+\Omega_j \begin{pmatrix} 0 \\ 0 \\ p^{\mathrm{int}} \partial_t \alpha_k \end{pmatrix}_j - \sum \begin{pmatrix} 0 \\ p_{\mathrm{int}} \alpha_k \\ 0 \end{pmatrix}_m n_m S_m = \Omega_j \begin{pmatrix} 0 \\ \rho_k \alpha_k g \\ \rho_k \alpha_k u_k g \end{pmatrix}_j .$$

Here the subscript $k = g$ or l representing gas phase or liquid phase. The functions $\overline{ab}$, $\overline{bc}$ and $\overline{cd}$ are the effective length of each type of interfaces. We define

$$\overline{ab} = \min\left(\alpha_g^L, \alpha_g^R\right),$$

$$\overline{bc} = \max\left(0, \alpha_l^R - \alpha_g^L\right) \qquad \text{or}$$

$$\overline{bc} = \max\left(0, \alpha_g^R - \alpha_l^L\right),$$

$$\overline{cd} = \min\left(\alpha_l^L, \alpha_l^R\right).$$

Current Works

1. Using the 4th order Runge - Kutta method with spatial difference uses 3rd order MUSCL extrapolation to solve a six-equation two fluid model.
2. Deriving AUSMD approximated Riemann flux of two-component flow.

- Journal of Computational Physics, March, 2016
- the 3D case has been published in Computers and Fluids. August ,2016

Two-Fluid Model

We use two kinds of conservative governing equation to solve numerical simulation in each phase then establish three conservative equations consist of mass, momentum and energy.

$$\frac{\partial Q}{\partial t}+\frac{\partial F}{\partial x}=H$$

$$Q=\begin{bmatrix}\alpha_g\rho_g\\ \alpha_l\rho_l\\ \alpha_g\rho_g u_g\\ \alpha_l\rho_l u_l\\ \alpha_g\rho_g E_g\\ \alpha_l\rho_l E_l\end{bmatrix}\quad F=\begin{bmatrix}\alpha_g\rho_g u_g\\ \alpha_l\rho_l u_l\\ \alpha_g\rho_g u_g^{\,2}+\alpha_g p\\ \alpha_l\rho_l u_l^{\,2}+\alpha_l p\\ \alpha_g\rho_g u_g H_g\\ \alpha_l\rho_l u_l H_l\end{bmatrix}\quad H=\begin{bmatrix}0\\ 0\\ g_x\alpha_g\rho_g+p_{\text{int}}\nabla\alpha_g\\ g_x\alpha_l\rho_l+p_{\text{int}}\nabla\alpha_l\\ -p\frac{\partial\alpha_g}{\partial t}+g_x\alpha_g\rho_g u_g\\ -p\frac{\partial\alpha_l}{\partial t}+g_x\alpha_l\rho_l u_l\end{bmatrix}$$

Two-Fluid model

The common two-fluid model (RELAP5-3D ,CATHARE codes) is widely used in Thermal Hydraulic codes

--but brings numerical difficulties. One of the issues is its non-hyperbolicity, making the system an ill-posed problem, presented as numerical instability.

The ways to cure the non-hyperbolicity, always requires additional terms or assumptions,

- the interfacial pressure force (Stuhmiller, 1997)
- virtual mass force (Bestion, 1990)
- two-pressure model or volume fraction transport equation (Baer and Nunziato (1986), Saurel, Abgrall (1999), (Niu, 2001),…….), five equation model……..

Interfacial pressure term

$$p_{\text{int}} = p - \sigma \frac{\alpha_g \rho_g \alpha_l \rho_l}{\alpha_g \rho_l + \alpha_l \rho_g} \left(u_g - u_l \right)^2$$

Where σ is a parameter and the system becomes weakly hyperbolic if

$$\sigma \geq 1$$

Equation of state

Liquid Phase EOS ：

$$P_l = \frac{\gamma_l - 1}{\gamma_l} \rho_l C_{pl} T_l - p_\infty$$ Stiffened Gas E.O.S.

$\rho_l = 998.23 \ {kg}/{m^3}$ $\gamma_l = 1.932$ $C_{pl} = 8095.08 \ {J}/{kg \cdot K}$ $P_\infty = 1.1645 \times 10^9$ Pa

Gas Phase EOS ：

$$P_g = \rho_g R_g T_g$$ Ideal Gas E.O.S.

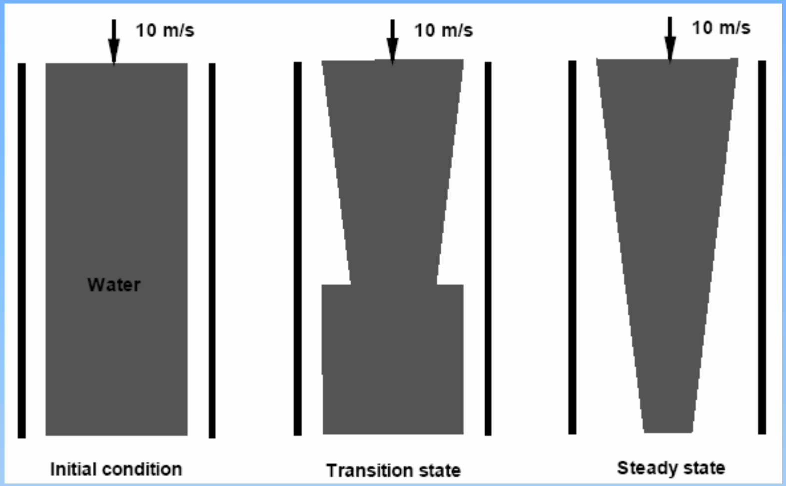
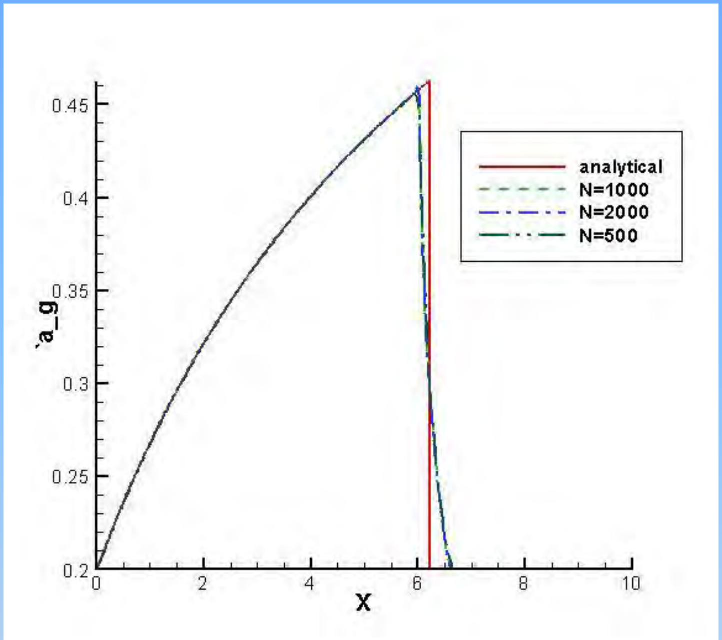


Illustration of Ransom’s water faucet problem.



Several different grids are used in the water faucet problem.

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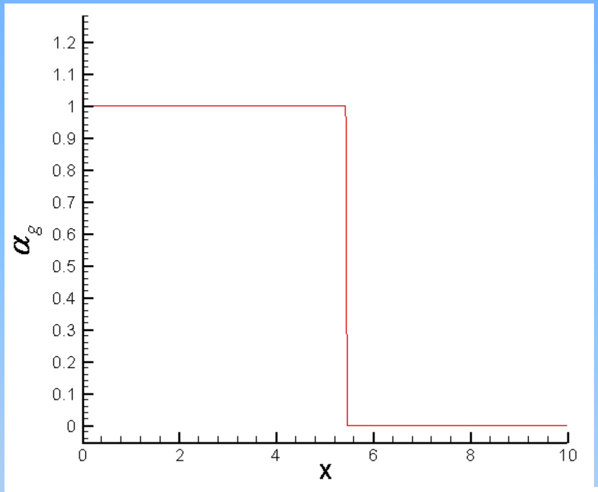
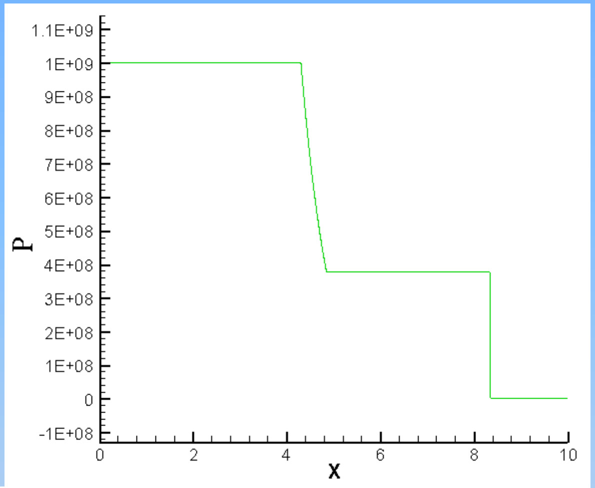
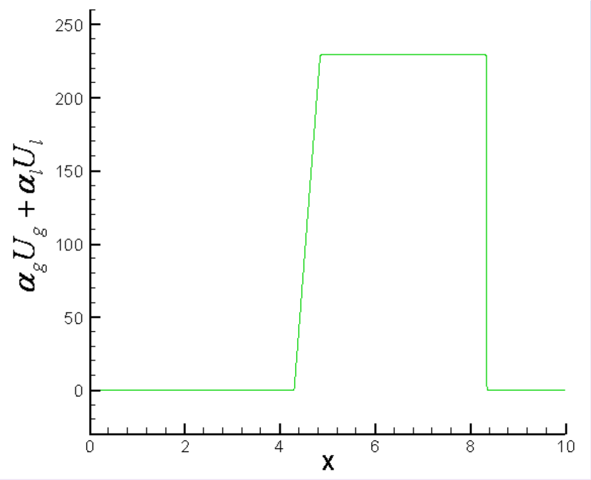
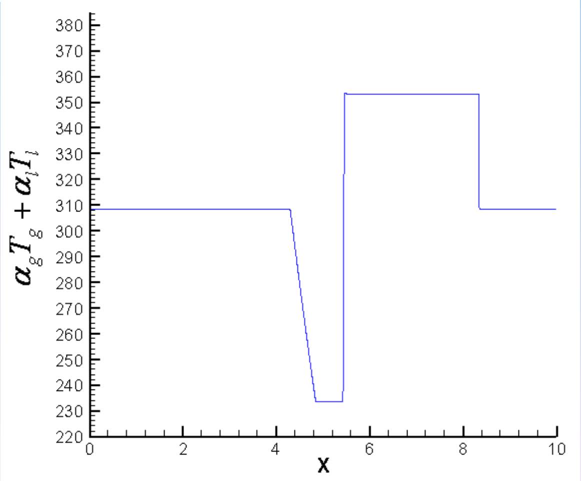
$$(p,\alpha_g,u_i,T_i)_L = (1.0\times 10^9\, pa,\ \ 1-\varepsilon,\ \ 0\, m/s, 308.15\, K^0)$$

$$(p,\alpha_g,u_i,T_i)_R = (1.0\times 10^5\, pa,\ \ \varepsilon,\ \ 0\, m/s, 308.15\, K^0)$$





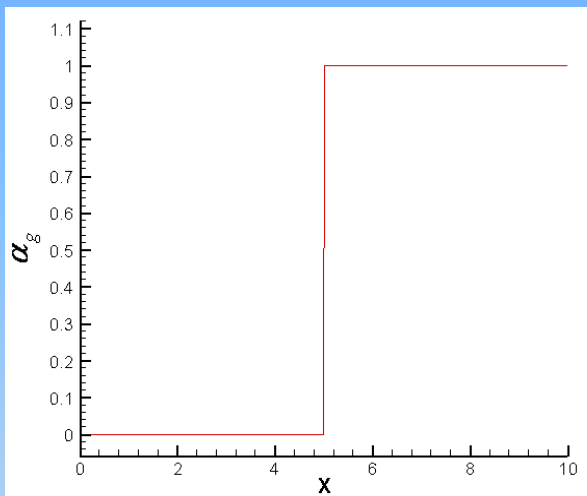


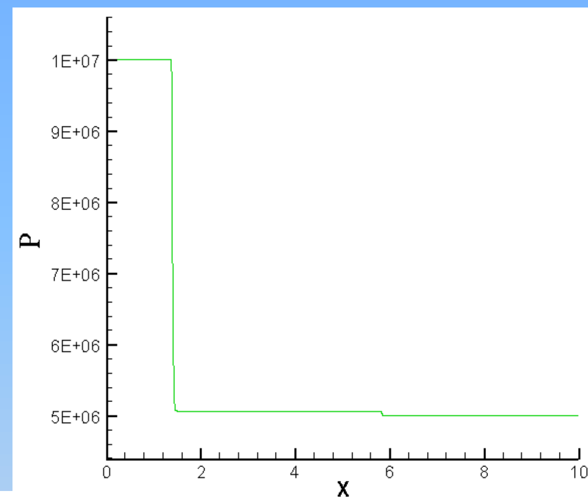


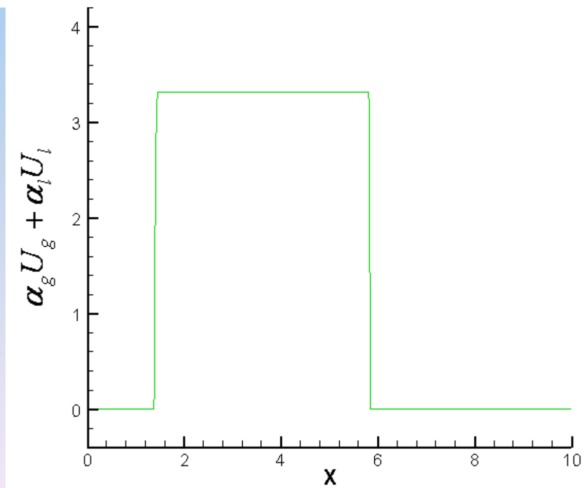
Case 3

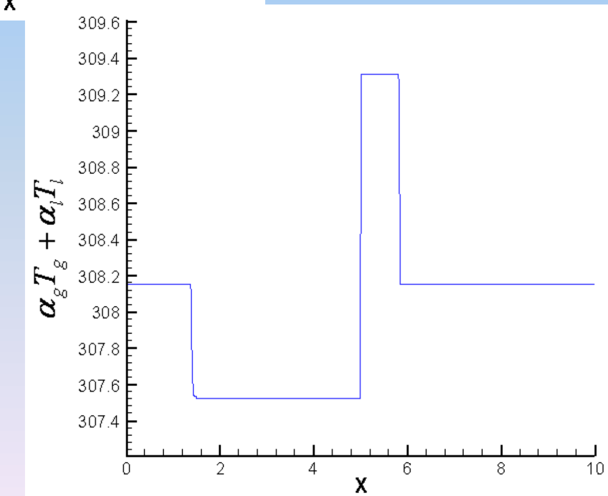
$$(p,\alpha_g,u_i,T_i)_L = (1.0\times10^7\,pa,\quad \varepsilon,\quad 0\,m/s, 308.15\,K^0)$$

$$(p,\alpha_g,u_i,T_i)_R = (5.0\times10^6\,pa,\ 1-\varepsilon,\ 0\,m/s, 308.15\,K^0)$$









Left:

$$P = 4.18375\times10^{6}\ Pa$$

$$u_i = 1.818957\times10^{3}\ m/s$$

$$v_i = 0\ m/s$$

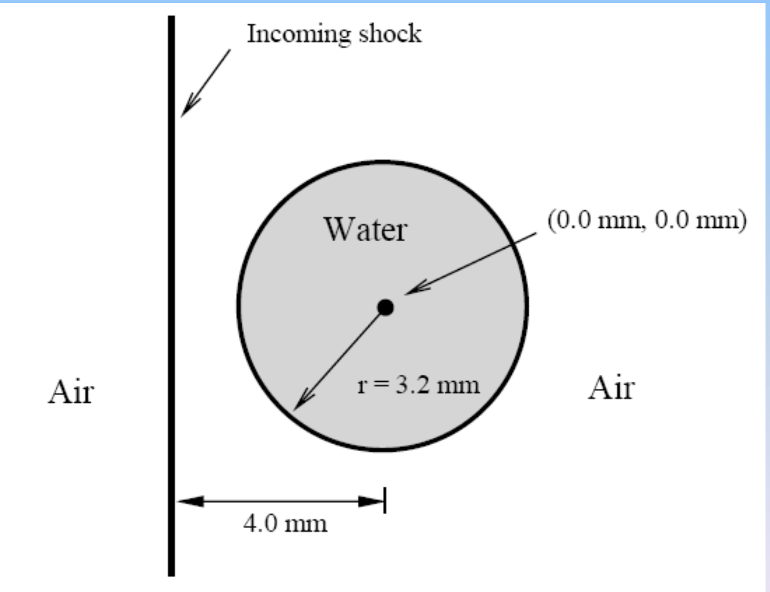
$$T_i = 2755.48\ K$$

Right:

$$P = 1.0\times10^{5}\ Pa$$

$$u_i = v_i = 0\ m/s$$

$$T_i = 346.98\ K$$



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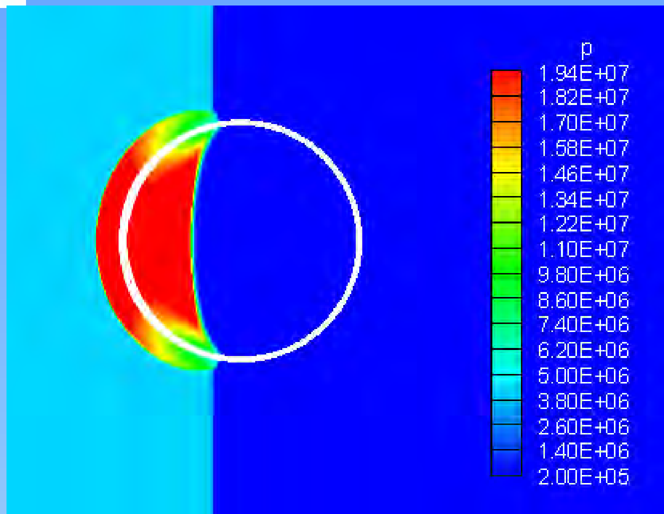
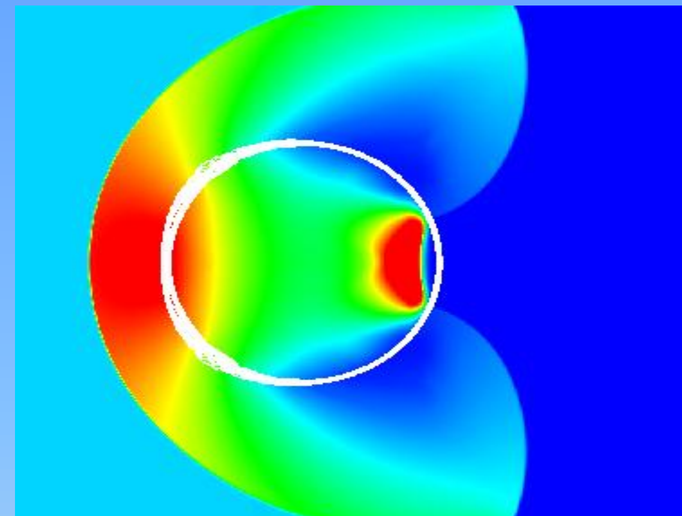
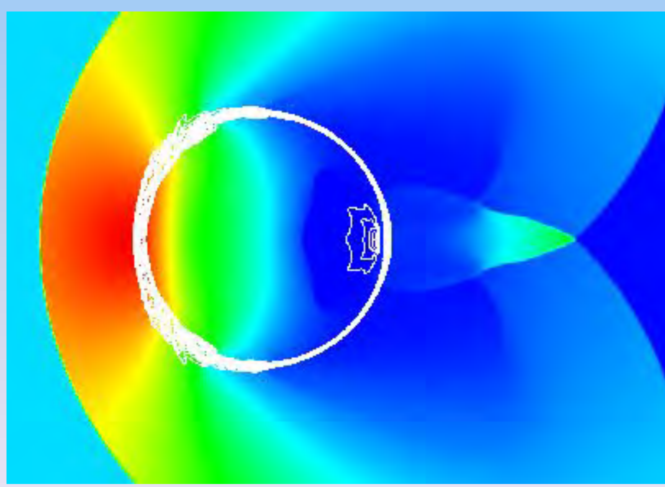


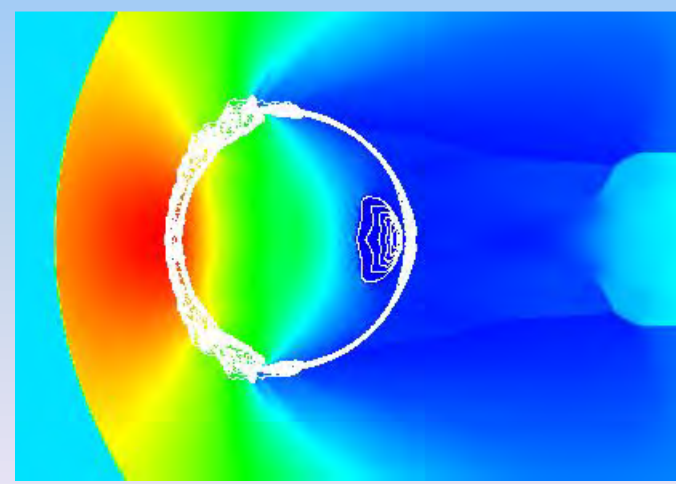
(1) t = 1.5 μs



(2) t = 4.0 μs



(3) t = 6.5 μs



(4) t =9.0 μs

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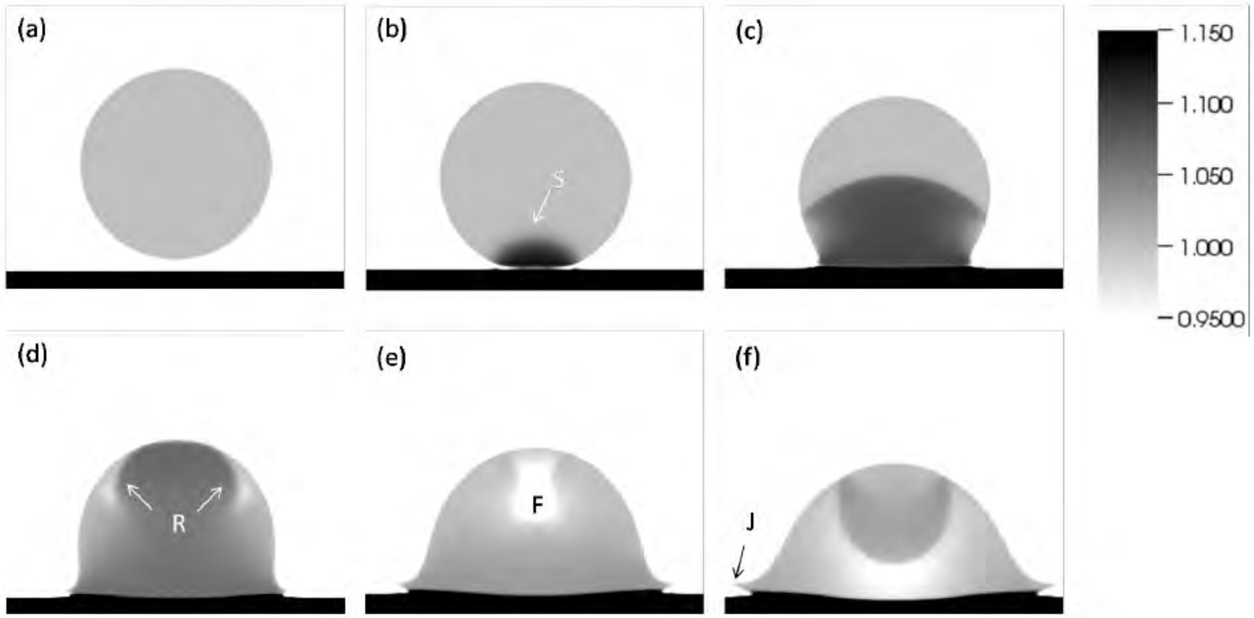

Figure 5: The density contours for droplet impact with a solid wall. The impact Mach number is 0.2, the target compliance is 0.04. The labels indicate S: Shock Wave, R: Reflected, F: Focus and J: Jetting.

Numerical Analysis of High Speed Droplet Impact

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2D high-speed water droplet impact dry wall

Initial Conditions：

Outside Droplet:

$$p = 1. \times 10^5\ Pa$$

$$u_i = v_i = 0\ \text{m/s}$$

$$T_i = 300\ \text{K}$$

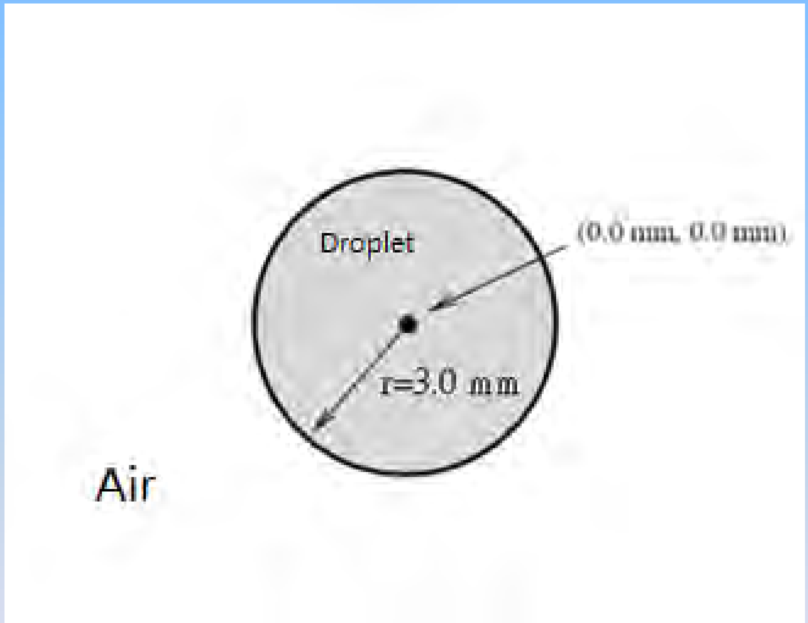
Inside Droplet:

$$p = 1.1 \times 10^5\ Pa$$

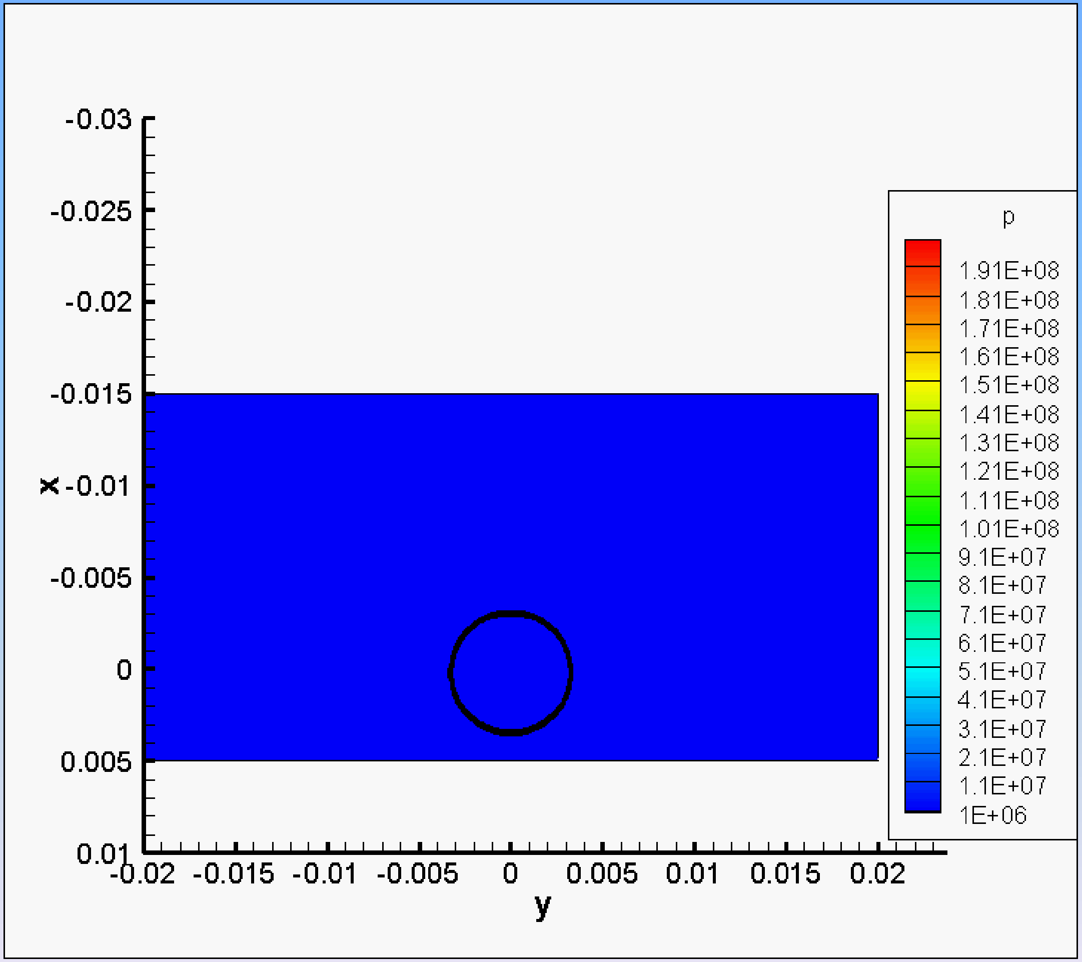
$$u_i = 200\text{-}500\ \text{m/s}$$

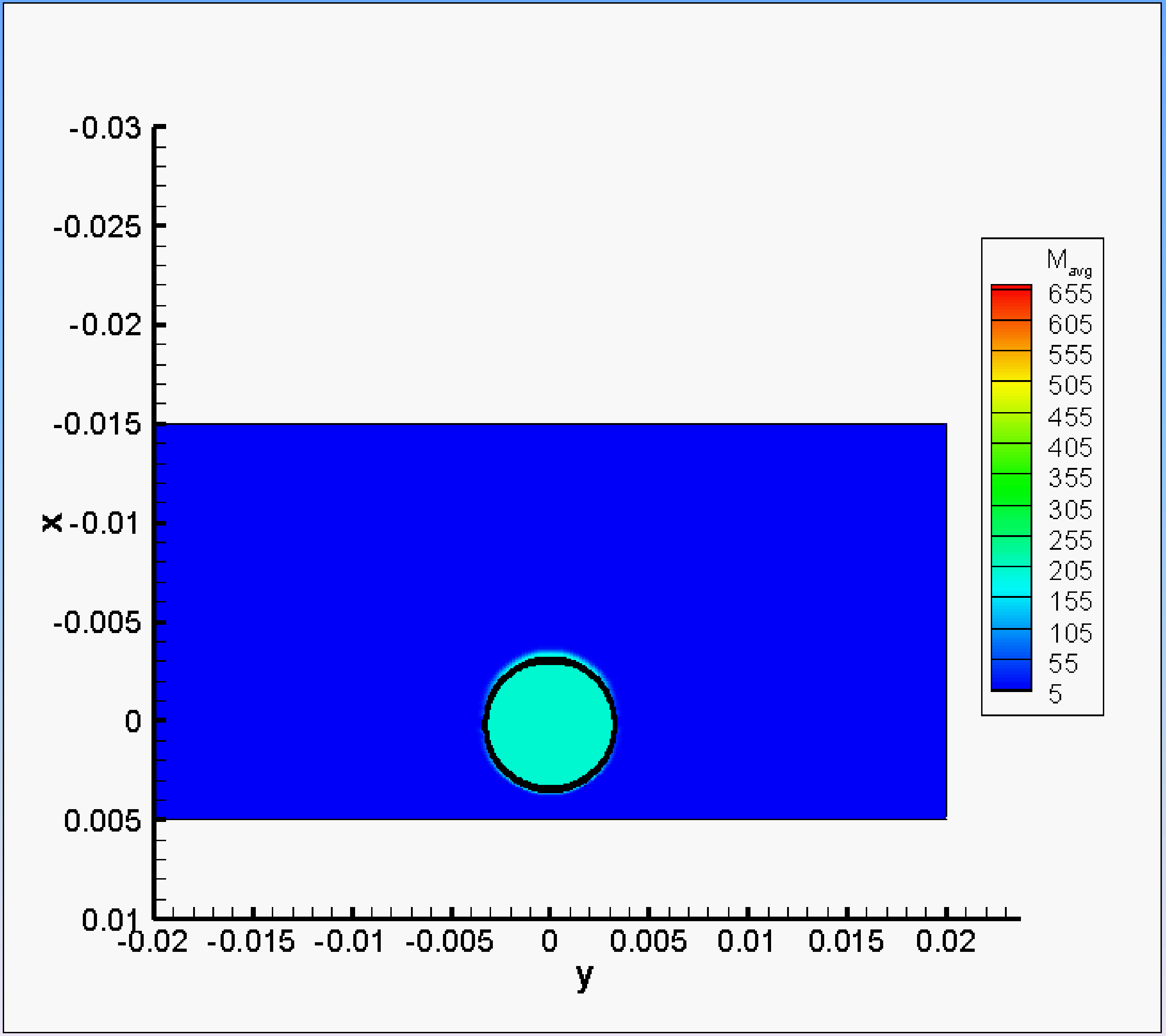
$$v_i = 0\ \text{m/s}$$

$$T_i = 300\ \text{K}$$

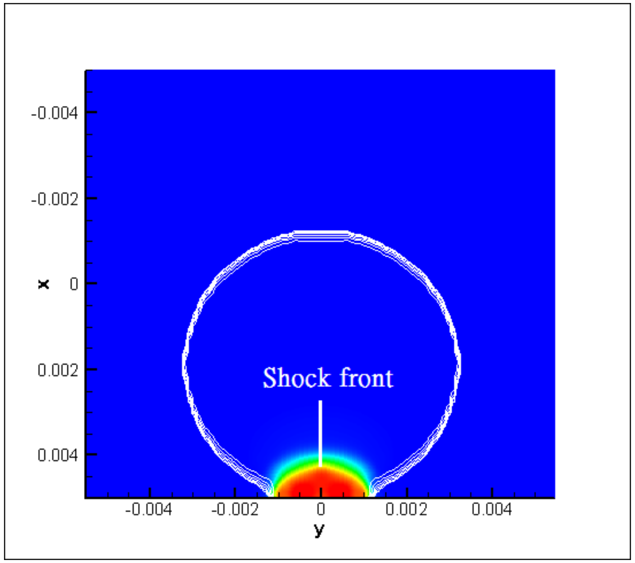


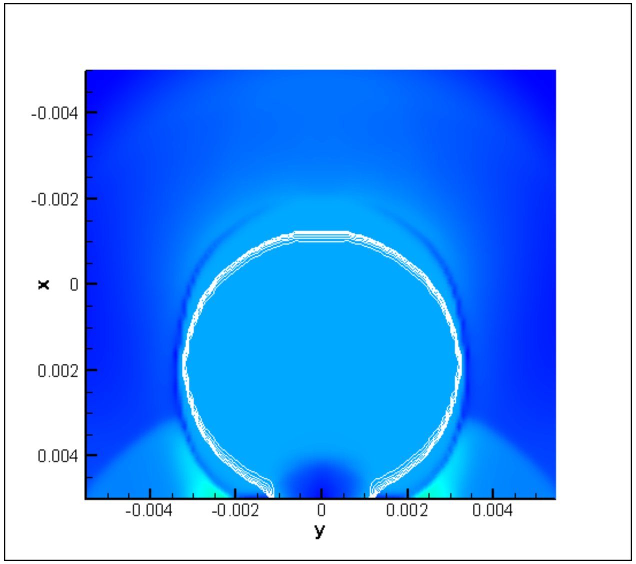
2D high-speed water droplet impact dry wall




-0.03
-0.025
-0.02
-0.015
x -0.01
-0.005
0
0.005
0.01
-0.02 -0.015 -0.01 -0.005 0 0.005 0.01 0.015 0.02
y
M_{avg}
655
605
555
505
455
405
355
305
255
205
155
105
55
5

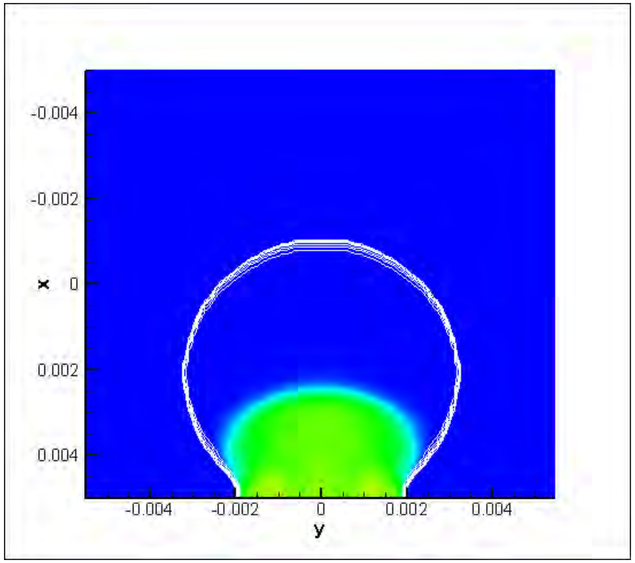
$t = 9.48\mu s$

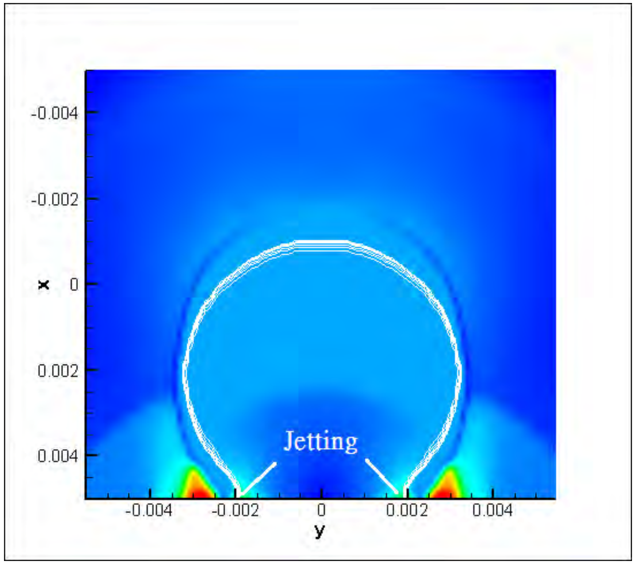




Pressure Velocity VOF

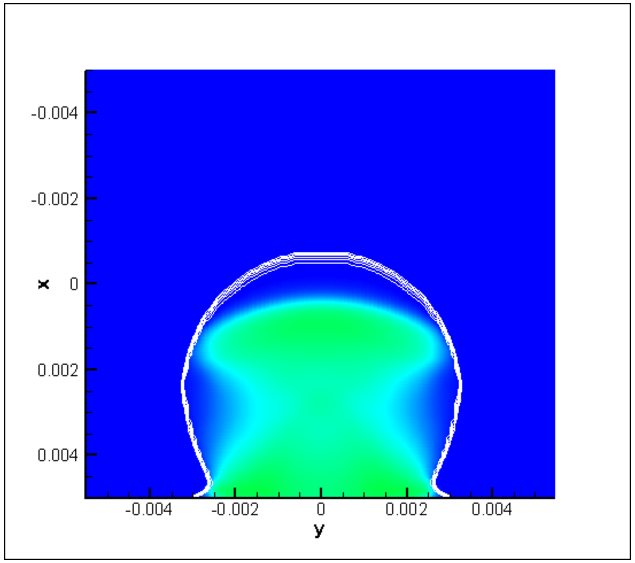
$t = 10.6\mu s$

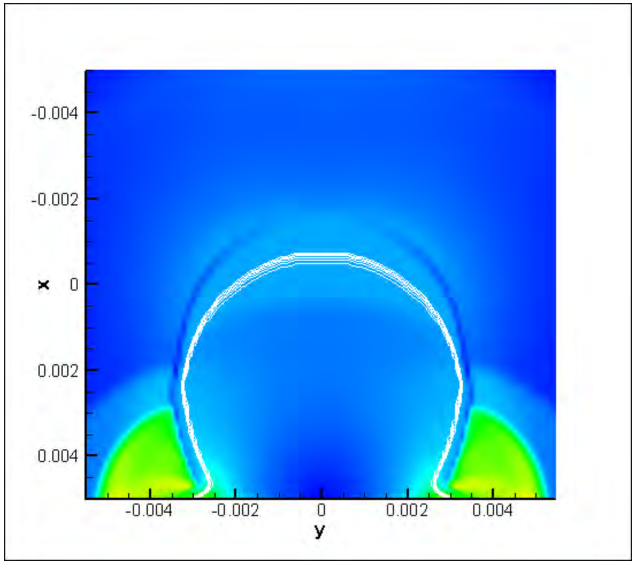




Pressure Velocity VOF

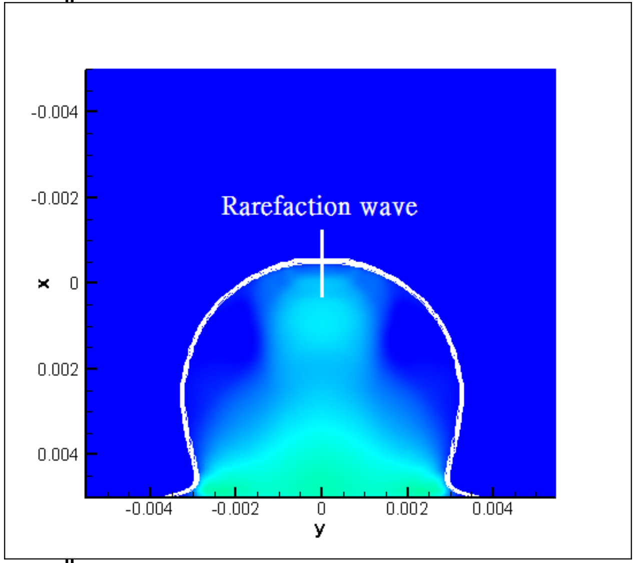
$t = 12.0\mu s$



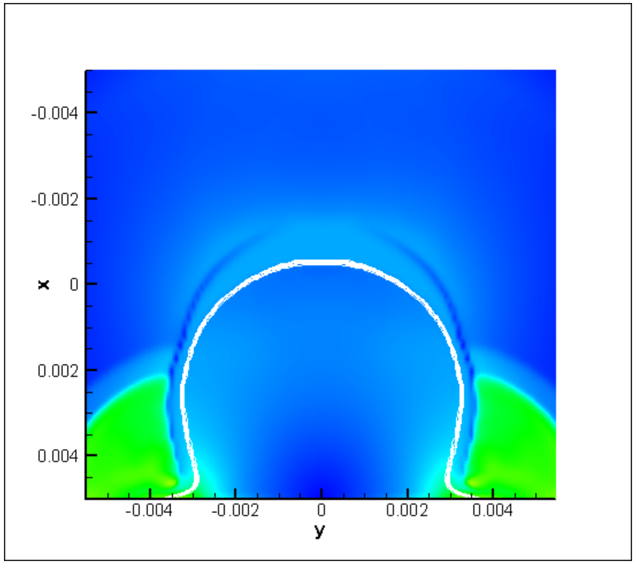


Pressure　　　　Velocity　　　　VOF

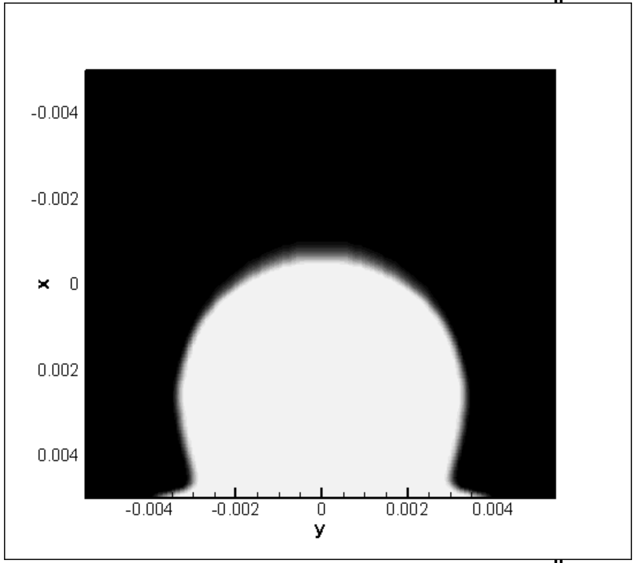
$t = 12.6\mu s$



Pressure

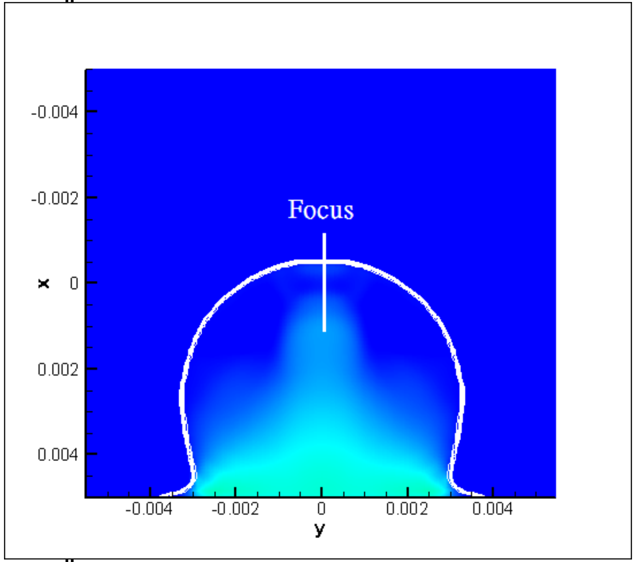


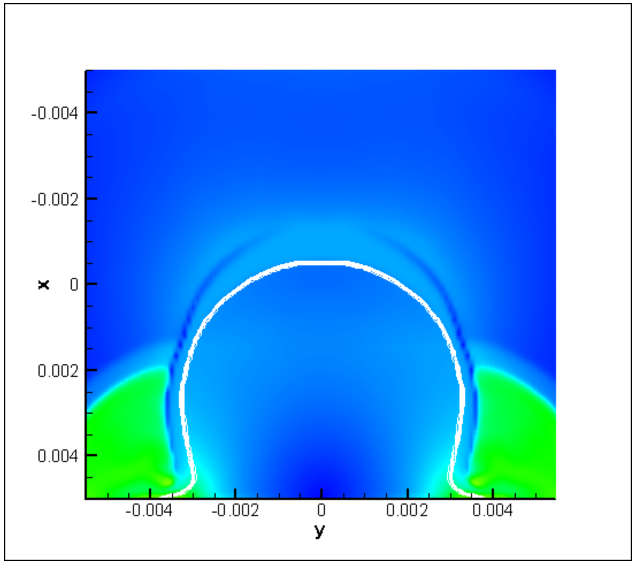
Velocity

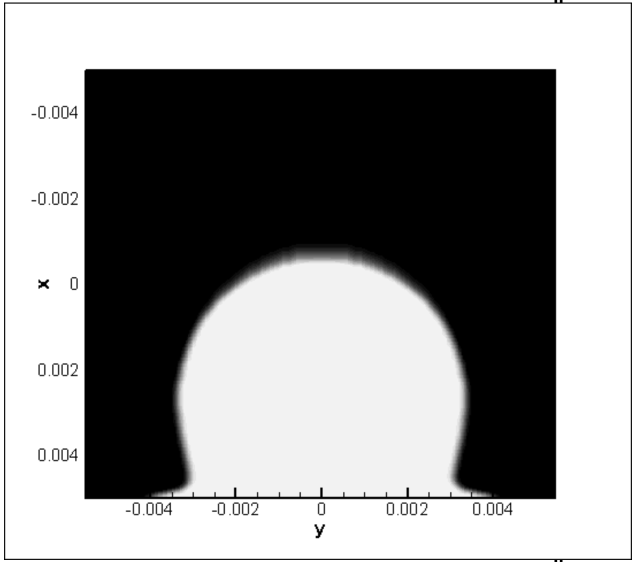


VOF

$t = 13.1\mu s$

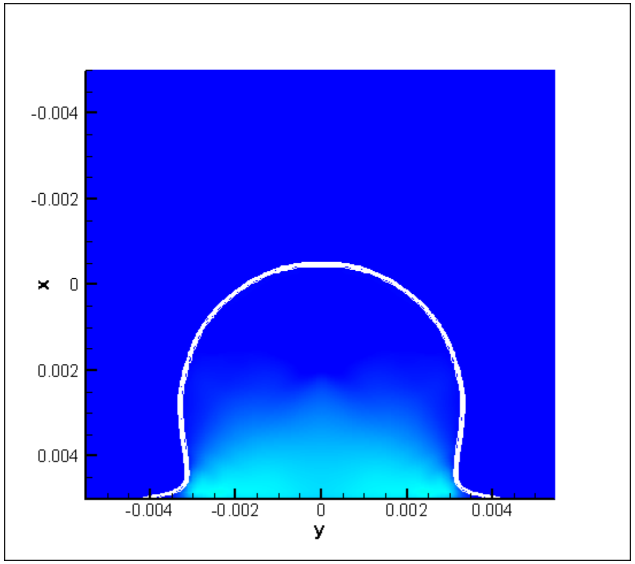




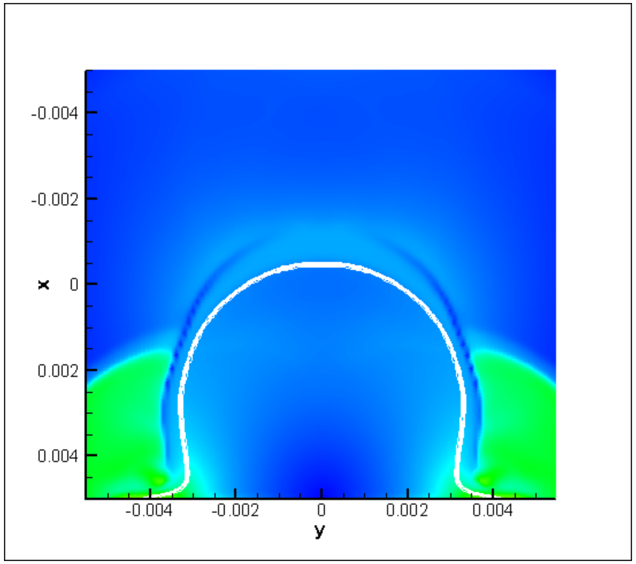


Pressure | Velocity | VOF

$t = 13.6\mu s$

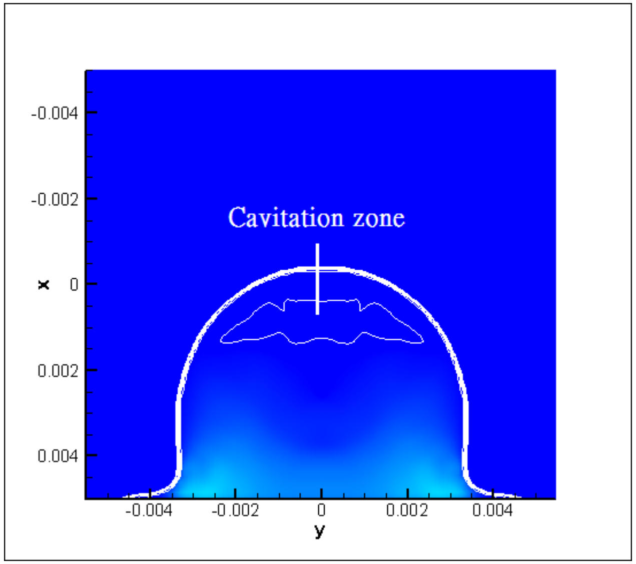


Pressure

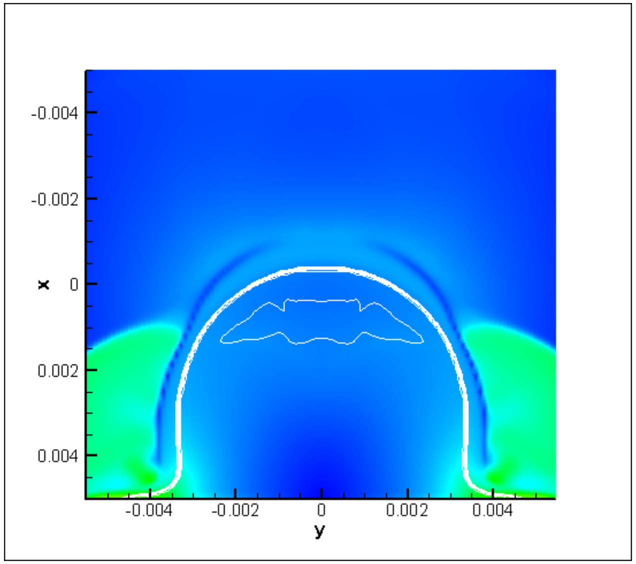


Velocity

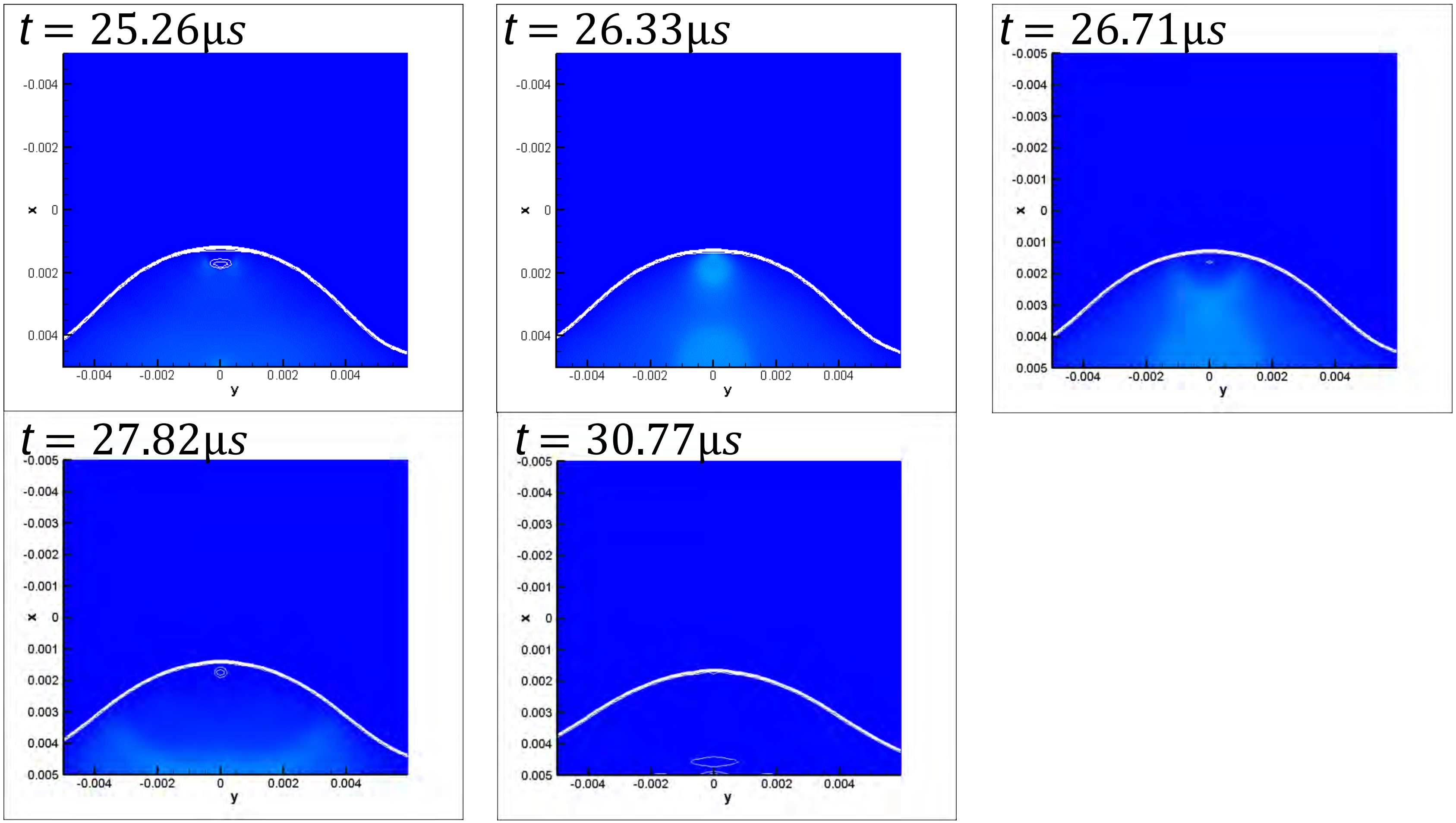
$t = 14.4\mu s$



Pressure



Velocity


t = 25.26μs
t = 26.33μs
t = 26.71μs
t = 27.82μs
t = 30.77μs
x
y

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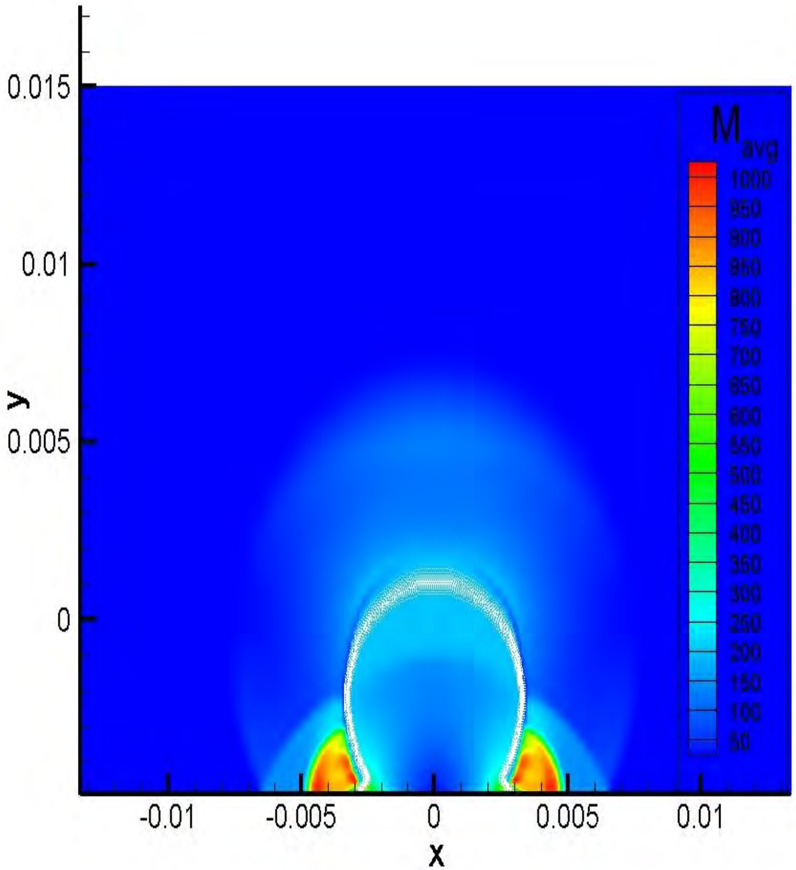
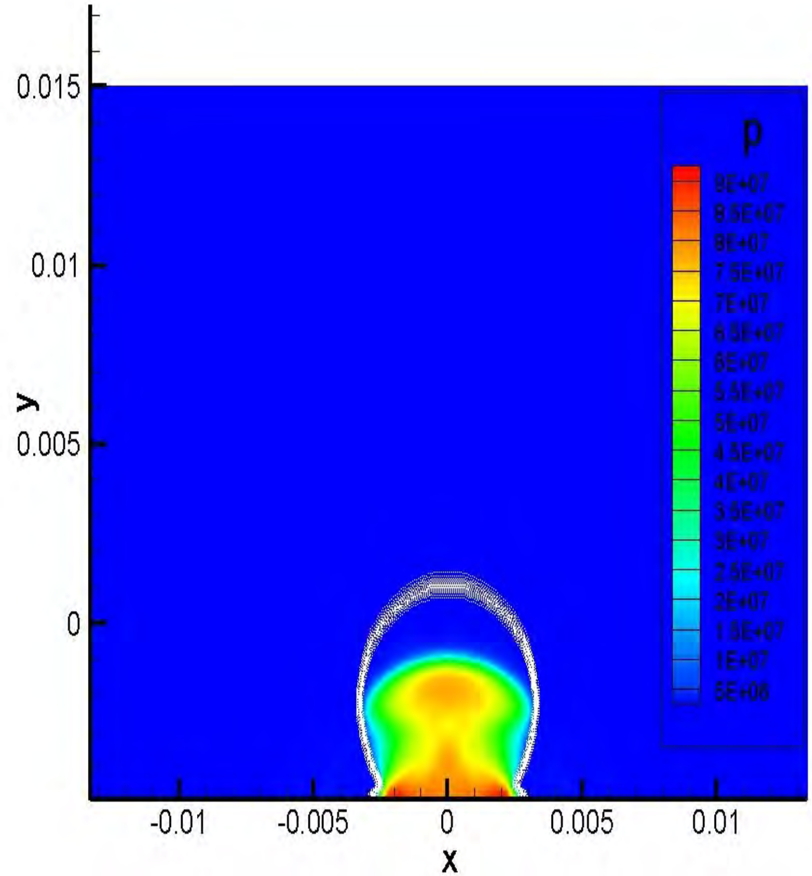
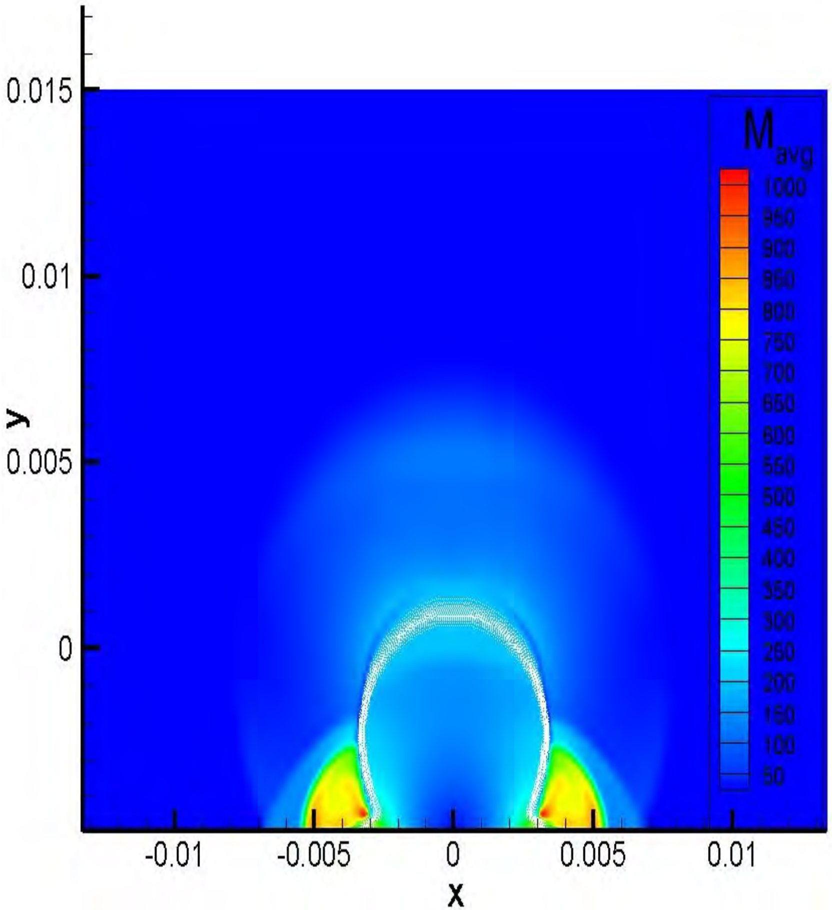
Wet Surface



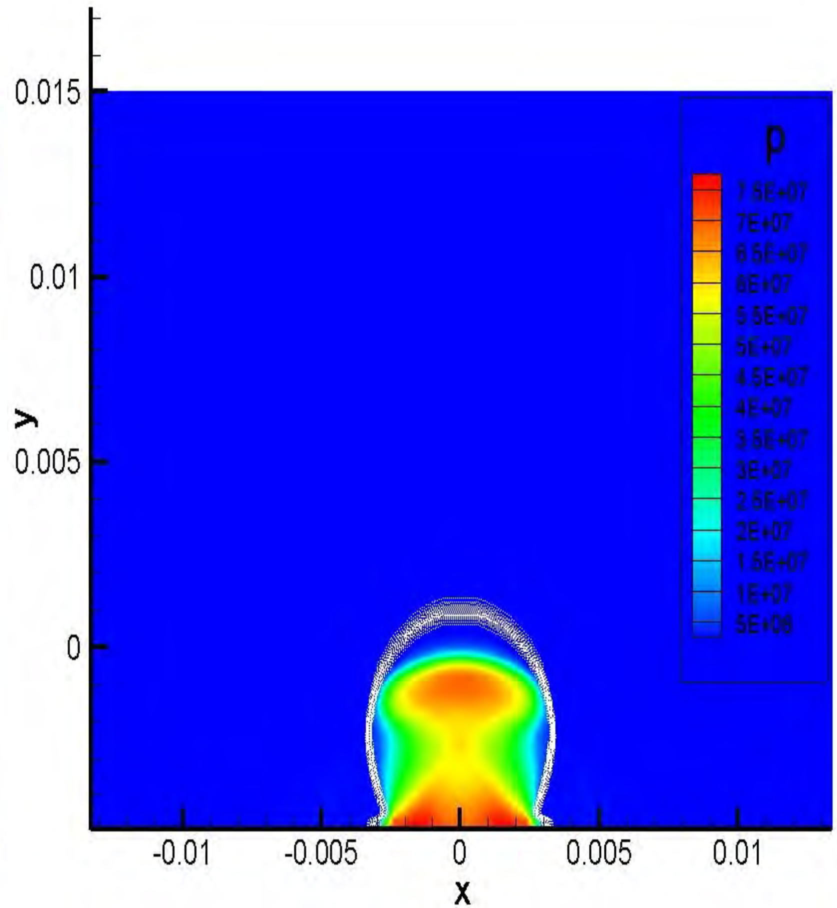
Mavg



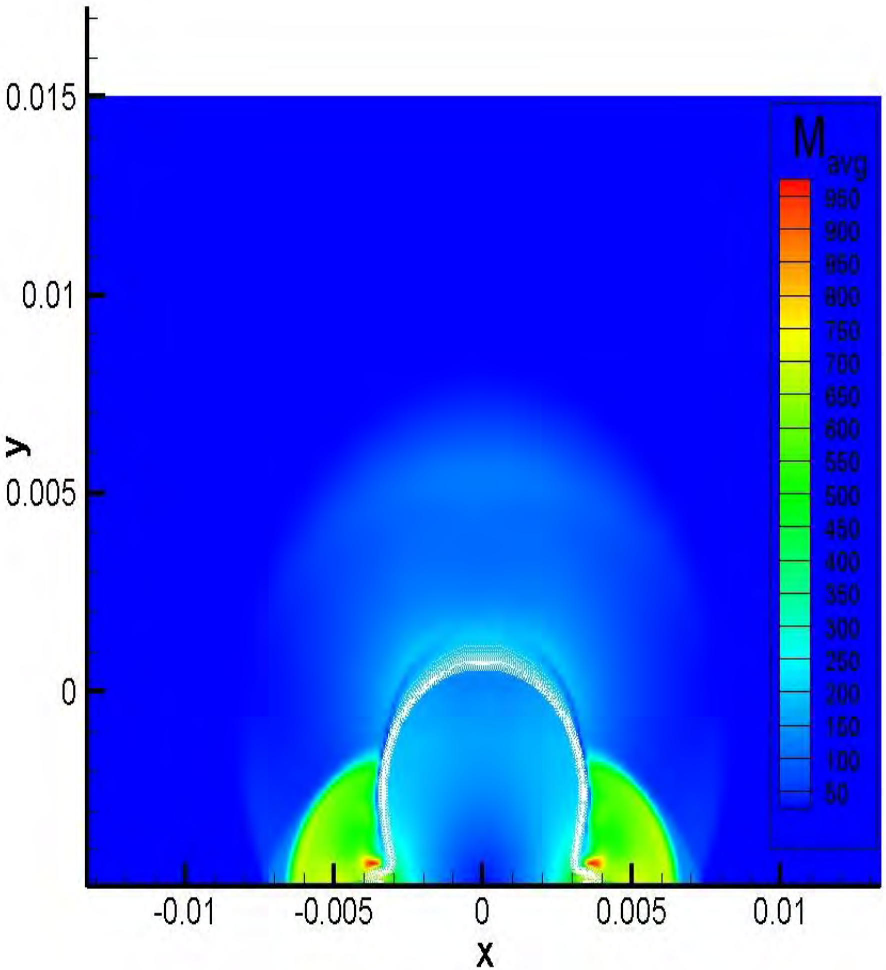
Pressure



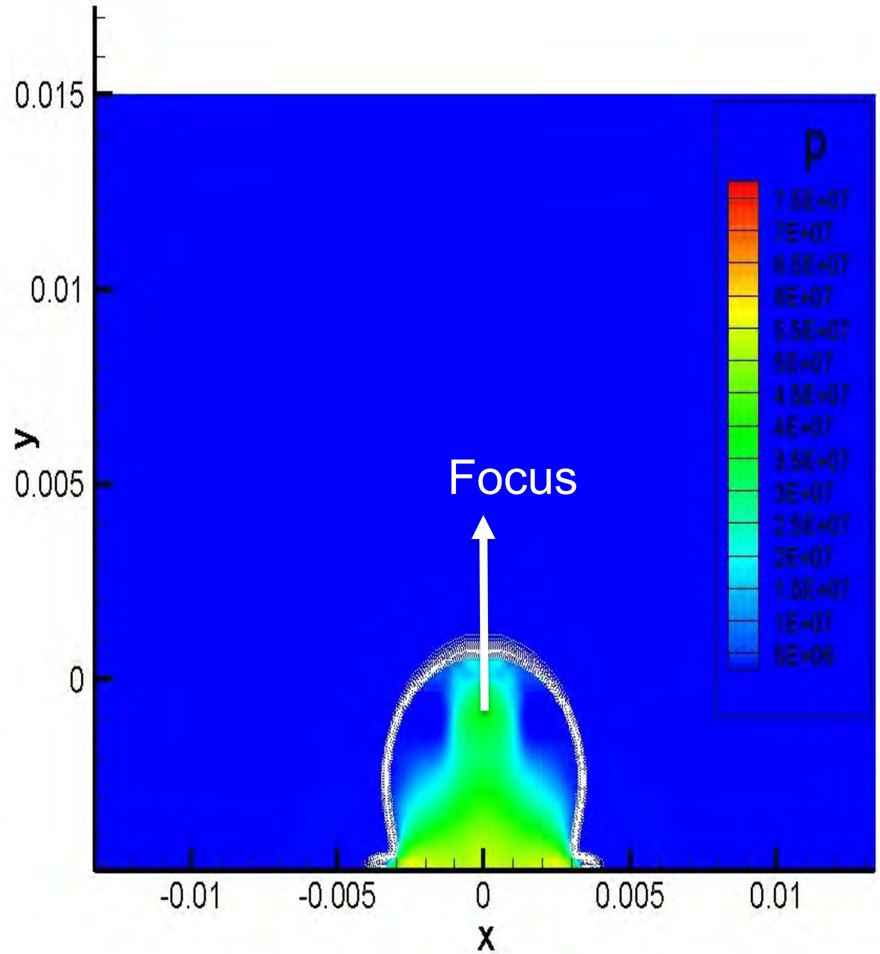
Mavg



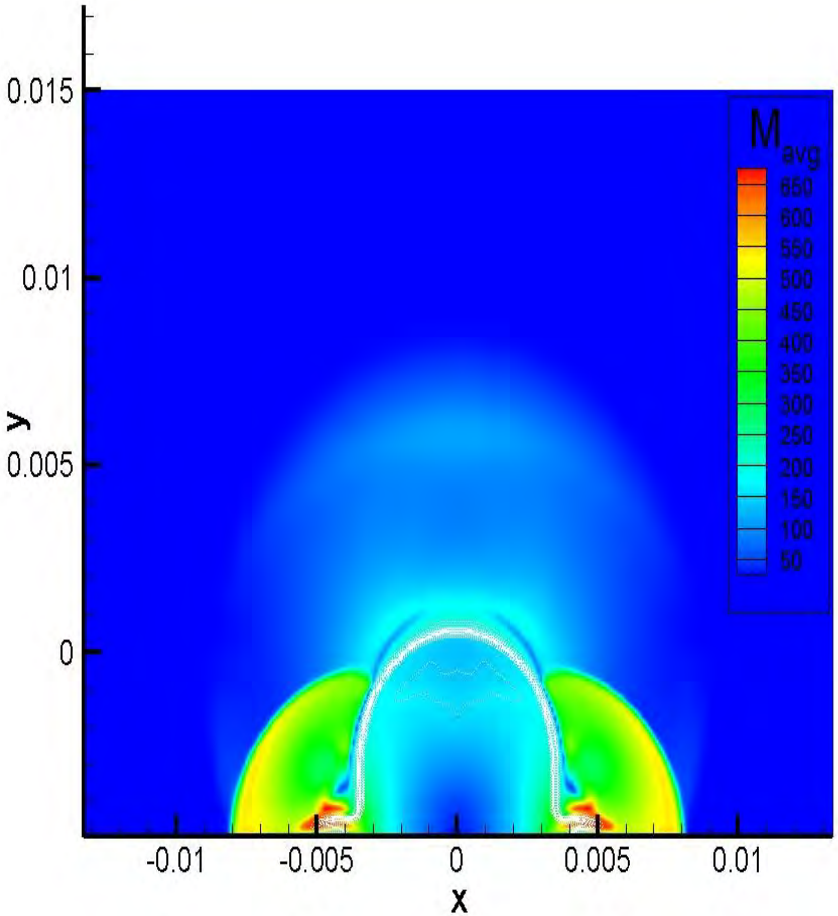
Pres
sure



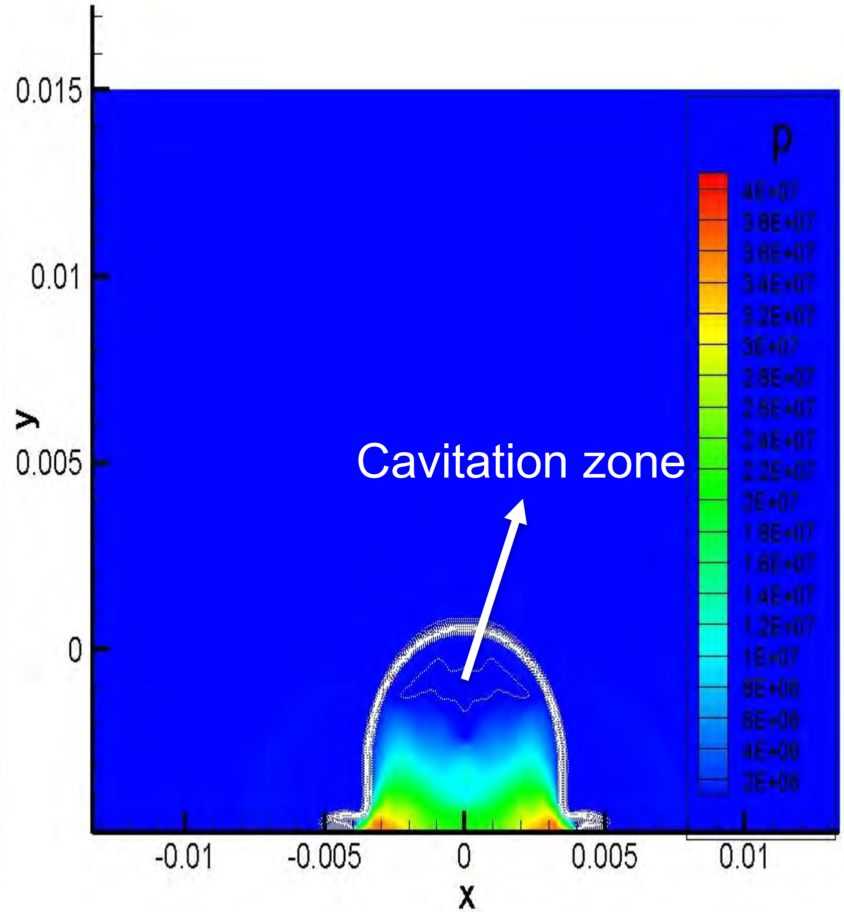
Mavg



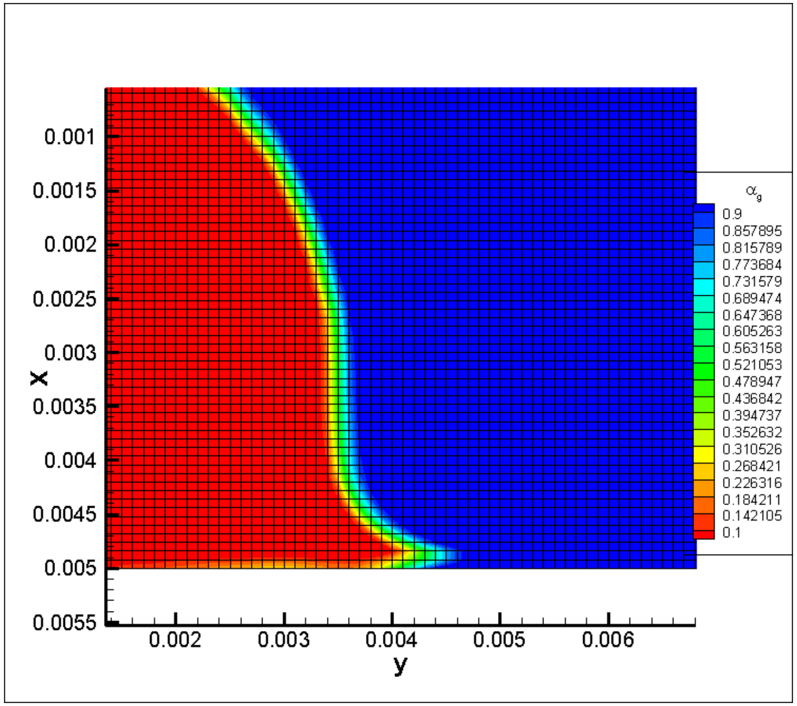
Pressure



Mavg

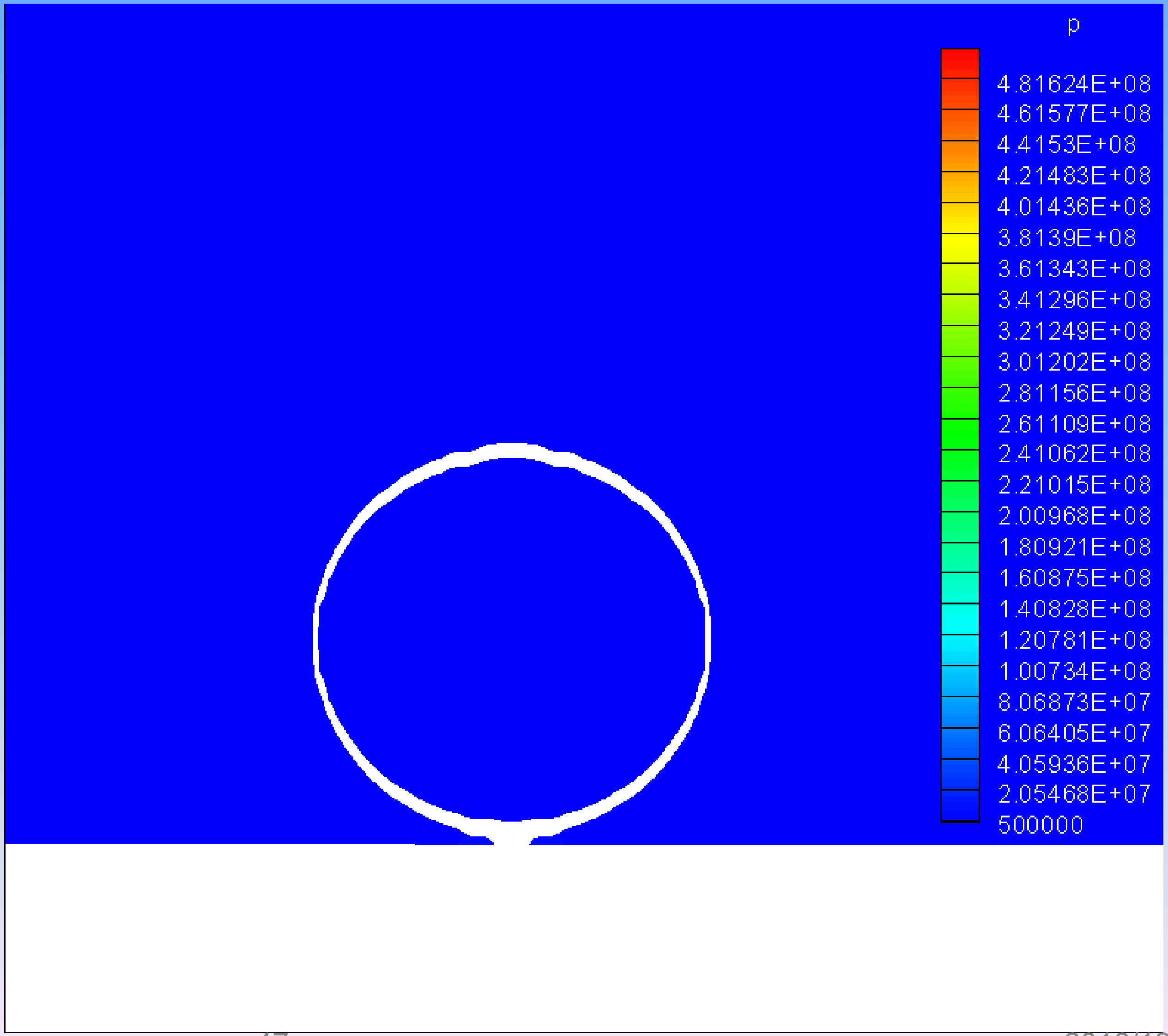


Pressure



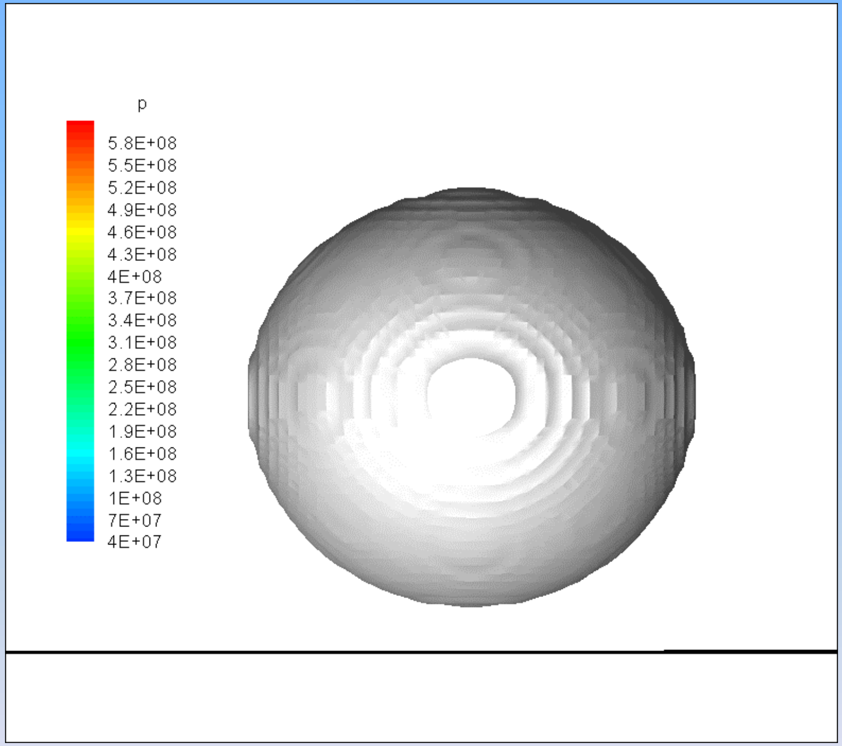
(a) interface capturing on uniform grid

3D high-speed water droplet impact wall, V=500m/s

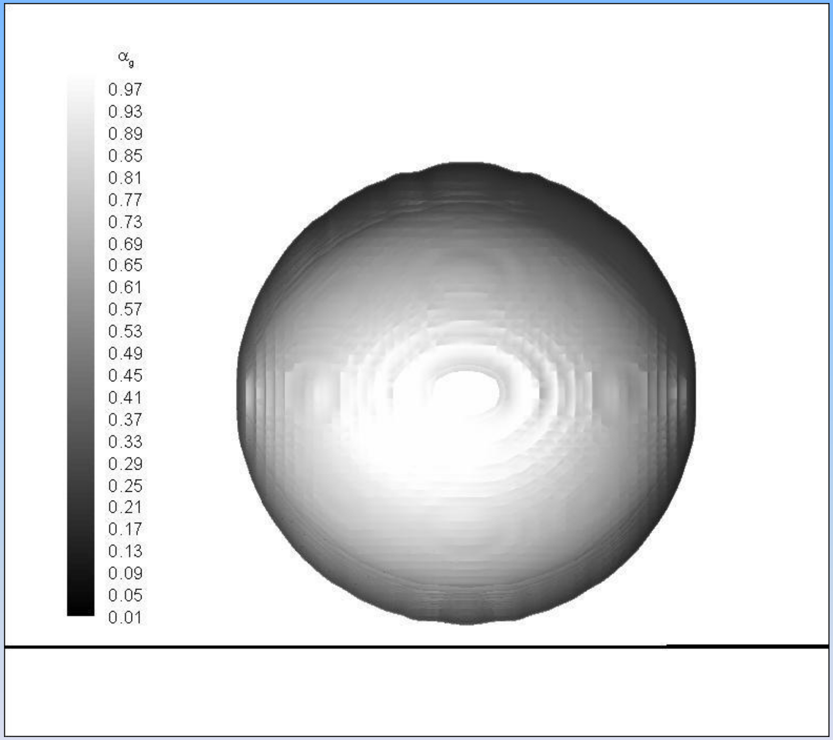


3D high-speed water droplet impact wall (V=500m/s)

$t = 3.0\ \mu s$



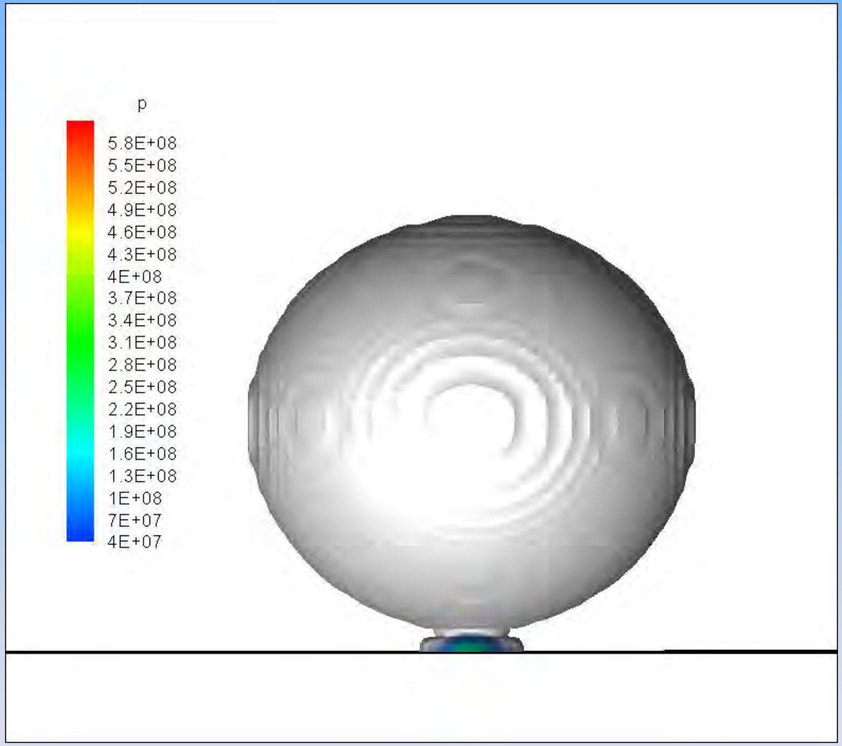
Pressure



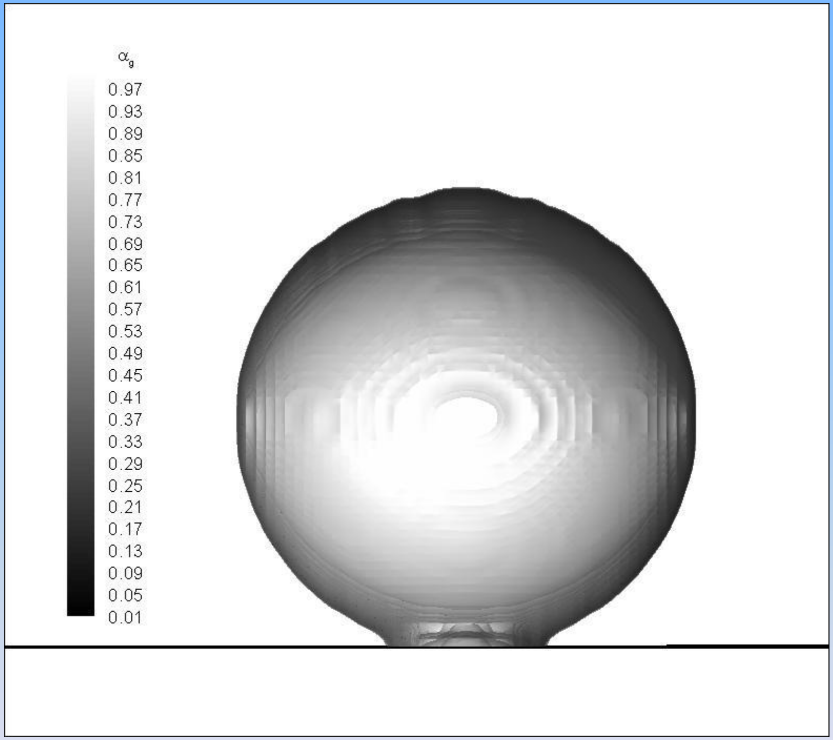
Volume of fraction

Case(VII):3D high-speed water droplet impact wall (V=500m/s, 0˚)

$t = 3.7\ \mu s$



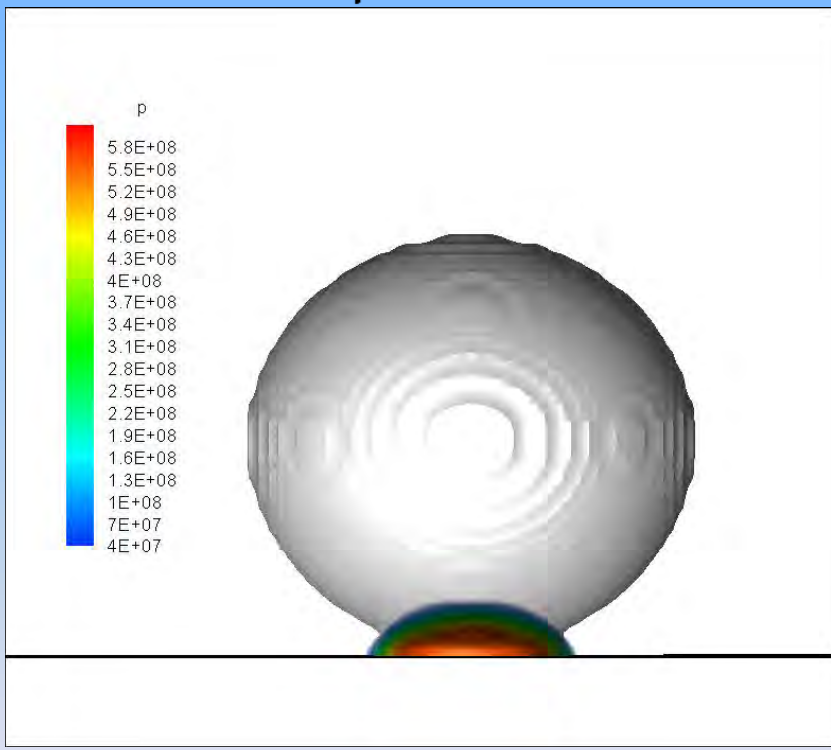
Pressure



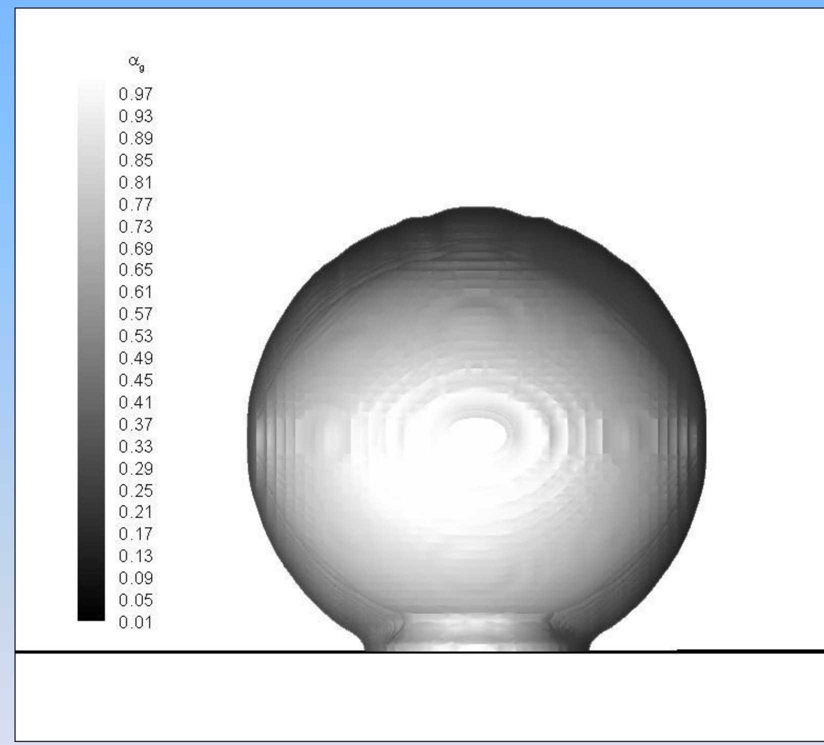
Volume of fraction

Case(VII):3D high-speed water droplet impact wall (V=500m/s, 0˚)

$t = 4.1\ \mu s$



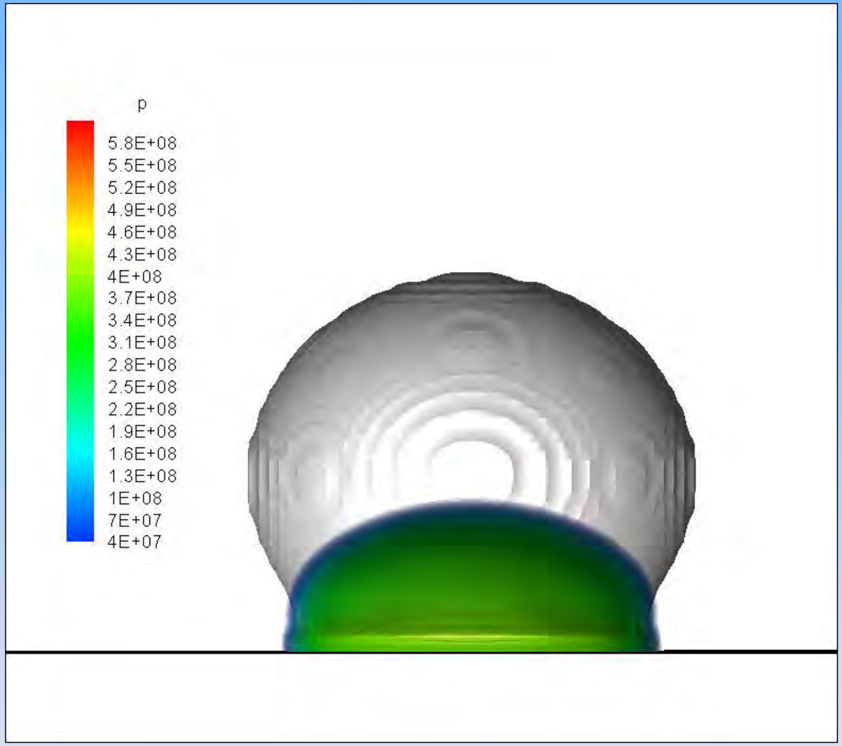
Pressure



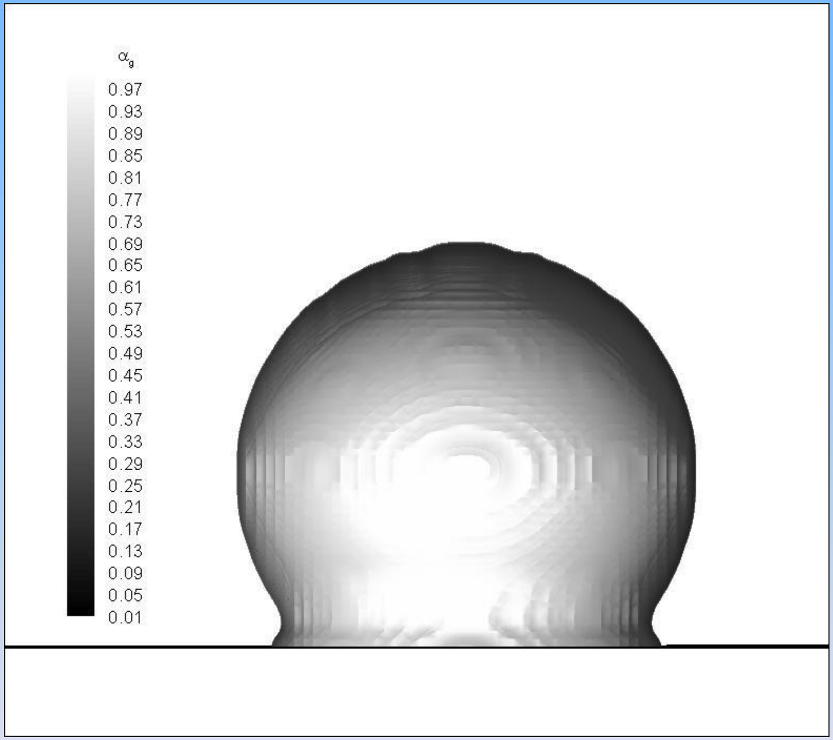
Volume of fraction

Case(VII):3D high-speed water droplet impact wall (V=500m/s, 0˚)

$t = 5.2\ \mu s$



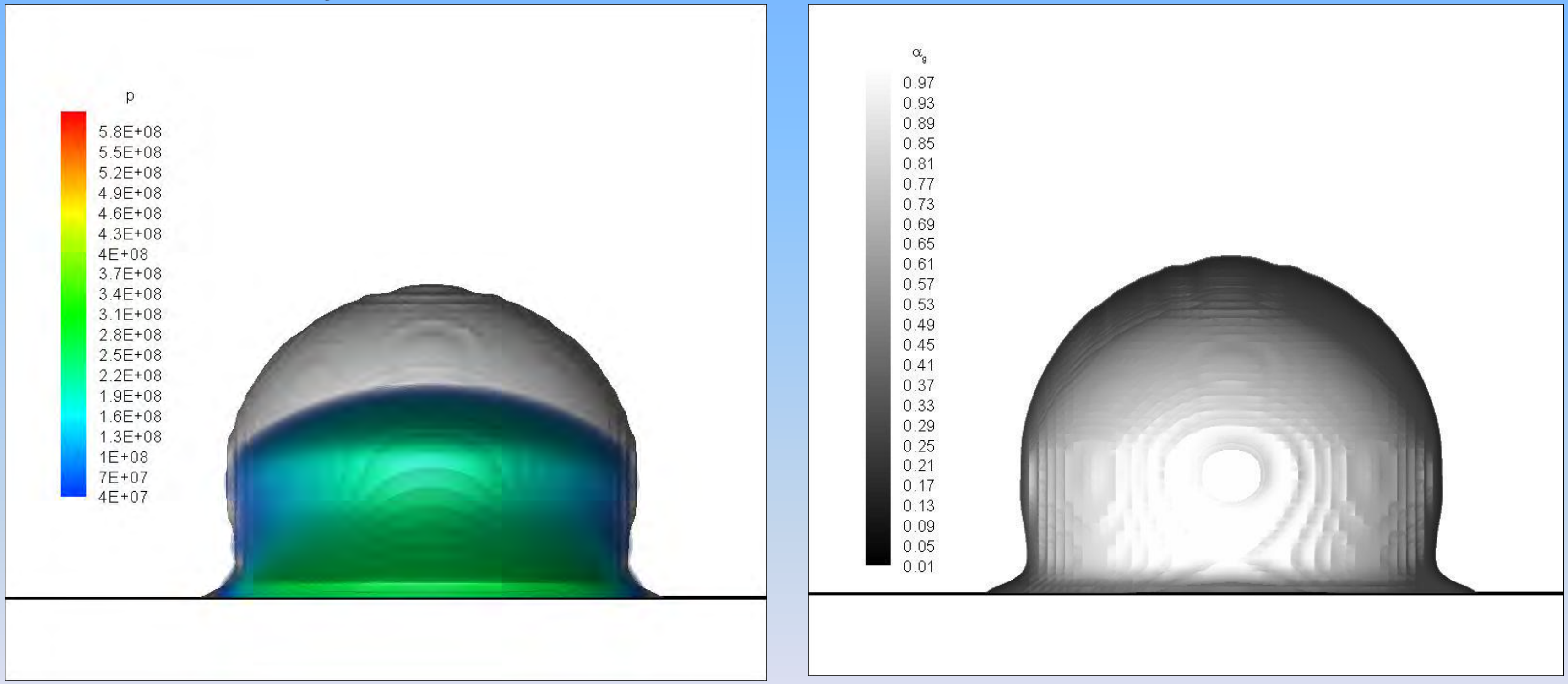
Pressure



Volume of fraction

Case(VII):3D high-speed water droplet impact wall (V=500m/s, 0˚)

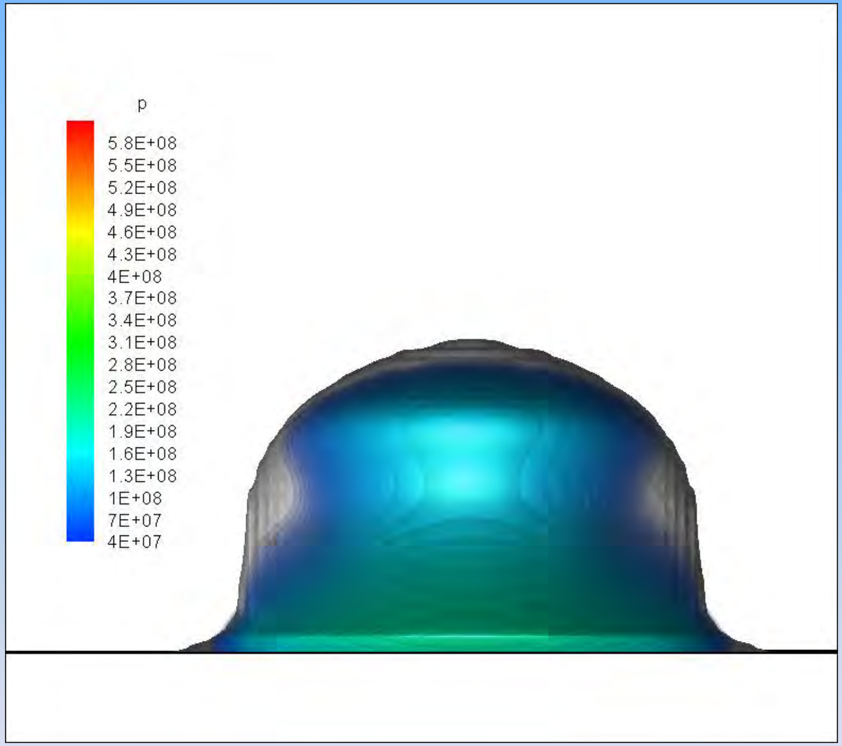
$t = 6.2\ \mu s$



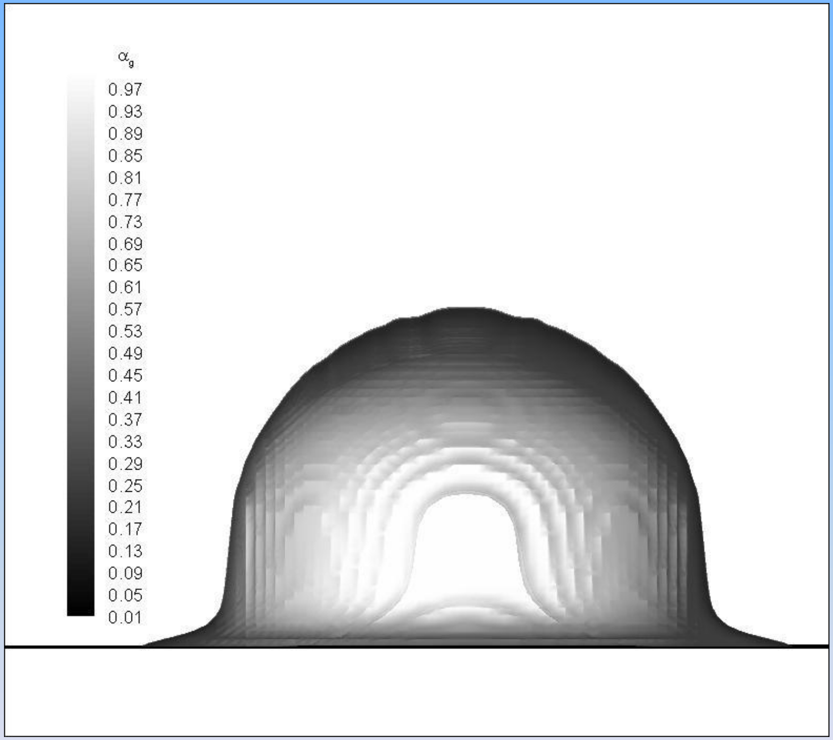
Pressure　　Volume of fraction

Case(VII):3D high-speed water droplet impact wall (V=500m/s, 0˚)

$t = 7.0\ \mu s$



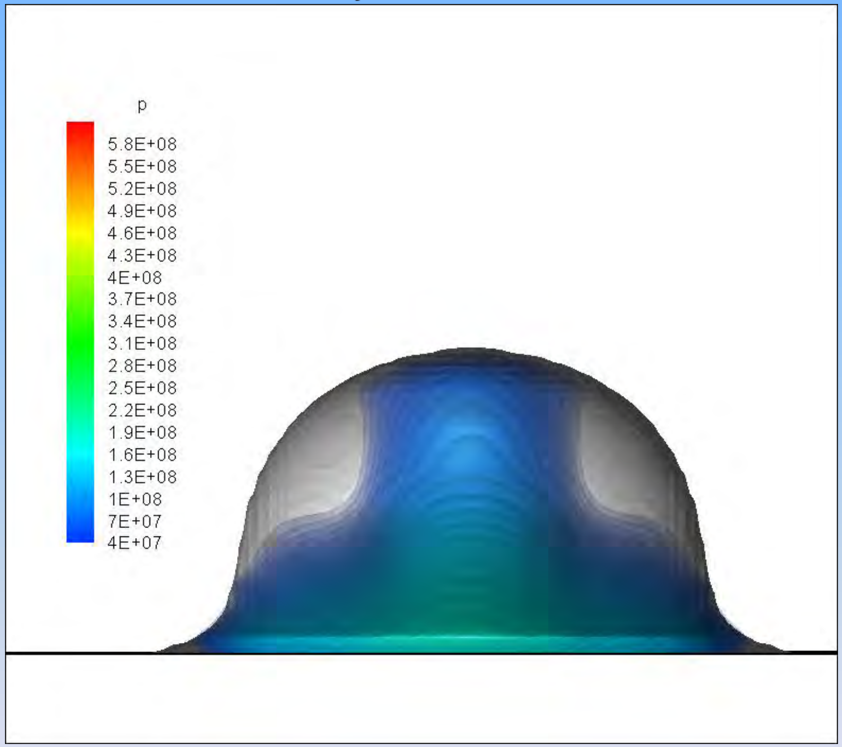
Pressure



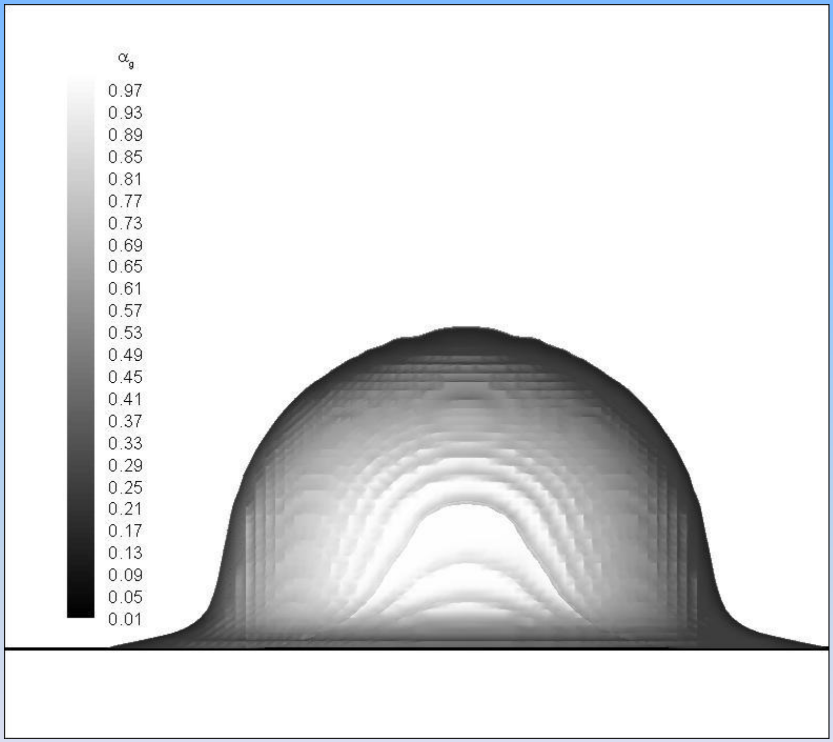
Volume of fraction

Case(VII):3D high-speed water droplet impact wall (V=500m/s, 0˚)

$t = 7.5\ \mu s$



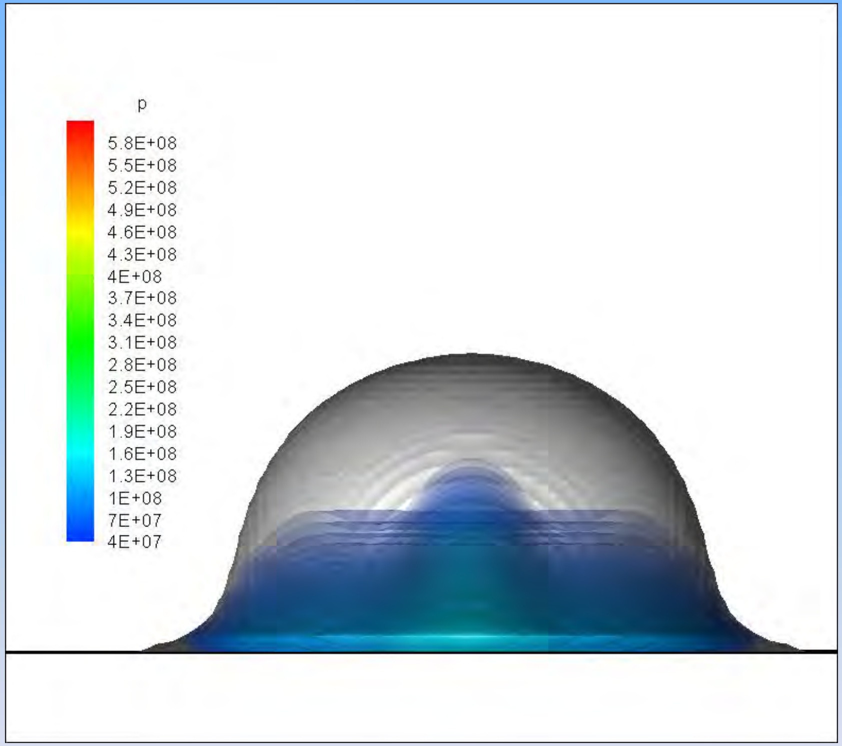
Pressure



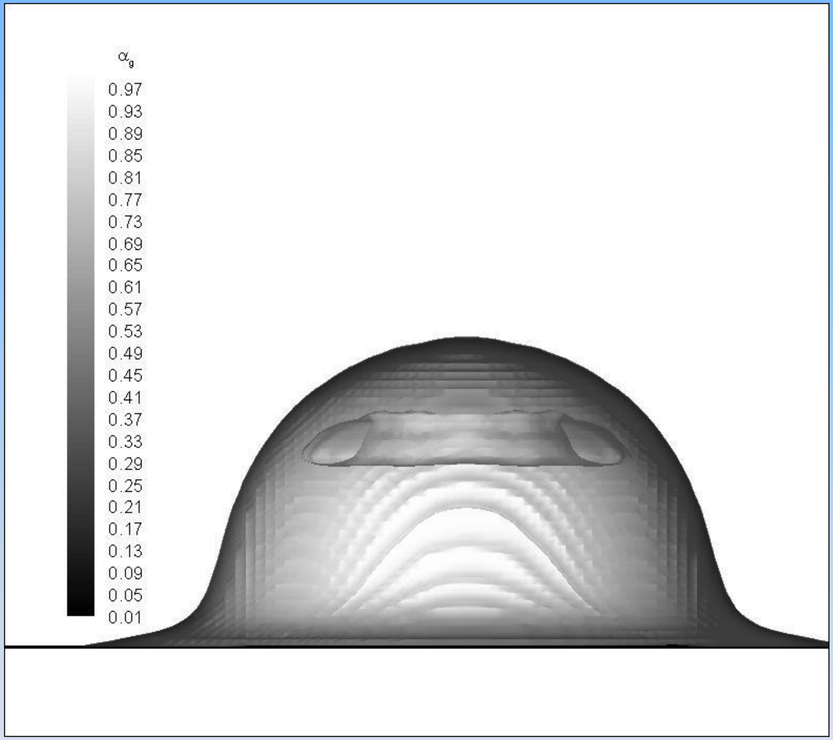
Volume of fraction

Case(VII):3D high-speed water droplet impact wall (V=500m/s, 0˚)

$t = 7.9\ \mu s$



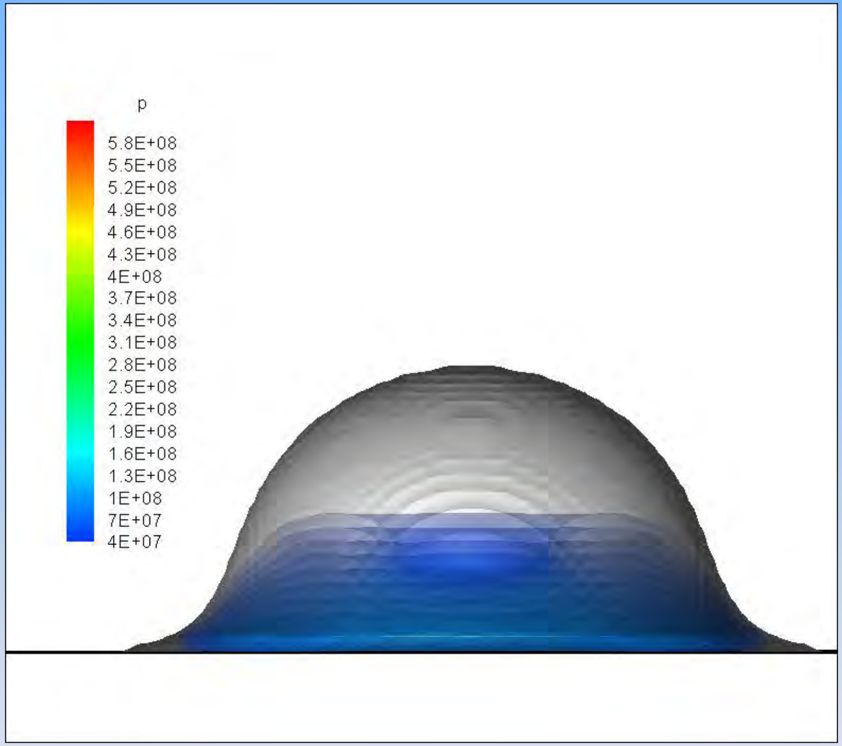
Pressure



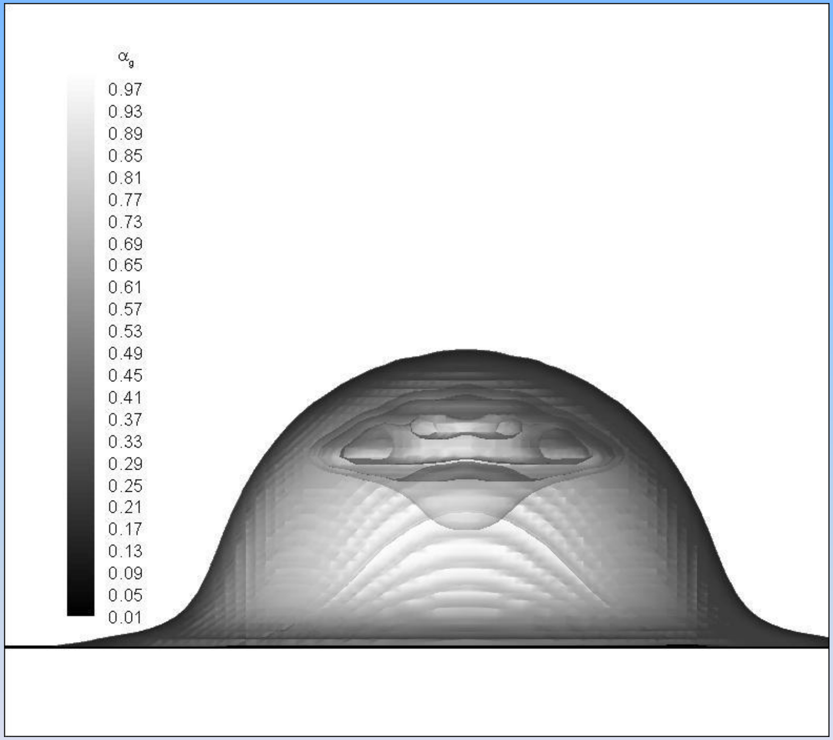
Volume of fraction

Case(VII):3D high-speed water droplet impact wall (V=500m/s, 0˚)

$t = 8.4\ \mu s$

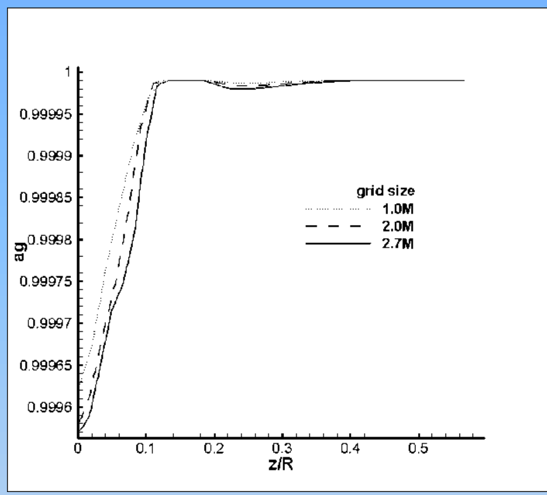


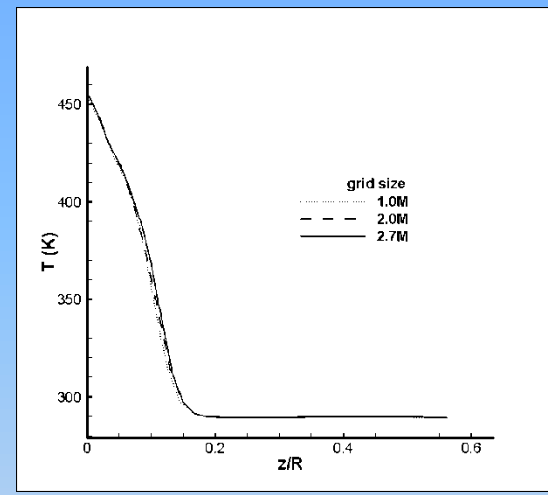
Pressure

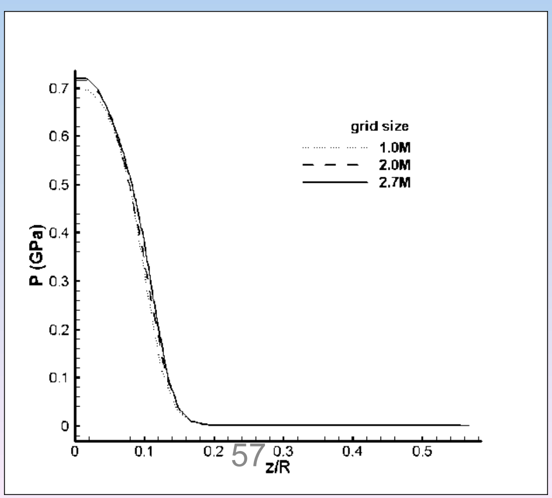


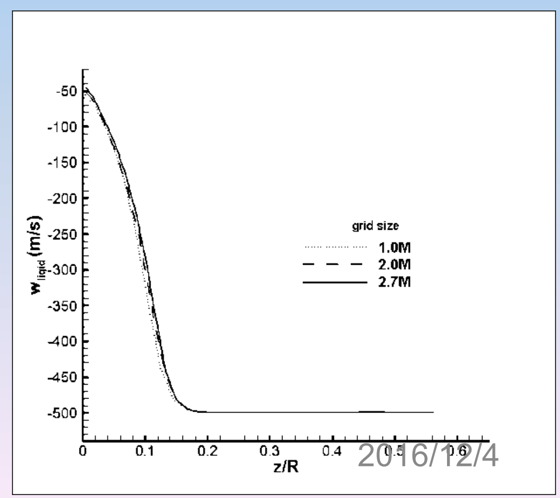
Volume of fraction

Grid Independence of 3D V=500m/s



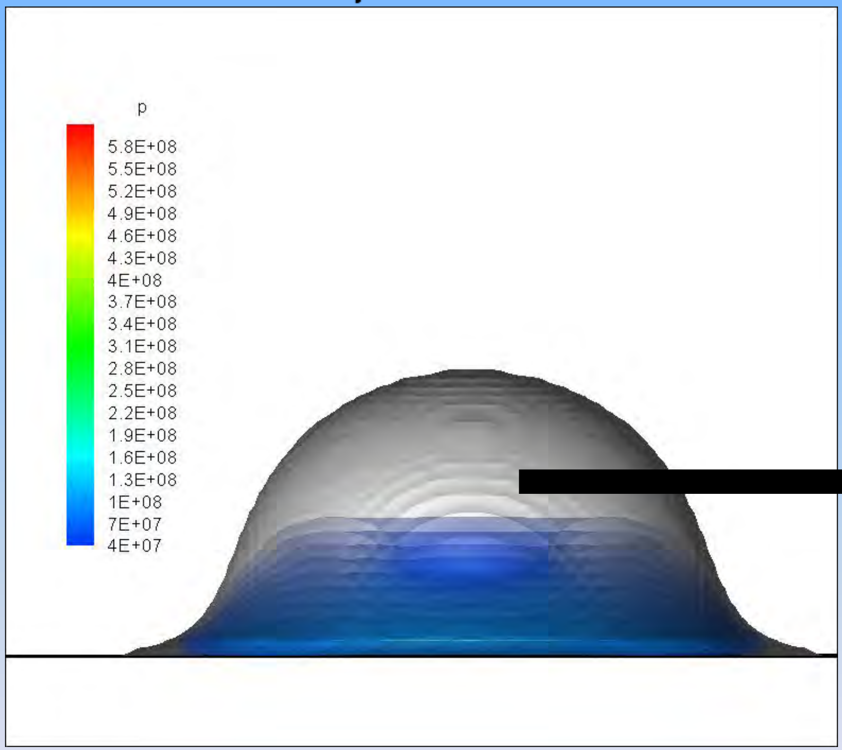




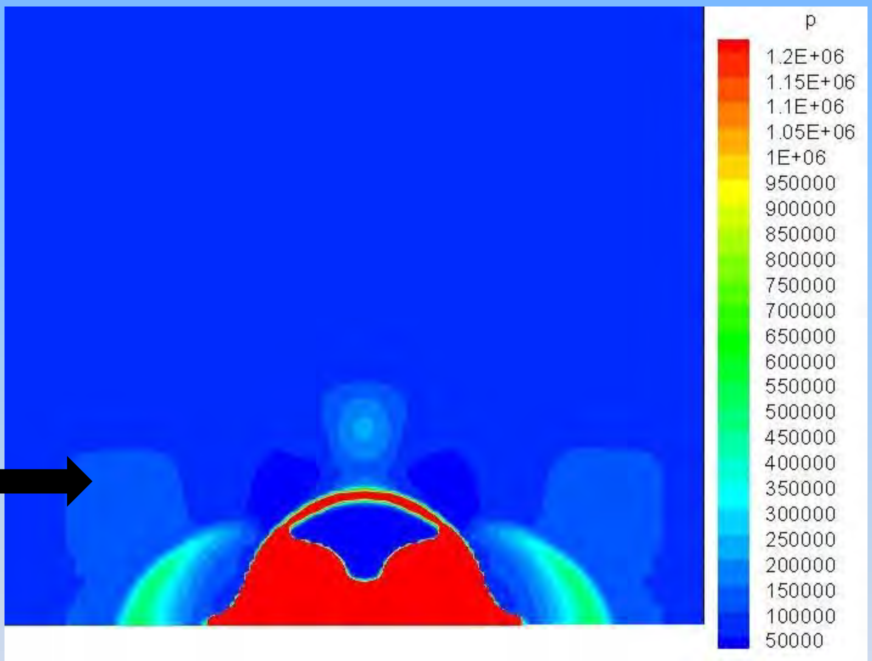


Case(VII):3D high-speed water droplet impact wall (V=500m/s, 0˚)

$t = 8.4\ \mu s$ α=1e-6



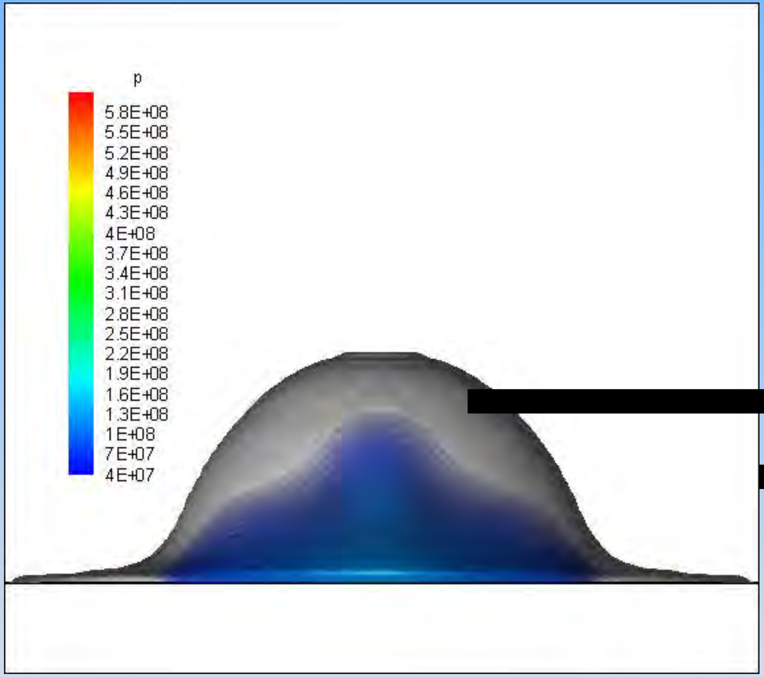
Pressure

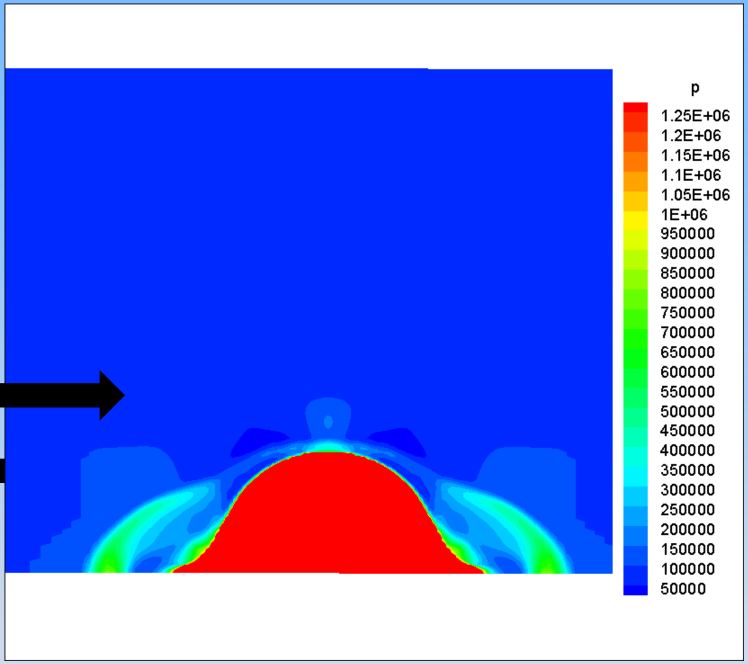


The lower pressure region sketch at the same time

Case(VII):3D high-speed water droplet impact wall (V=500m/s,)

$t = 8.4\ \mu s$ α=0.018

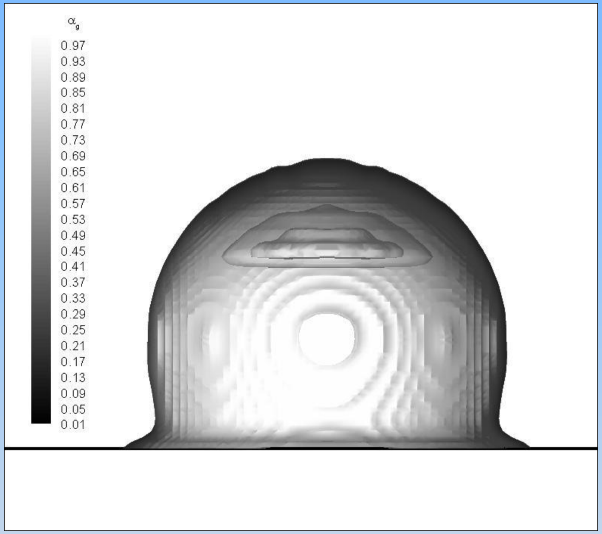




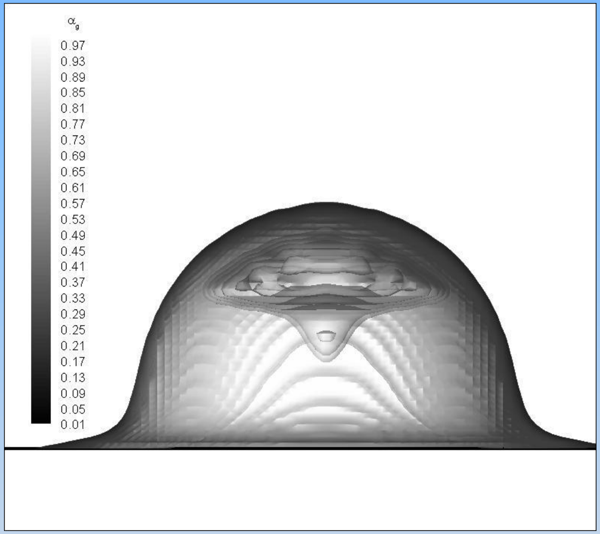
The lower pressure region sketch at the same time

Pressure

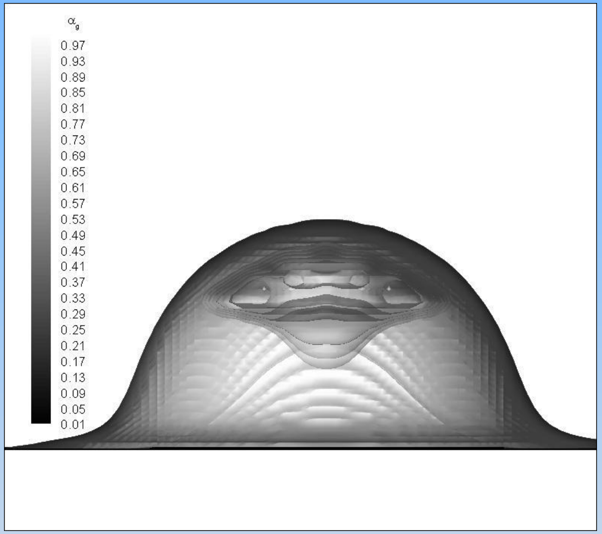
The cavitation area .vs. impact velocity



V = 200 m/s
$t = 15\ \mu s$

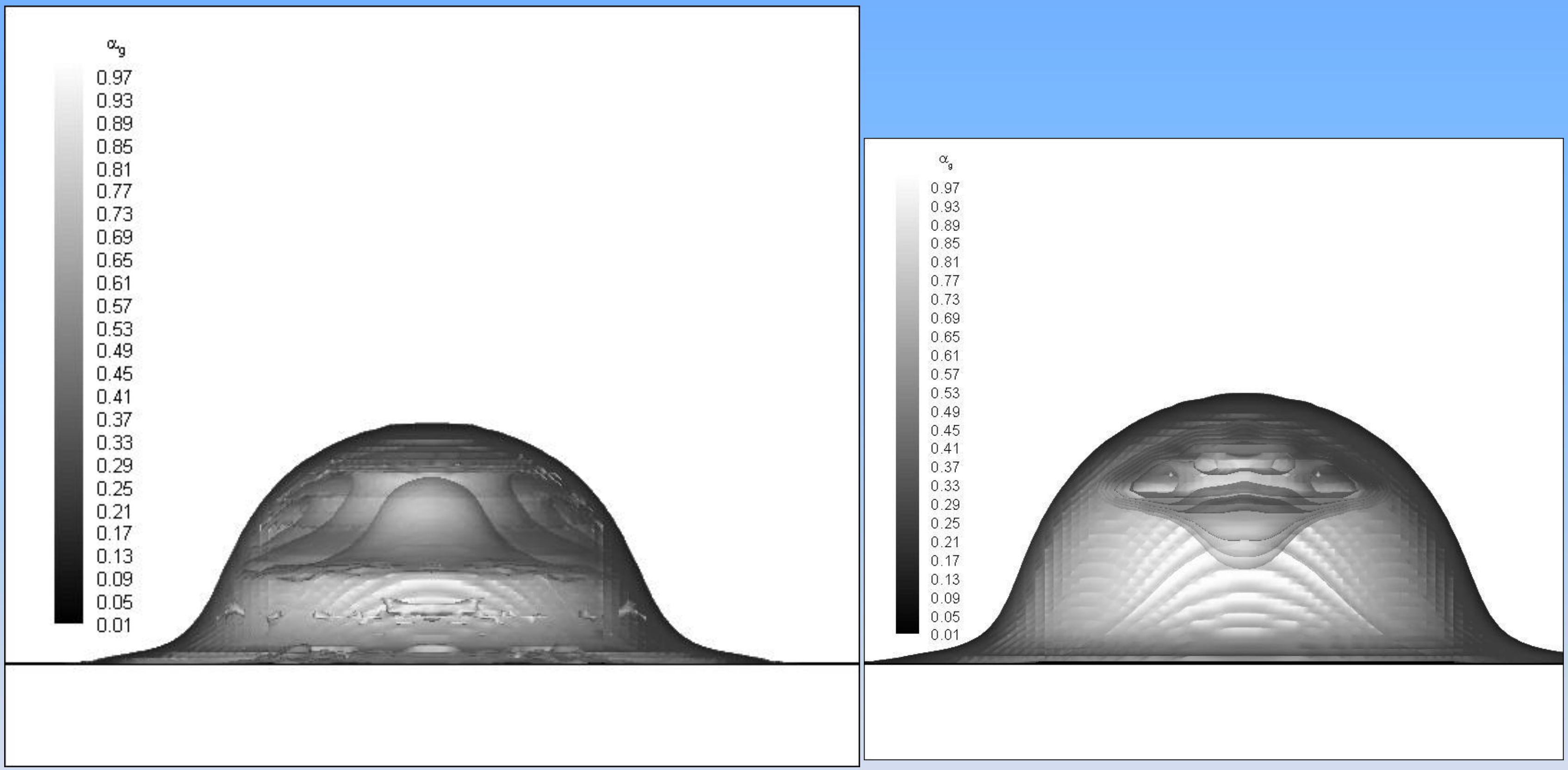


V = 400 m/s
$t = 9.6\ \mu s$



V = 500 m/s
$t = 8.4\ \mu s$

The cavitation area .vs. initial conditions



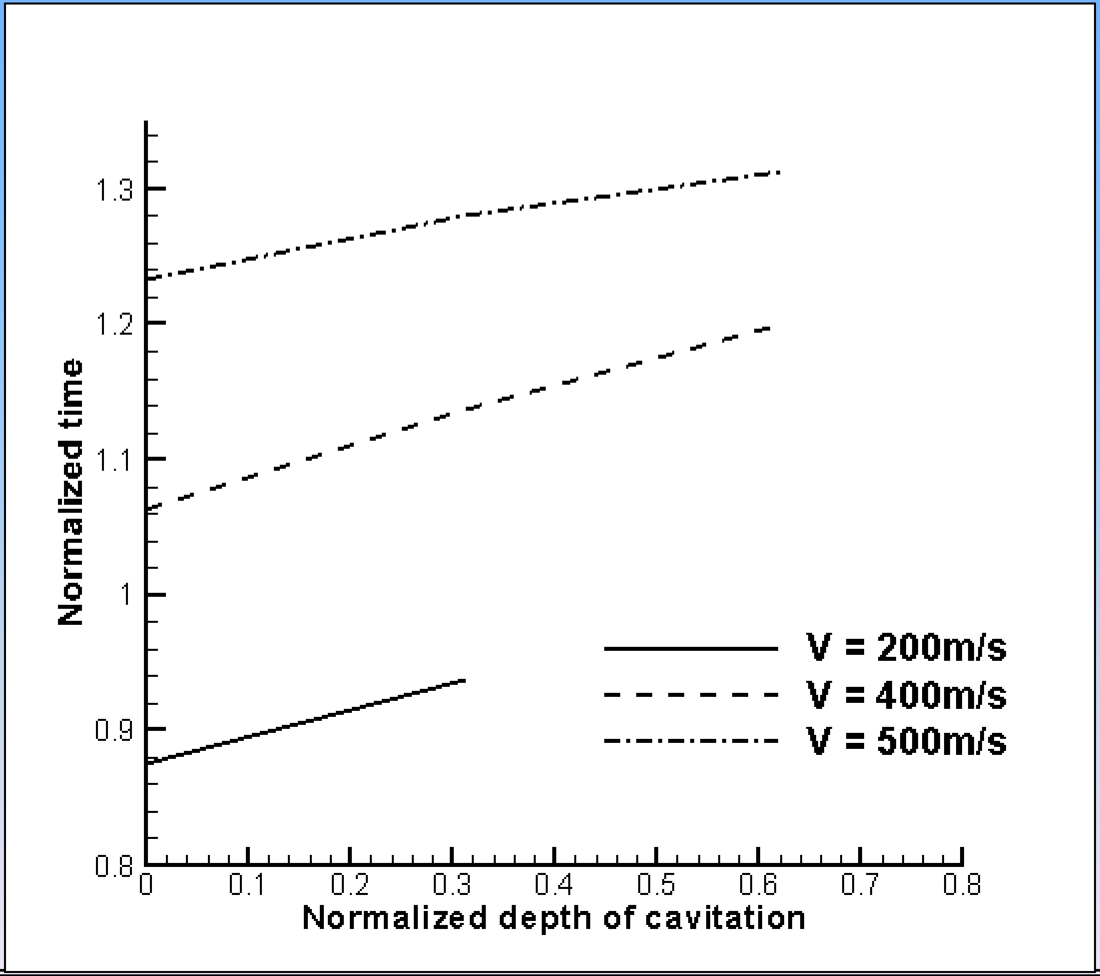
α=0.018 α=1e-6

- 3D droplet impact animation

THE END

Thanks For Your Listening

The Growth Rate of Cavitation .vs. Time



Normalized pressure versus Impact Mach No.