Unified Gas-kinetic Scheme for Multiscale Transport

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International Conference on Flow Physics and its Simulation

In memory of Prof. Jaw-Yen Yang

Contents

- Modeling, PDEs, and Computation
- Direct modeling for multiscale transport processes

  Unified Gas-kinetic Scheme (UGKS)

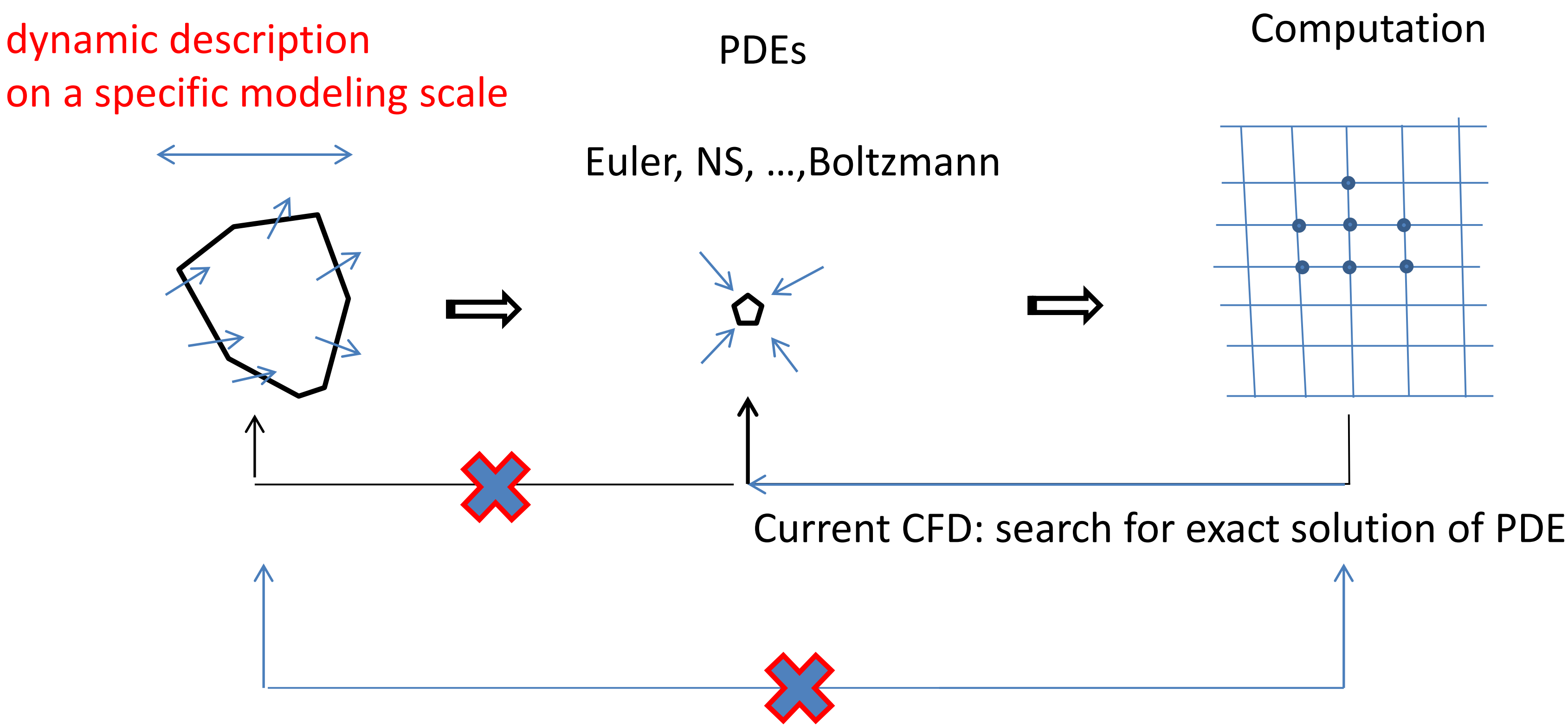
  rarefied gas dynamics

  radiative transfer

  plasma

  …
- **Conclusion**

Current CFD Methodology



No direct connection between the **mesh size scale** and the **physical modeling scale**

The 10 orders of magnitude hierarchy

(*air* under standard conditions, *1 atm* , *0° C*) $n_0 = 2.687 \times 10^{25}\, m^{-3}$


HYDRODYNAMIC

NAVIER-STOKES

$$\partial_t u + (u \cdot \nabla) u = -\frac{\nabla P}{\rho} + \nu \Delta u$$

(continuum description)

KINETIC

BOLTZMANN EQUATION

- Mean free-path: $\lambda_0 = \left(\sqrt{2}\,\pi\, d^2\, n_0\right)^{-1} = 6.1 \times 10^{-8}\, m$

MICROSCOPIC

MOLECULAR DYNAMICS

- Effective molecular diameter (via C-E formulas): $d \approx 3.7 \times 10^{-10}\, m$

Having failed to obtain any **CFL** picture, we have switched to a **CLL** one: Richard Courant, Hans Lewy, and Jean Leray at the Arden Conference Center around 1950. It may be thought as a first order approximation... (From Peter Lax files)

Richard Courant, Kurt Friedrichs, and Hans Lewy

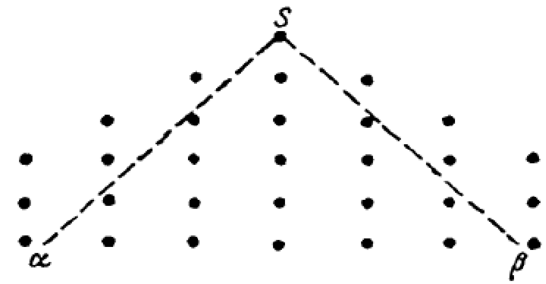


Fig. 9.

1928

Über die partiellen Differenzengleichungen der mathematischen Physik.

Von

R. Courant, K. Friedrichs und H. Lewy in Göttingen.

English Translation:

On the partial difference equations of mathematical physics

IBM JOURNAL · MARCH 1967

Problems involving the classical linear partial differential equations of mathematical physics can be reduced to algebraic ones of a very much simpler structure by replacing the differentials by difference quotients on some (say rectilinear) mesh. This paper will undertake an elementary discussion of these algebraic problems, in particular of the behavior of the solution as the mesh width tends to zero. For present purposes we limit ourselves mainly to

cases. We will show by typical examples that the passage to the limit is indeed possible, i.e., that the solution of the difference equation converges to the solution of the corresponding differential equation; in fact we will find

ative. Nowhere do we assume the existence of the solution to the differential equation problem—on the contrary, we obtain a simple existence proof by using the limiting process.[1] For the case of elliptic equations convergence is

obtained independently of the choice of mesh, but we will find that for the case of the initial value problem for hyperbolic equations, convergence is obtained only if the ratio of the mesh widths in different directions satisfies certain inequalities which in turn depend on the position of the characteristics relative to the mesh.

(CFL condition)

力学丛书·典藏版—6

计算流体动力学—偏微分方程的数值解法

程心一　著

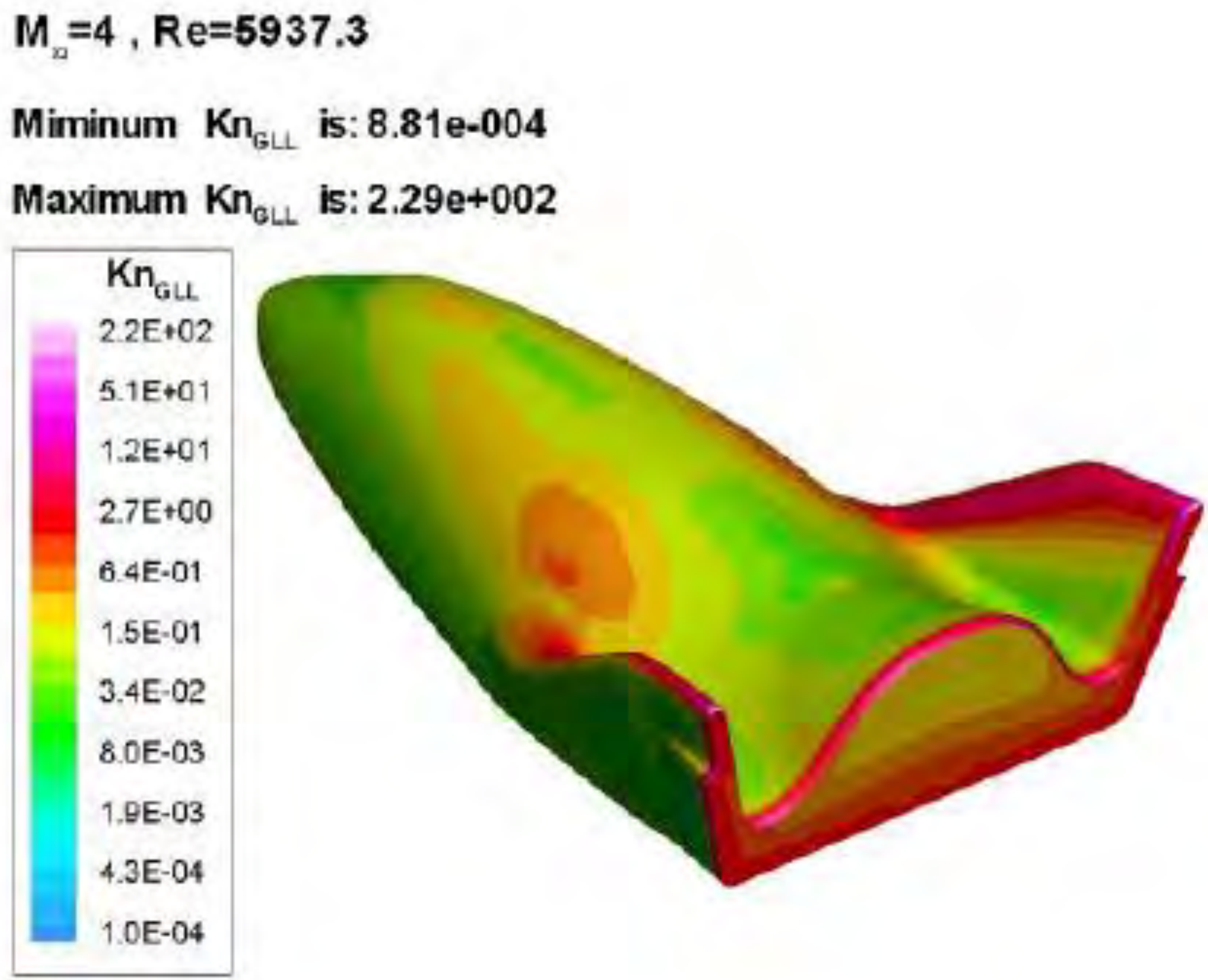
LX

科学出版社

Current CFD principle -> Numerical PDEs

Limitations:

1. To get convergent solution of the PDEs as mesh size and time step go to zero. With limited cell size and time step, theoretically never know the exact underlying governing equations.
2. All PDEs are valid in their modeling scales. It is hard to study multiple scale problem if there is no such a governing equation valid in all scales.
3. In certain scales, we don't have the corresponding PDEs at all.


M∞=4 , Re=5937.3
Miminum Kn_GLL is: 8.81e-004
Maximum Kn_GLL is: 2.29e+002
Kn_GLL
2.2E+02
5.1E+01
1.2E+01
2.7E+00
6.4E-01
1.5E-01
3.4E-02
8.0E-03
1.9E-03
4.3E-04
1.0E-04

D.W. Jiang

Near Space flow modeling

(Boltzmann or DSMC)

free molecular

100km

Transitional

Near space
No valid governing equations

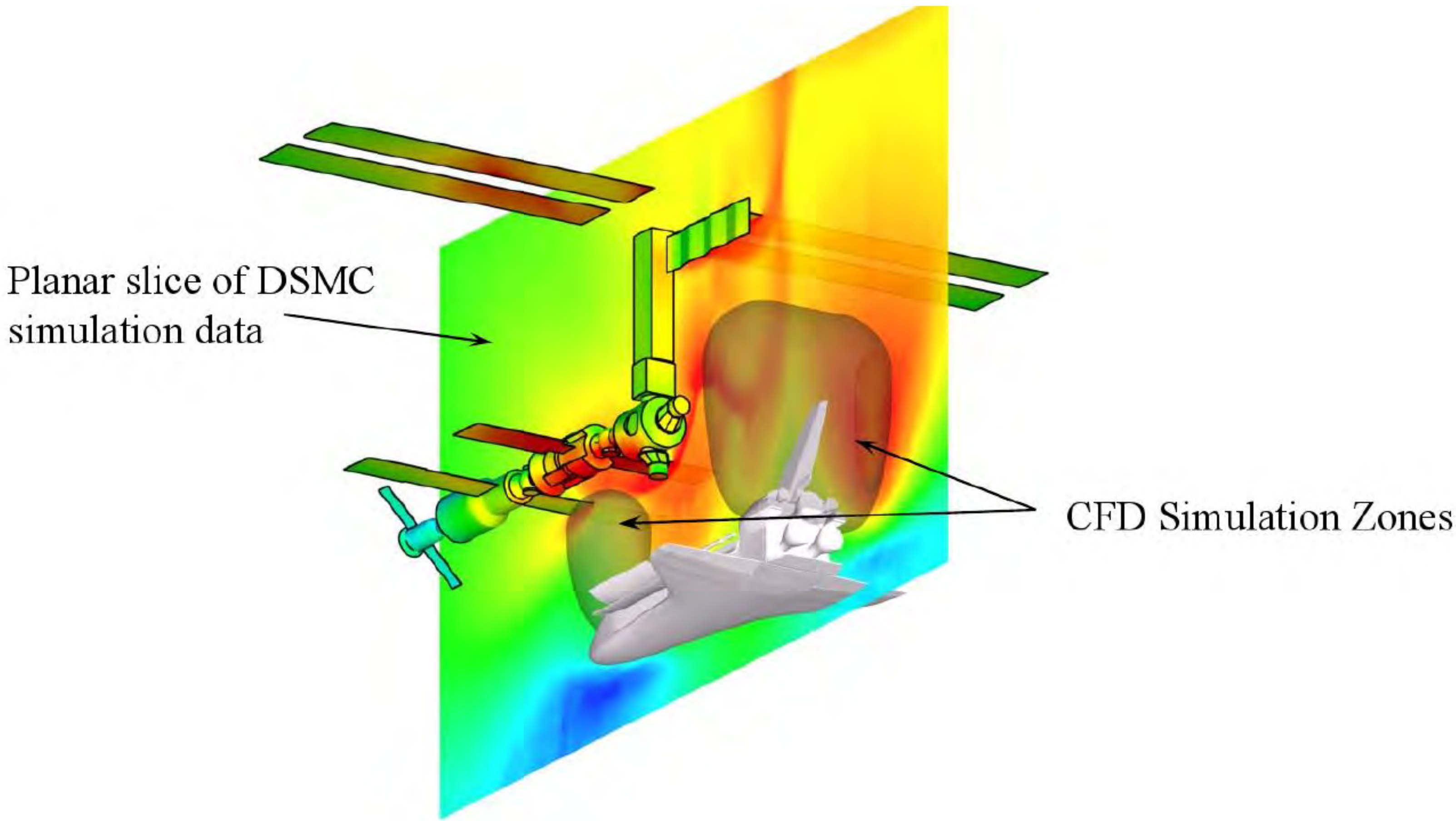
60km

slip

20km

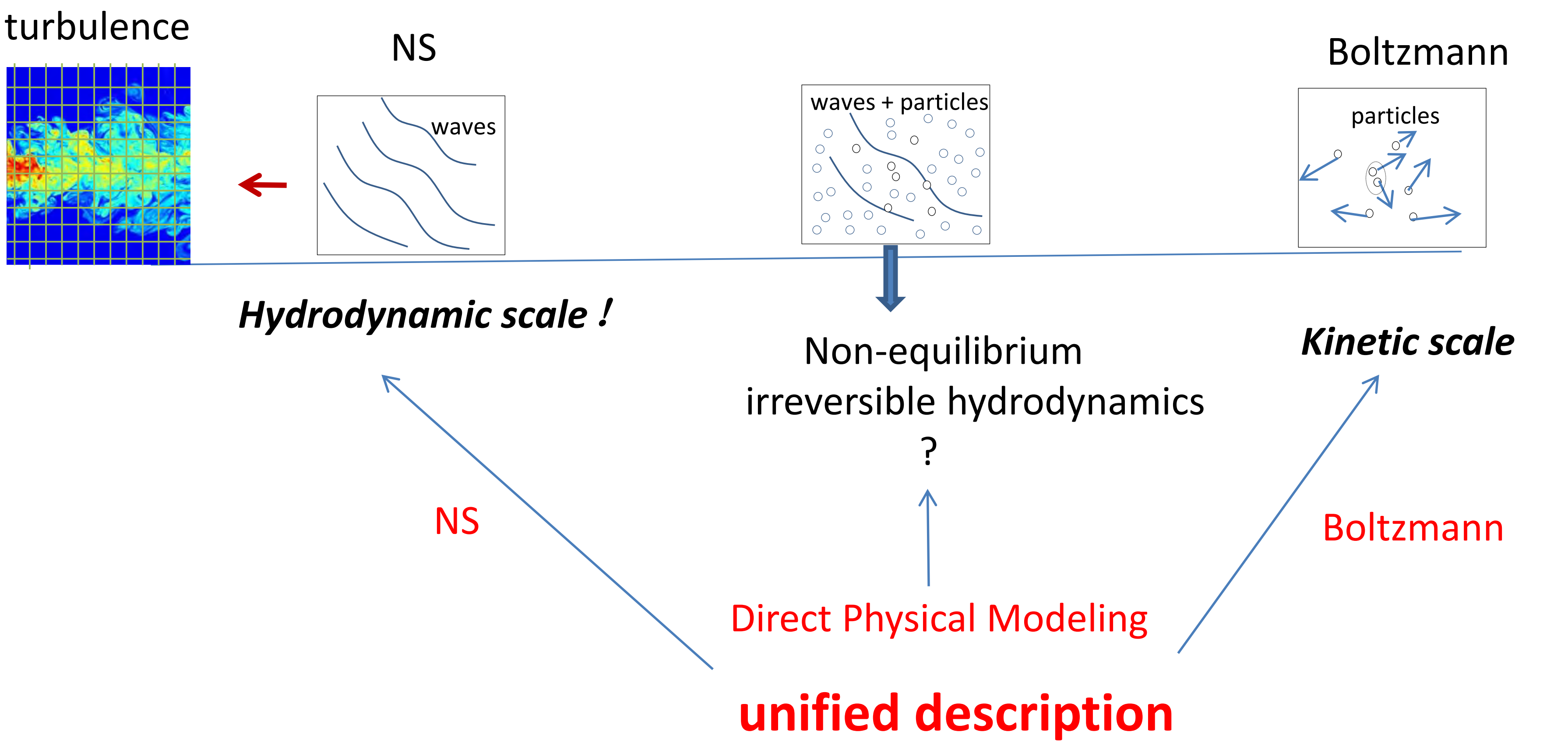
continuum

(Navier-Stokes)



I. Boyd

Turbulent modeling <-> Navier-Stokes <-> ? ? <-> Boltzmann -> Molecular Dynamics



Theoretical fluid dynamics

Computational fluid dynamics

Euler equations

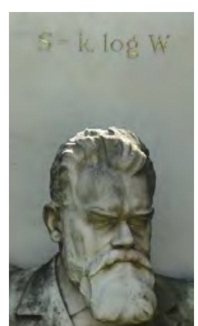
Navier-Stokes equations

Distinct governing equations in different scale modeling

…

…

Boltzmann equation

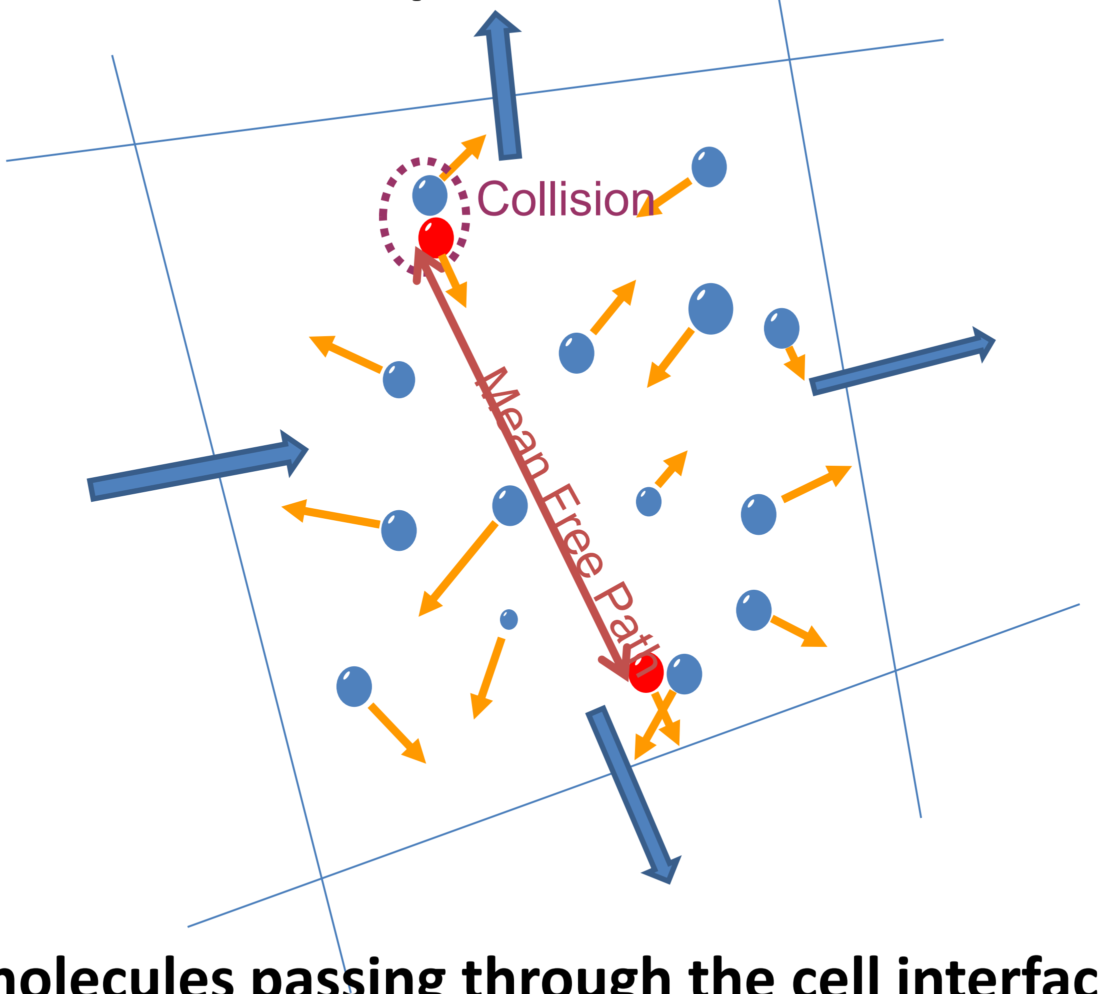


Aim:

A continuous spectrum of governing Equations

Direct Modeling for CFD

Computation: a description of flow motion in a discretized space and time



The way of gas molecules passing through the cell interface depends on the cell resolution and particle mean free path

Direct modeling in discretized space

f : gas distribution function,

W : conservative macroscopic variables

Fundamental Governing Equations

Micro:

$$f_j^{n+1} = f_j^n + \frac{1}{\Delta x}\int_{t^n}^{t^{n+1}} [uf_{x_{j-1/2}}(t) - uf_{x_{j+1/2}}(t)]dt + \frac{1}{\Delta x}\int_{t^n}^{t^{n+1}}\int_{x_{j-1/2}}^{x_{j+1/2}} Q(f,f)dxdt$$

Marco:

$$W_j^{n+1} = W_j^n + \frac{1}{\Delta x}\int_{t^n}^{t^{n+1}}\int u\,\psi(f_{j-1/2} - f_{j+1/2})du\,d\xi dt$$

Modeling: **Interface distribution function**

Inner cell collision term

Unified Gas-kinetic Scheme (UGKS)

(modeling for both continuum and rarefied flows)

General framework of Unified Gas-kinetic Scheme (UGKS)

Update of distribution function (micro):

$$f_{j,k}^{n+1} = f_{j,k}^{n} + \frac{1}{\Delta x}\int_{t^n}^{t^{n+1}} [uf_{j-1/2,k}(t) - uf_{j+1/2,k}(t)]dt + \frac{1}{\Delta x}\int_{t^n}^{t^{n+1}}\int_{x_{j-1/2}}^{x_{j+1/2}} Q(f,f)dxdt$$

Update of conservative variables (marco):

$$W_j^{n+1} = W_j^n + \frac{1}{\Delta x}\int_{t^n}^{t^{n+1}}\int u_k \psi (f_{j-1/2,k} - f_{j+1/2,k}) du_k d\xi dt$$

Both interface flux and inner cell collision term need to be modeled

K. Xu and J.C. Huang

Collision term treatment:

$$f_{j,k}^{n+1} = f_{j,k}^{n} + \frac{1}{\Delta x}\int_{t^n}^{t^{n+1}} (u_k \hat{f}_{j-1/2,k} - u_k \hat{f}_{j+1/2,k})dt$$
$$+ AQ(f_j^n, f_j^n)_k + B\frac{\tilde{M}(f_j^{n+1})_k - f_{j,k}^{n+1}}{\tau_s^{n+1}},$$

1. $A + B \sim \Delta t$ in order to have a consistent collision term treatment.
2. The scheme is stable in the whole flow regime.
3. In the rarefied flow regime, the scheme gives the Boltzmann solution.
4. In continuum regime, the scheme can efficiently recover the Navier-Stokes solutions.

$$Q(f,f) = \int_{R^3}\int_{S^2}(f_*'f' - f_*f)\,|u_r|\sigma\, d\Omega du_*.$$

Interface flux modeling:

The flux evaluation is based on the integral solution of the kinetic model:

$$f_{j+1/2,k} = \frac{1}{\tau}\int_0^t g(x', t', u_k, \xi) e^{-(t-t')/\tau} dt' + e^{-t/\tau} f_{0,k}(x_{j+1/2} - u_k t)$$

Hydrodynamic evolution

Kinetic scale evolution

UGKS: Evolution Processes

(micro-scale)

$$f_{j,k}^{n+1} = f_{j,k}^{n} + \frac{1}{\Delta x}\int_{t^n}^{t^{n+1}} [uf_{j-1/2,k}(t) - uf_{j+1/2,k}(t)]dt + \frac{1}{\Delta x}\int_{t^n}^{t^{n+1}}\int_{x_{j-1/2}}^{x_{j+1/2}} Q(f,f)dxdt$$

Scale up

taking conservative moments:

$$\psi = (1, u_k, \frac{1}{2}(u_k^2 + \xi^2))^T$$

Scale down

g^{n+1}

mass, momentum and energy

(macro-scale)

$$W_j^{n+1} = W_j^n + \frac{1}{\Delta x}\int_{t^n}^{t^{n+1}}\int u_k \psi (f_{j-1/2,k} - f_{j+1/2,k}) du_k d\xi dt$$

Numerical path: $f^{n} \rightarrow W^{n+1} \rightarrow g^{n+1} \rightarrow f^{n+1}$

The update of gas distribution function becomes

$$f_{j,k}^{n+1} = f_{j,k}^{n} + \frac{1}{\Delta x}\int_{t^n}^{t^{n+1}} [uf_{j-1/2,k}(t) - uf_{j+1/2,k}(t)]dt + \frac{1}{\Delta x}\int_{t^n}^{t^{n+1}}\int_{x_{j-1/2}}^{x_{j+1/2}} \frac{g-f}{\tau} dxdt$$

$$= f_{j,k}^{n} + \frac{1}{\Delta x}\left(\int_{t^n}^{t^{n+1}} u_k(\tilde{g}_{j-1/2,k} - \tilde{g}_{j+1/2,k})dt + \int_{t^n}^{t^{n+1}} u_k(\tilde{f}_{j-1/2,k} - \tilde{f}_{j+1/2,k})dt\right)$$

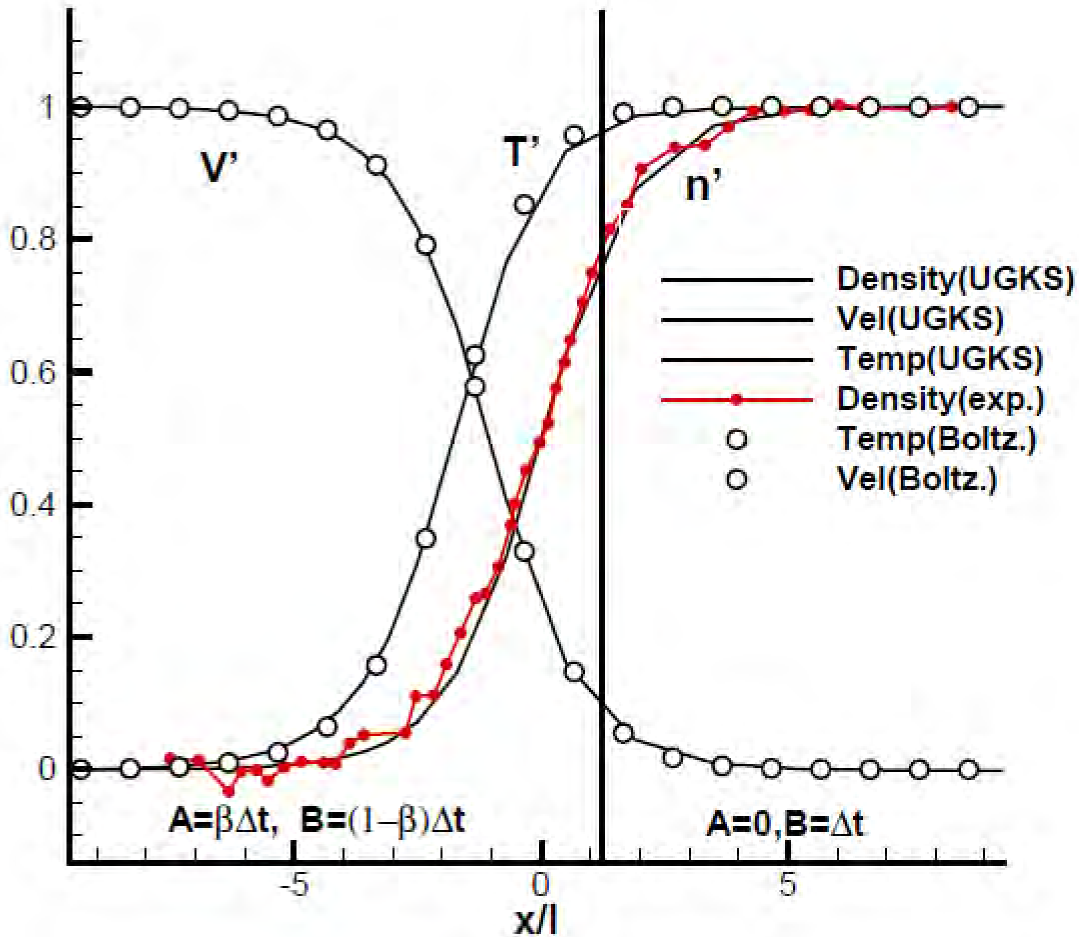
$$+ \frac{\Delta t}{2}\left(\frac{g_{j,k}^{n+1} - f_{j,k}^{n+1}}{\tau^{n+1}} + \frac{g_{j,k}^{n} - f_{j,k}^{n}}{\tau^{n}}\right)$$

with the solution:

$$\left(1 + \frac{\Delta t}{2\tau_j^{n+1}}\right) f_{j,k}^{n+1} = f_{j,k}^{n} + \frac{1}{\Delta x}\left(\int_{t^n}^{t^{n+1}} u_k(\tilde{g}_{j-1/2,k} - \tilde{g}_{j+1/2,k})dt + \int_{t^n}^{t^{n+1}} u_k(\tilde{f}_{j-1/2,k} - \tilde{f}_{j+1/2,k})dt\right)$$

$$+ \frac{\Delta t}{2}\left(\frac{g_{j,k}^{n+1}}{\tau_j^{n+1}} + \frac{g_{j,k}^{n} - f_{j,k}^{n}}{\tau_j^{n}}\right)$$

M=2.8 shock structure
(UGKS vs Experiments)



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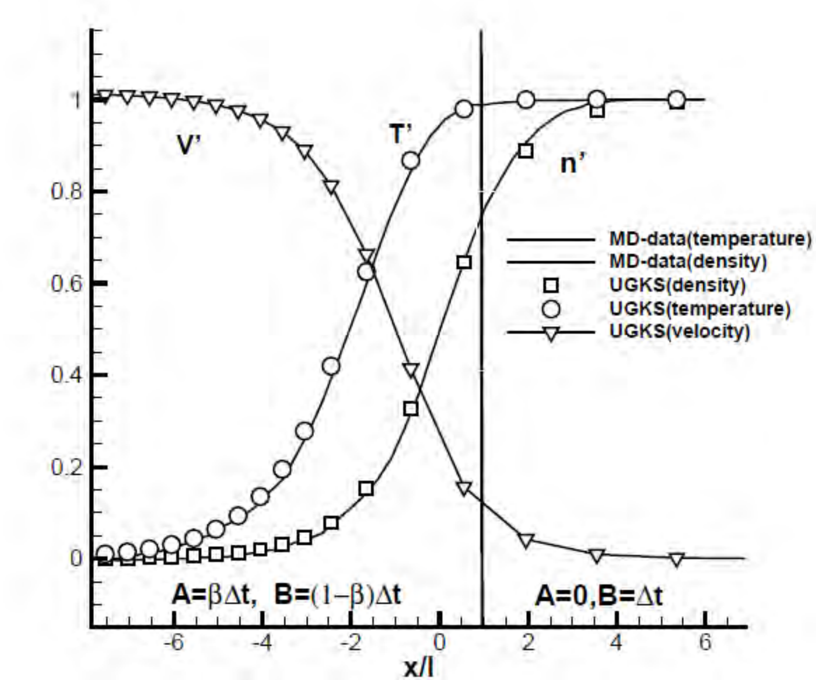
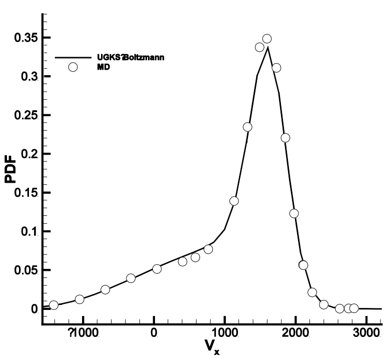
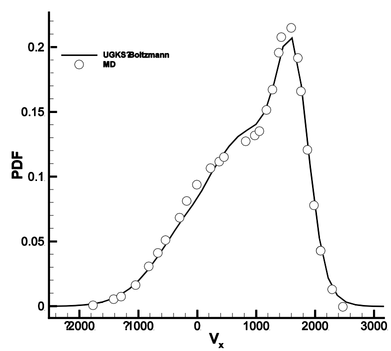
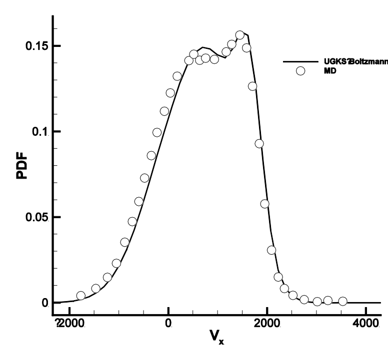
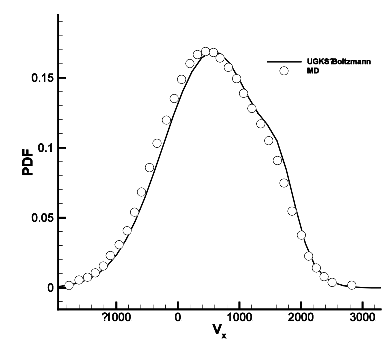
M=5 shock structure: UGKS vs MD











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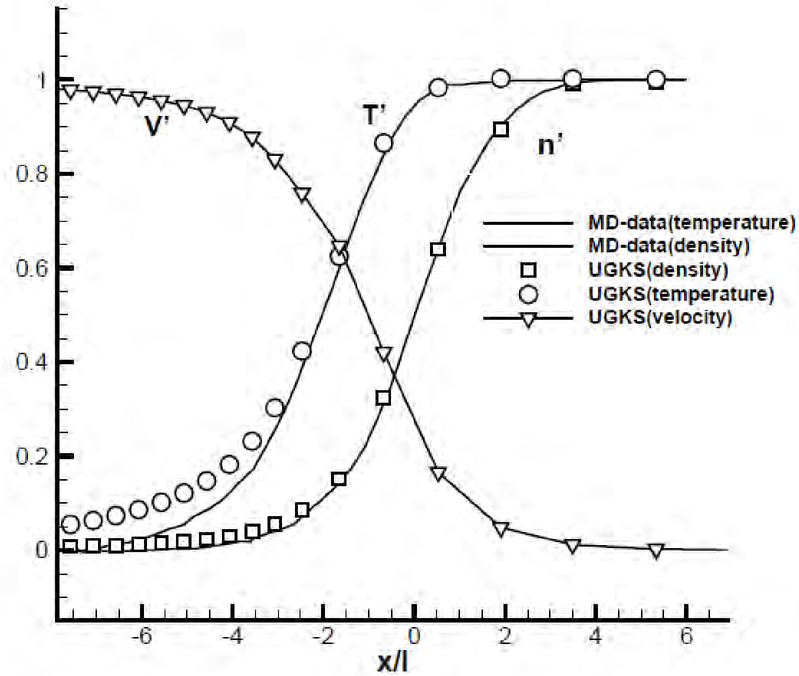
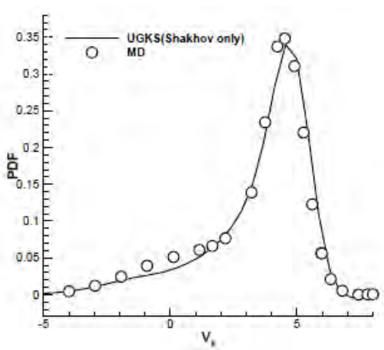
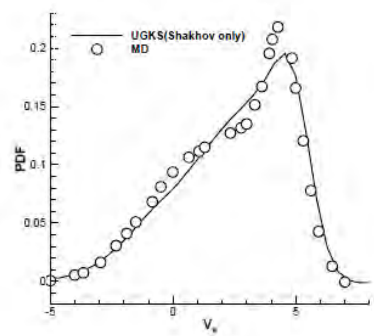
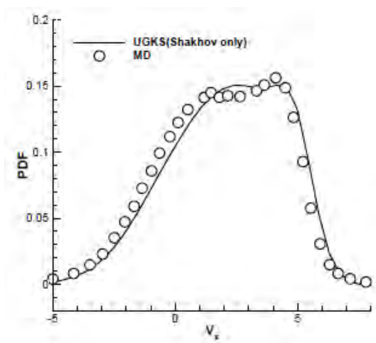
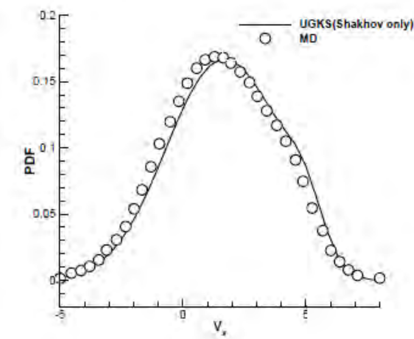
Shakhov model only:

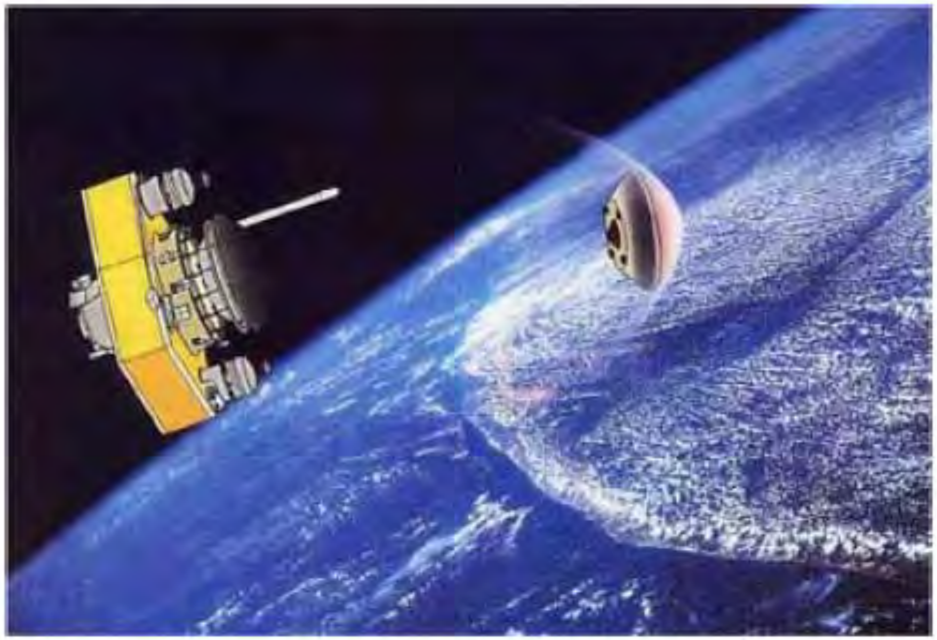








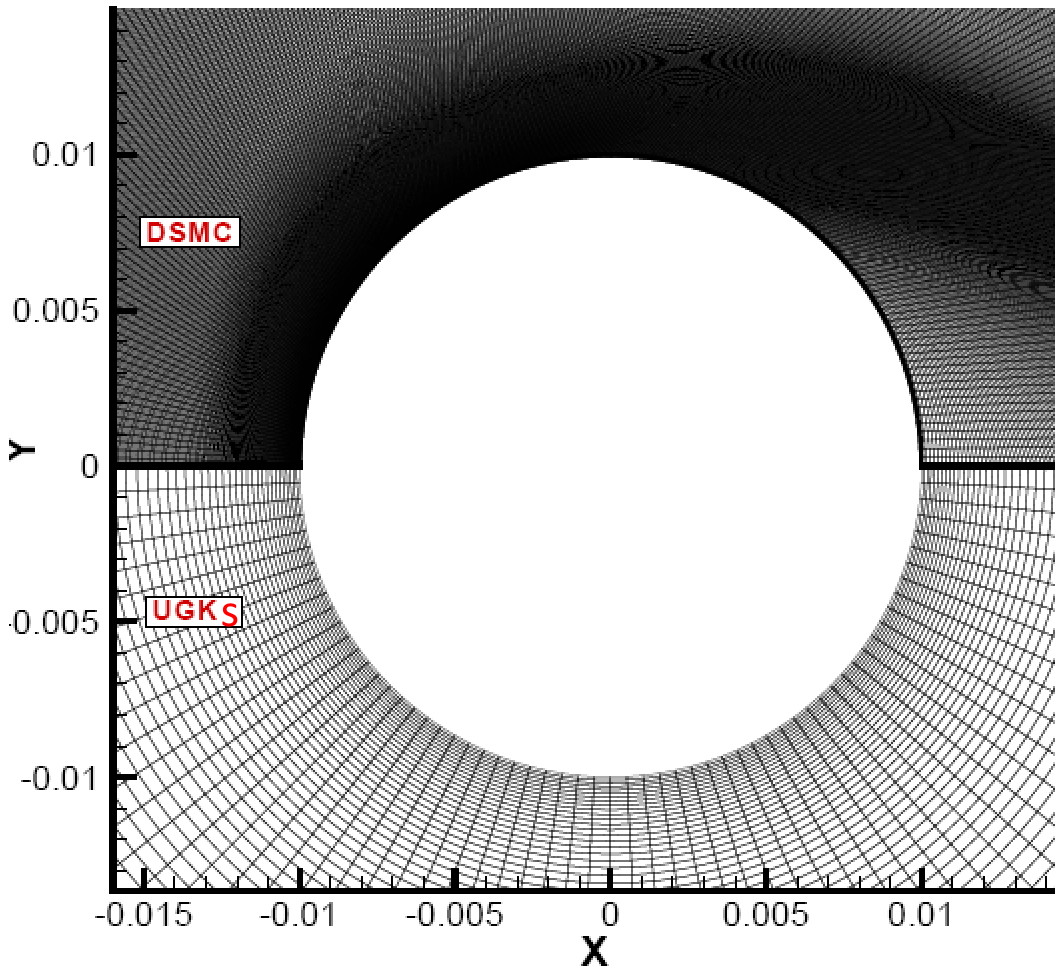




•Low-density gas dynamics

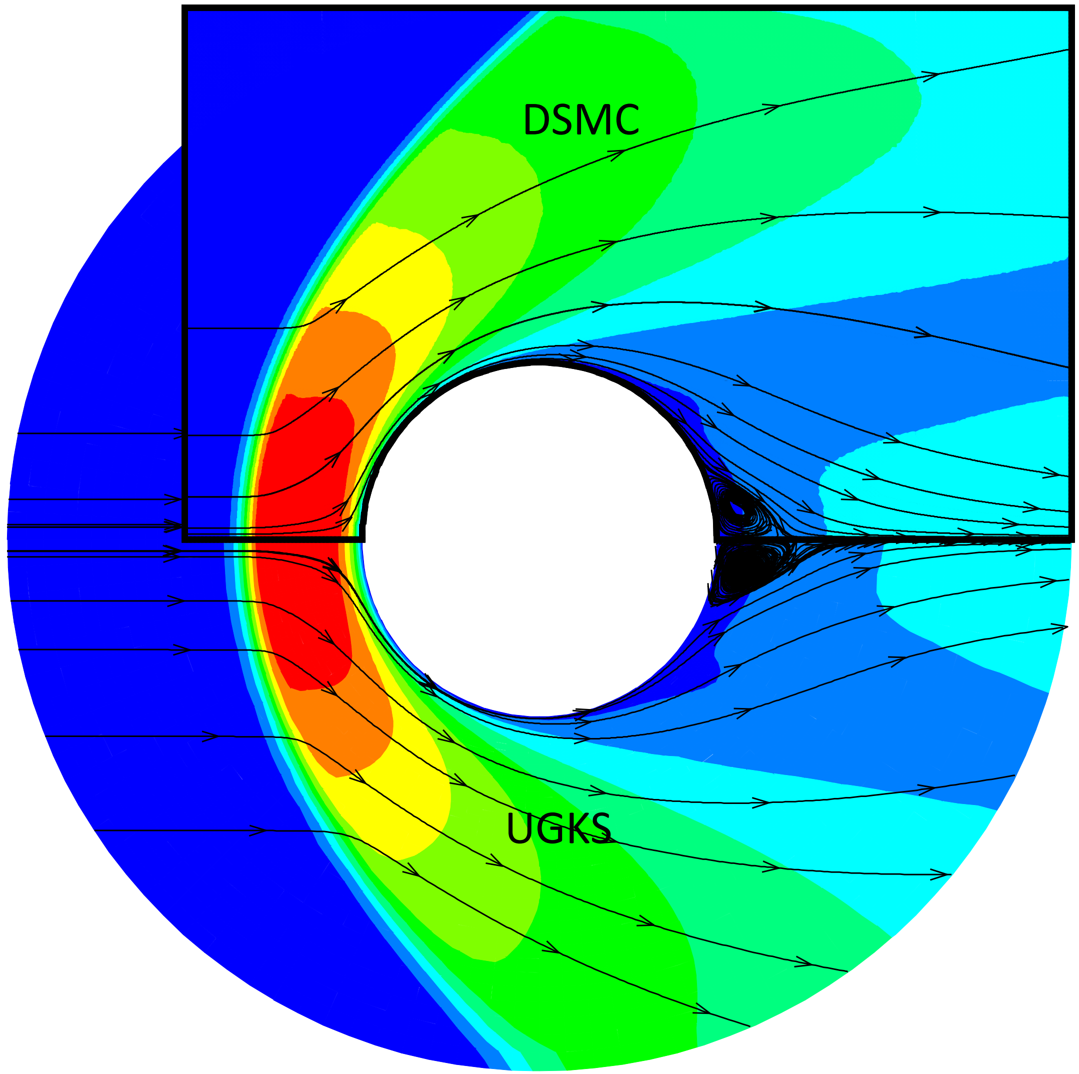
• Gas flows in micro-devices / nozzle

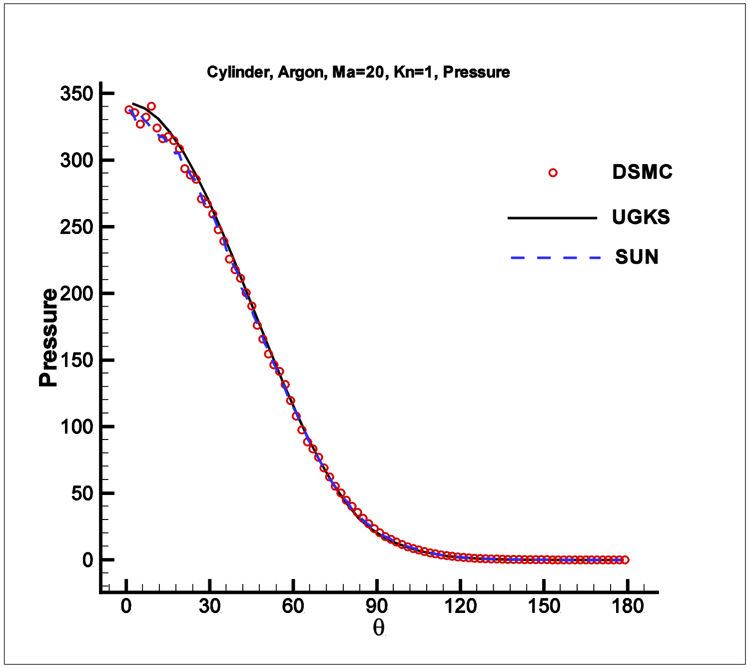
M=20, Kn=0.01

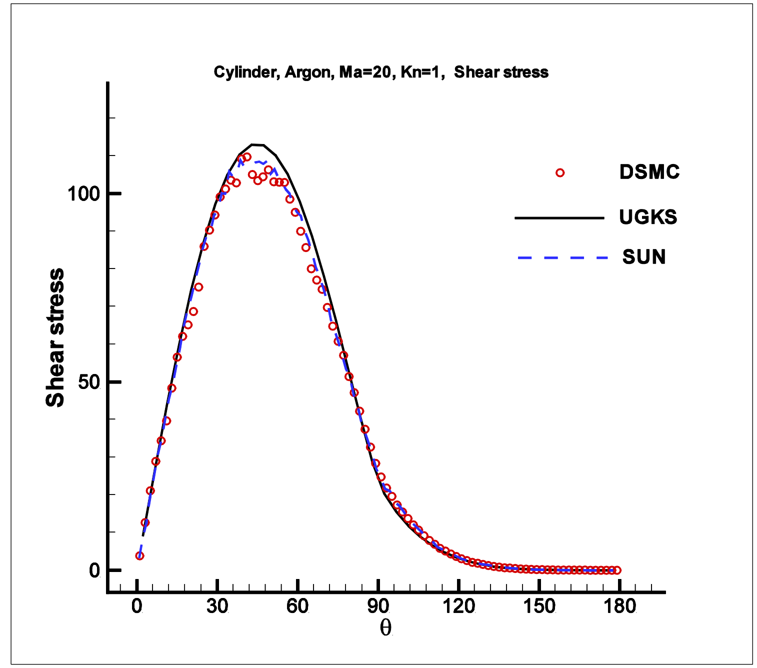


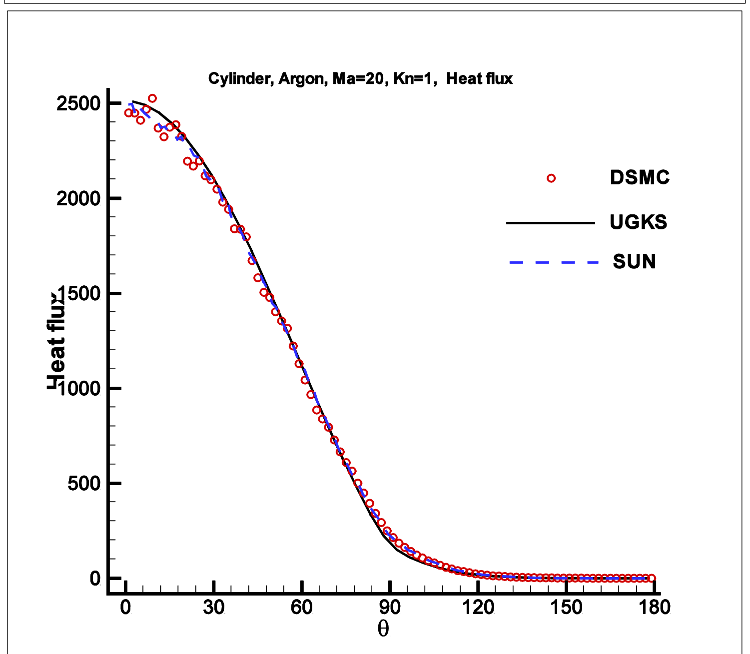
P.B. Yu

Cylinder, Argon, Ma=20,Kn=0.01




Cylinder, Argon, Ma=20, Kn=1, Pressure
Pressure
350
300
250
200
150
100
50
0
0
30
60
90
120
150
180
θ
DSMC
UGKS
SUN

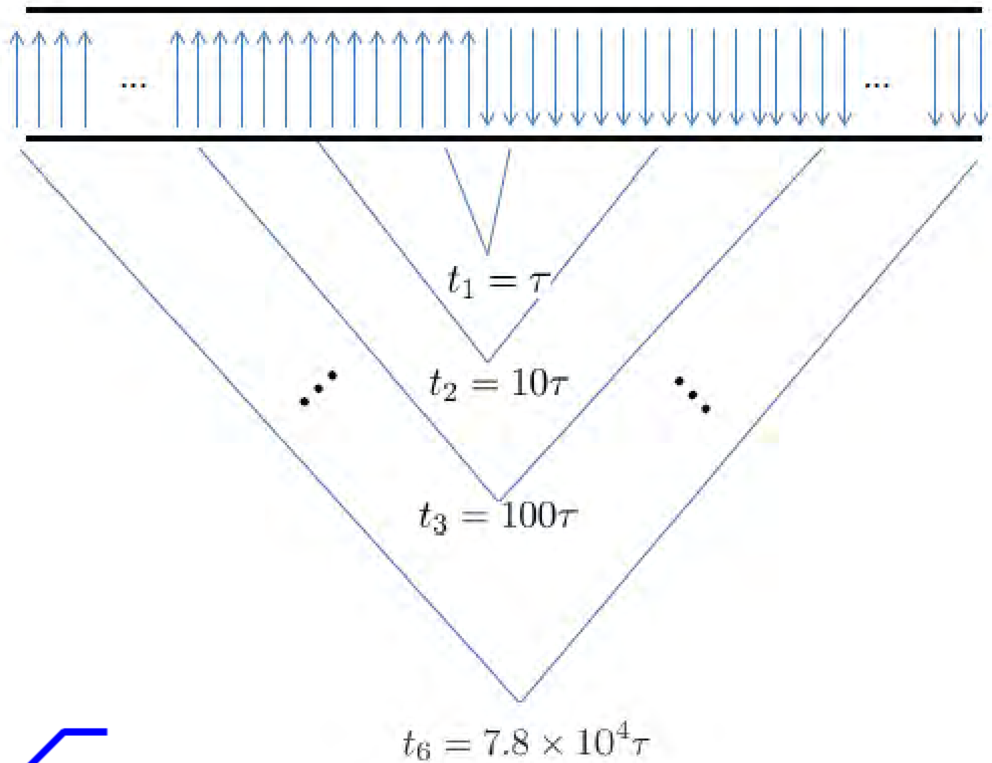

Cylinder, Argon, Ma=20, Kn=1, Shear stress
Shear stress
100
50
0
0
30
60
90
120
150
180
θ
DSMC
UGKS
SUN


Cylinder, Argon, Ma=20, Kn=1, Heat flux
Heat flux
2500
2000
1500
1000
500
0
0
30
60
90
120
150
180
θ
DSMC
UGKS
SUN

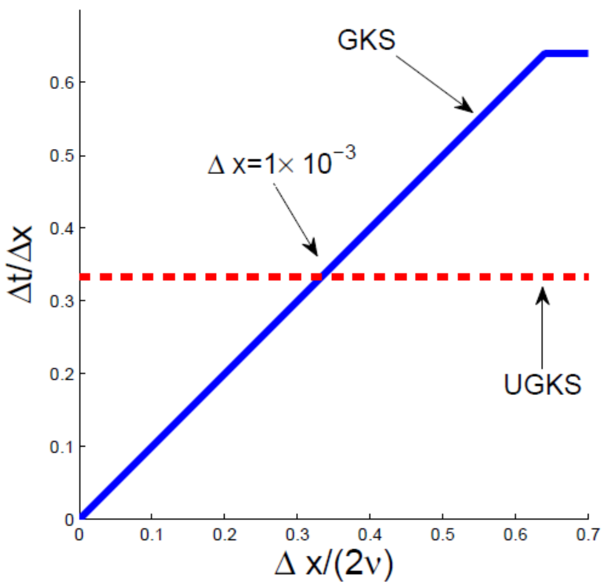
M=20, Kn=1

A continuous spectrum of gas dynamics description

Shear Layer



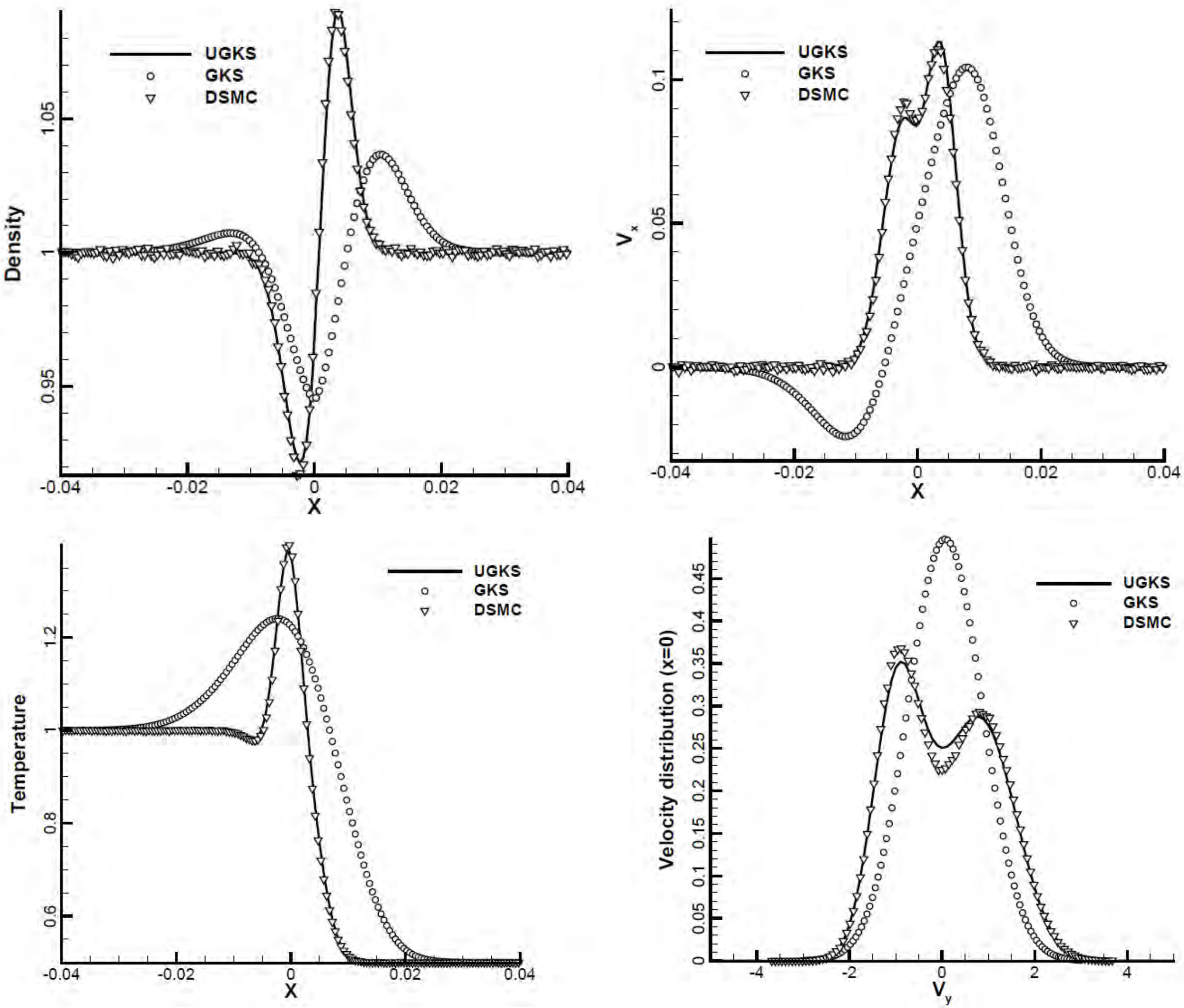
Time step:



$t = 5.12 \times 10^{-3}\ (t = \tau_{ph})$

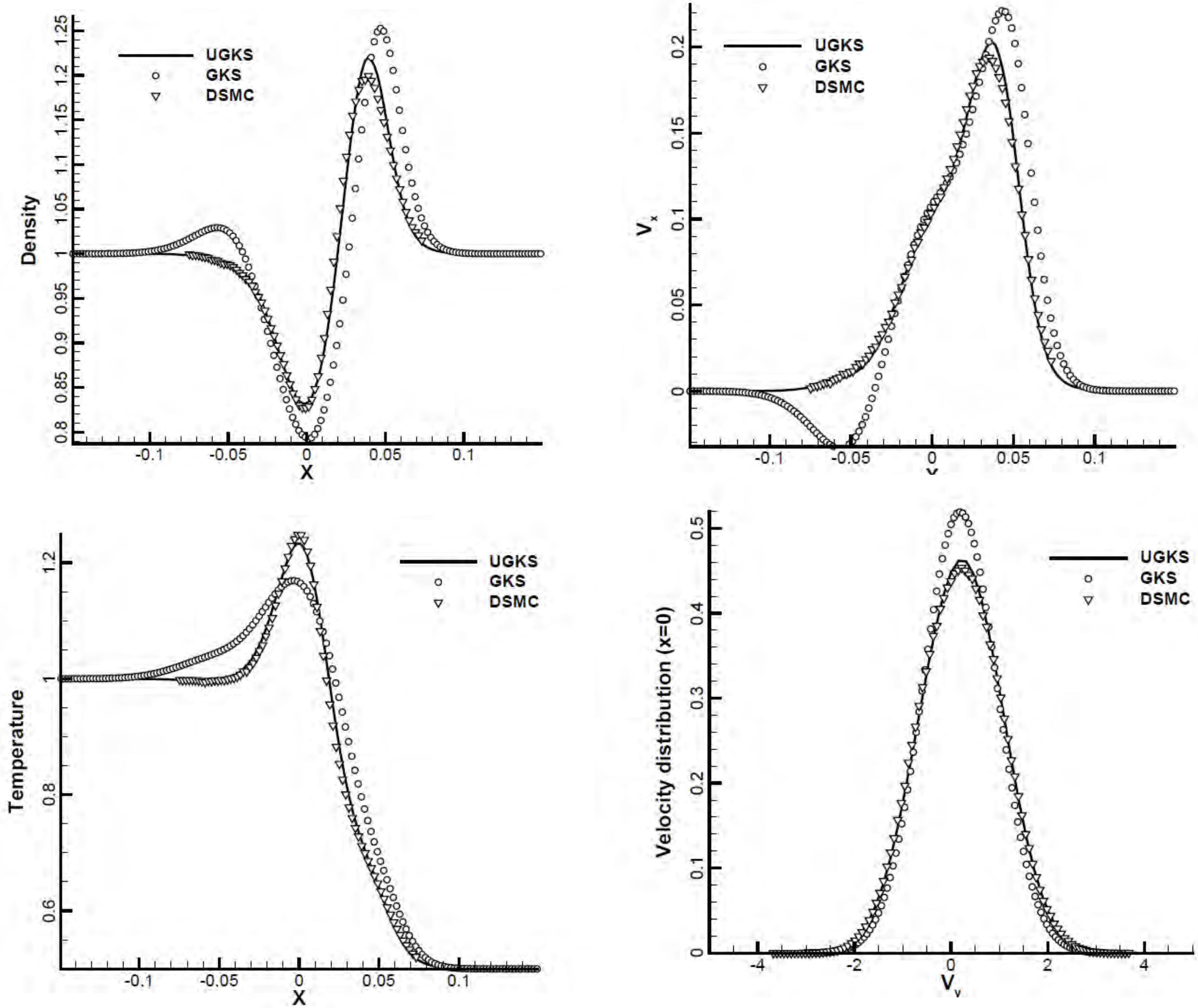
GKS $dx/\ell = 0.1$, $dt = 1.96 \times 10^{-3}\tau_{ph}$

UGKS $dx/\ell = 0.1$, $dt = 1.65 \times 10^{-2}\tau_{ph}$



$t = 5.12 \times 10^{-2}$ ($t = 10\tau_{ph}$) GKS $dx/\ell = 0.4$, $dt = 1.56 \times 10^{-2}\tau_{ph}$

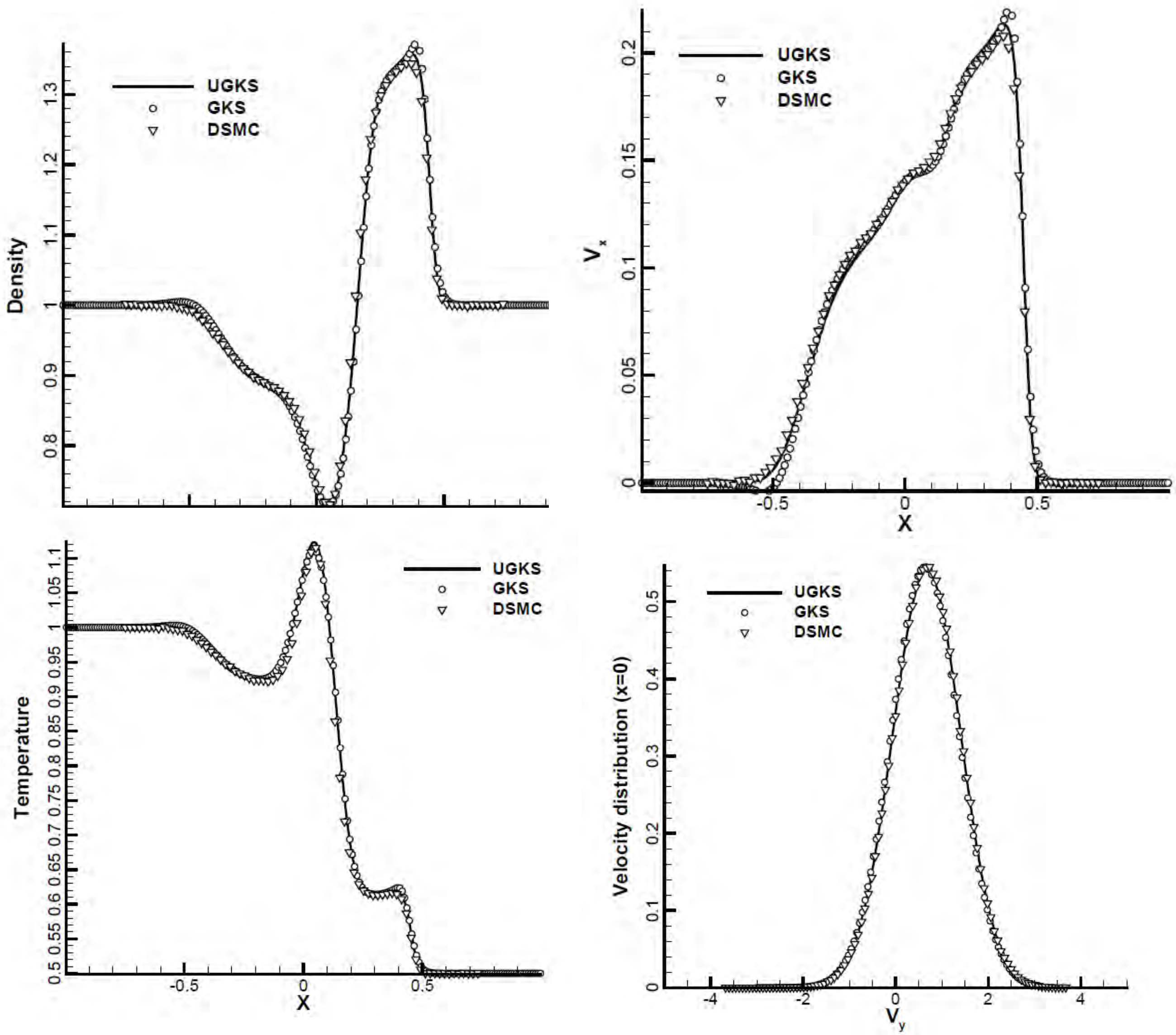
UGKS $dx/\ell = 0.4$, $dt = 6.61 \times 10^{-2}\tau_{ph}$



$$t = 5.12 \times 10^{-1} \ (t = 100\tau_{ph})$$

GKS $dx/\ell = 2,\ dt = 3.91 \times 10^{-1}\tau_{ph}$

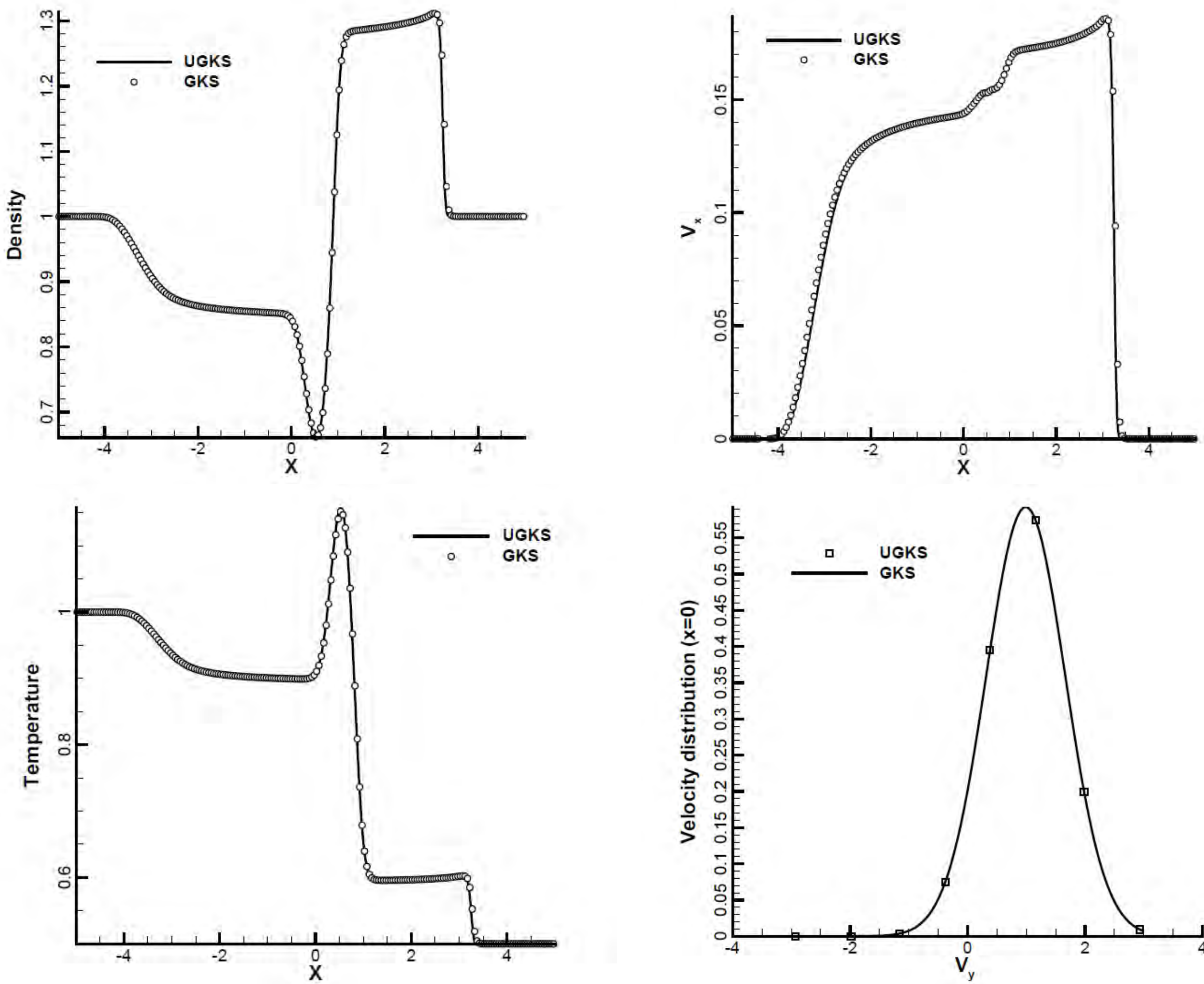
UGKS $dx/\ell = 2,\ dt = 3.78 \times 10^{-1}\tau_{ph}$



$t = 4\ (t = 781.76\tau_{ph})$

GKS $dx/\ell = 10,\ dt = 7.81\tau_{ph}$

UGKS $dx/\ell = 10,\ dt = 1.89\tau_{ph}$

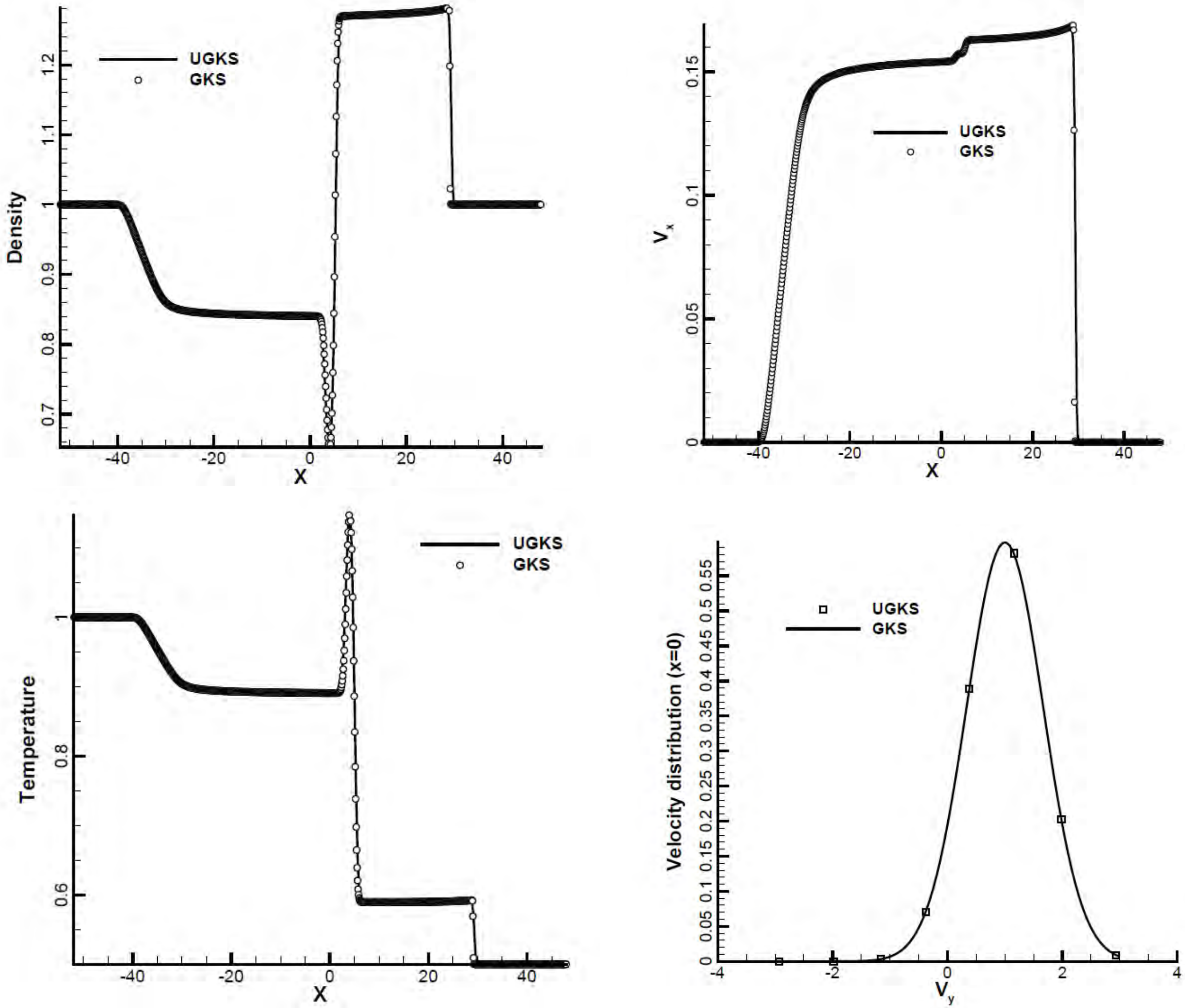


8x8 Gauss points

$t = 40 \ (t = 7817.64\tau_{ph})$

GKS $dx/\ell = 10$, $dt = 7.81\tau_{ph}$

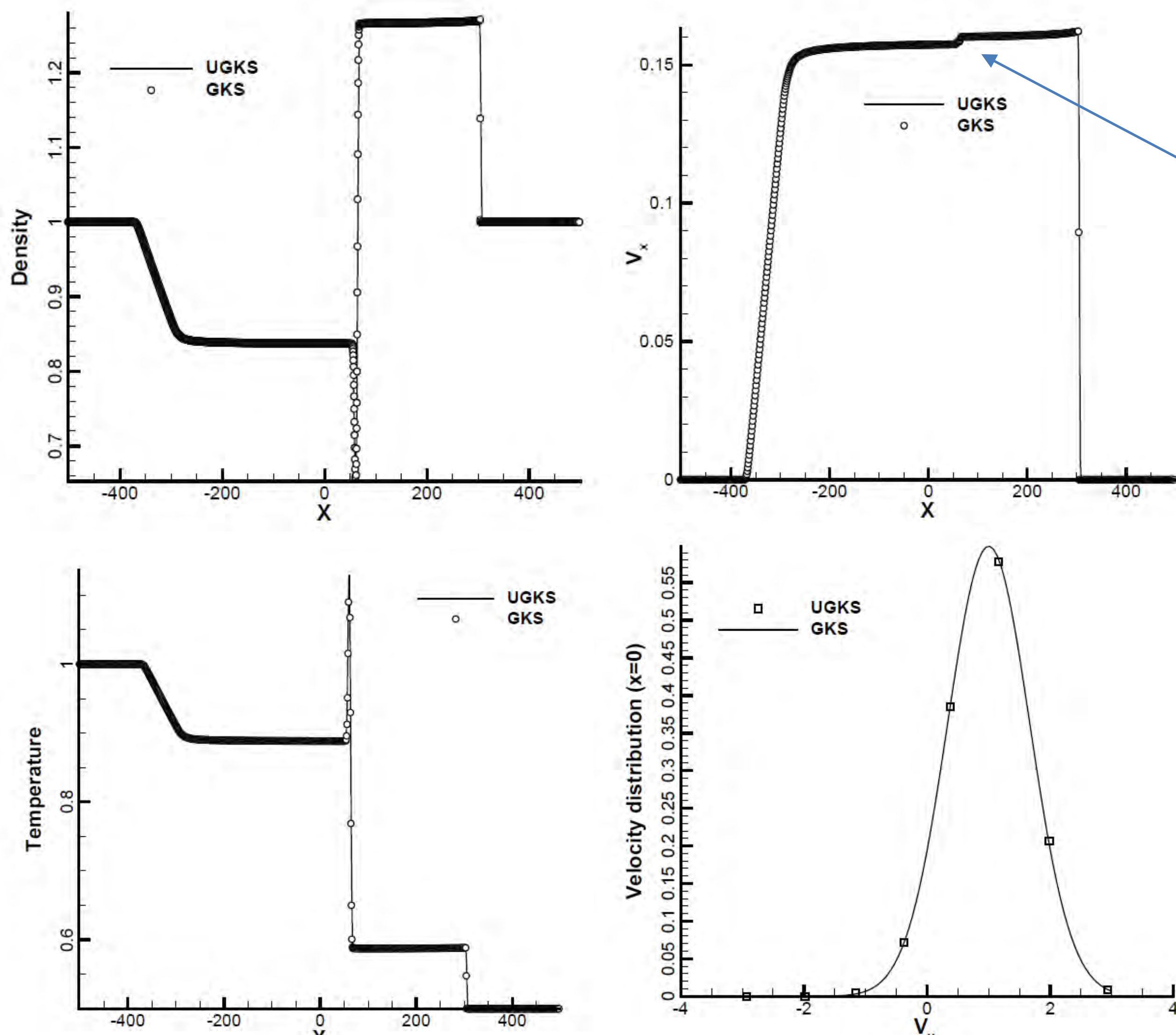
UGKS $dx/\ell = 50$, $dt = 8.25\tau_{ph}$



$t = 400 \; (t = 78176.40\tau_{ph})$

GKS $dx/\ell = 33.33, \; dt = 26.06\tau_{ph}$

UGKS $dx/\ell = 250, \; dt = 41.25\tau_{ph}$



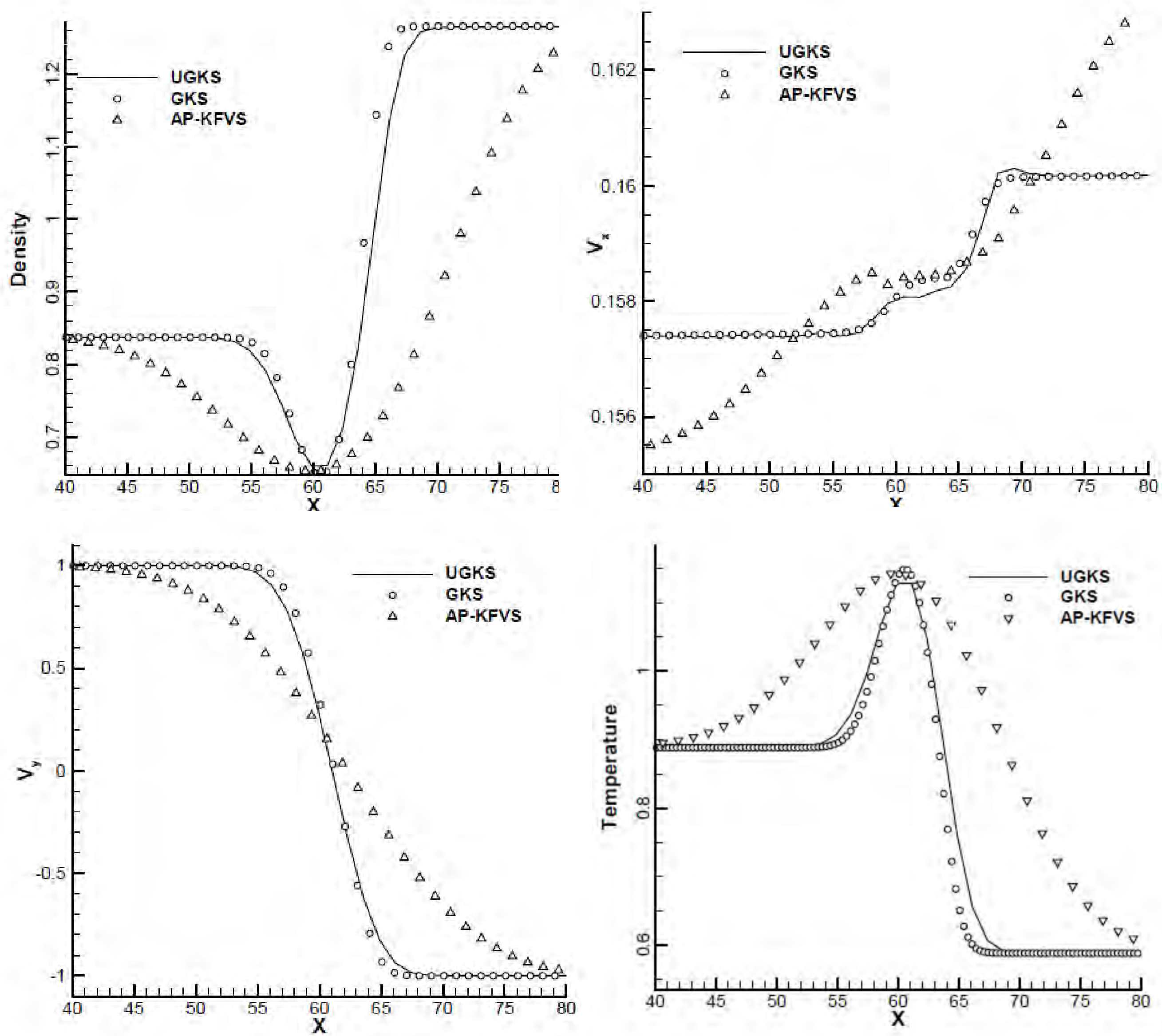
Enlarged view on next page

Comparison with other schemes: GKS (NS), UGKS, AP-KFVS

AP-KFVS $dx/\ell = 250,\ dt = 41.25\tau_{ph}$

GKS $dx/\ell = 33.33,\ dt = 26.06\tau_{ph}$

UGKS $dx/\ell = 250,\ dt = 41.25\tau_{ph}$



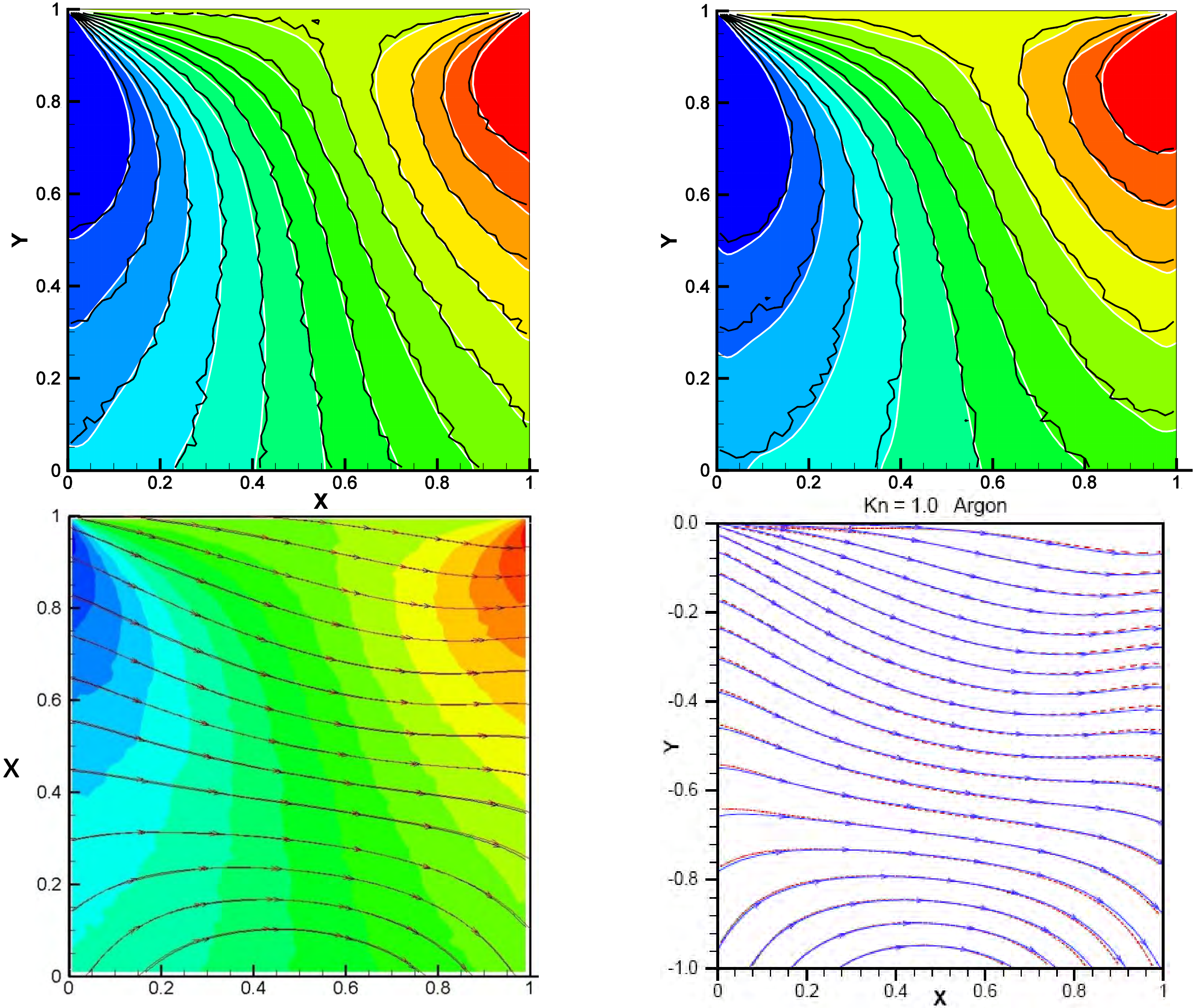
UGKS vs DSMC

Kn=10

Kn=1

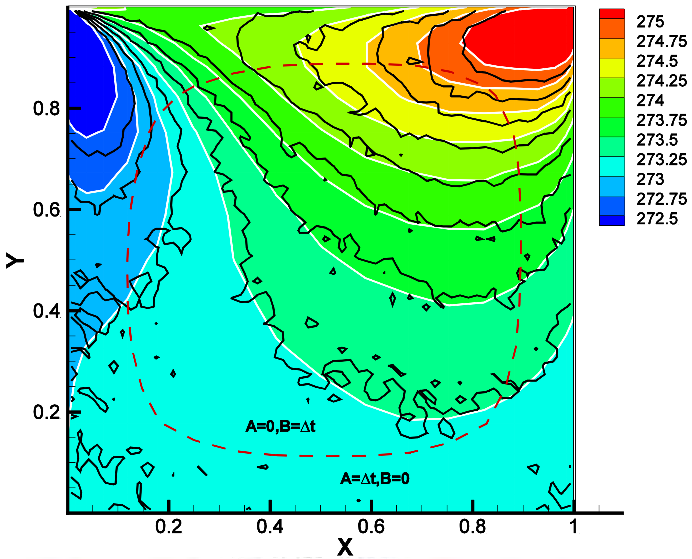
temp

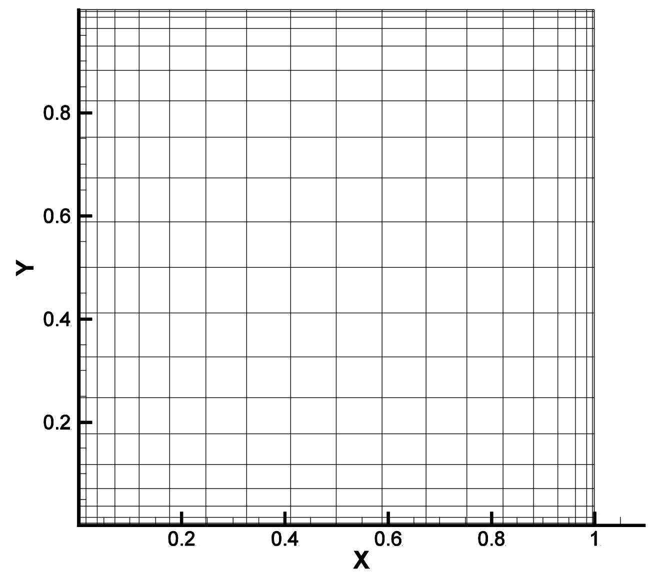
Heat flux



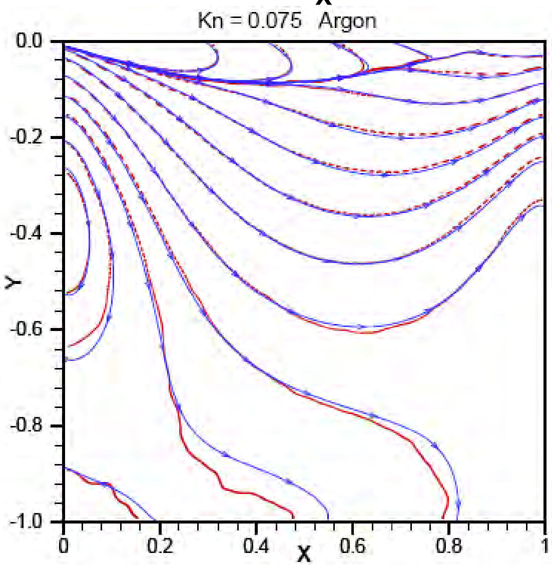
Kn=0.075 cavity case

temp





Heat flux



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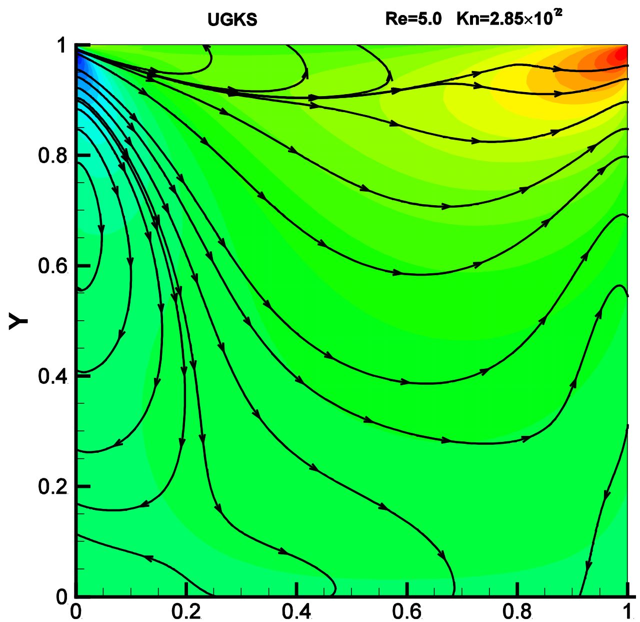
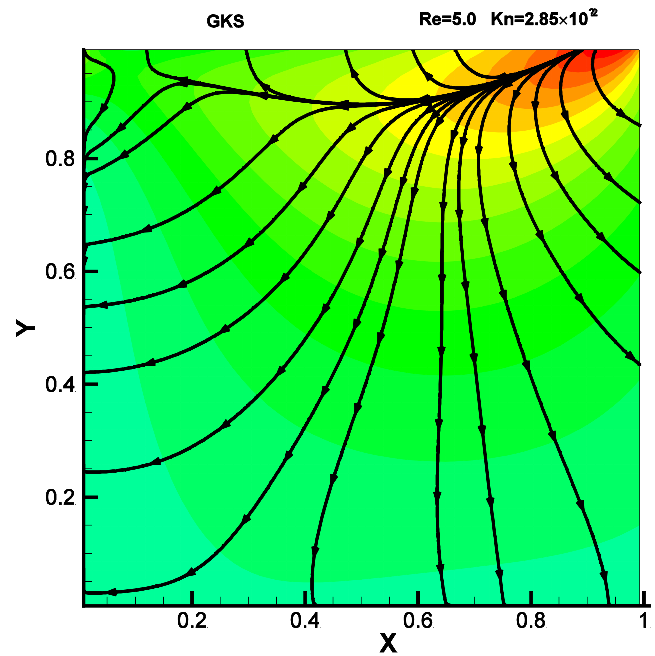
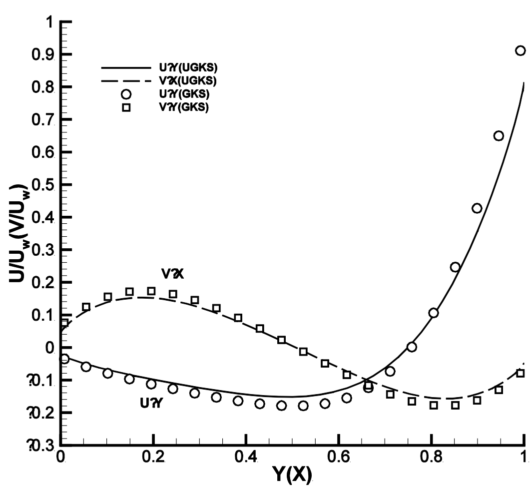
Re=5, Kn=0.0285

UGKS



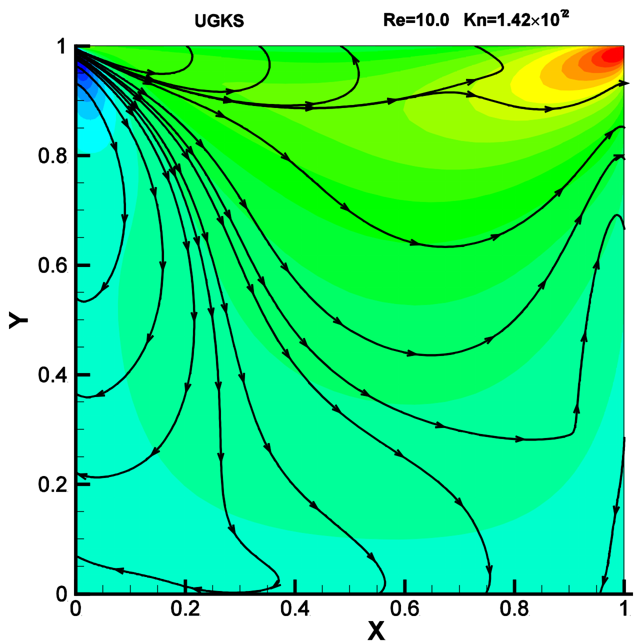


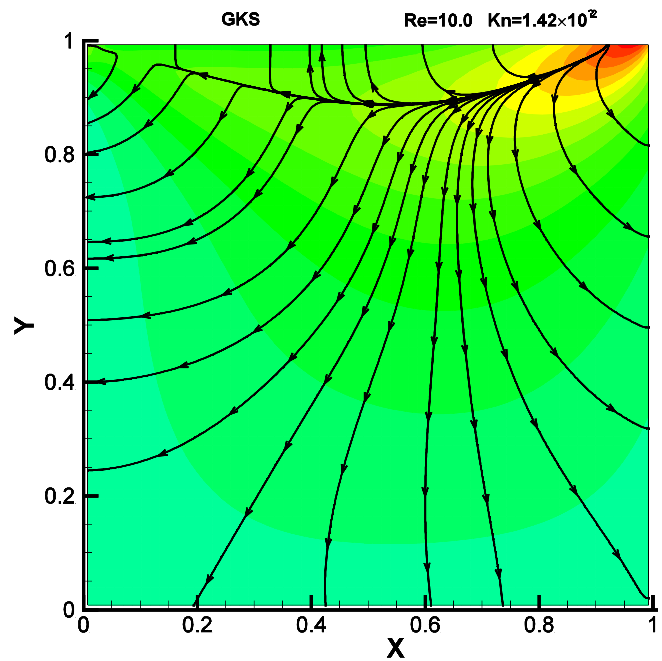
NS



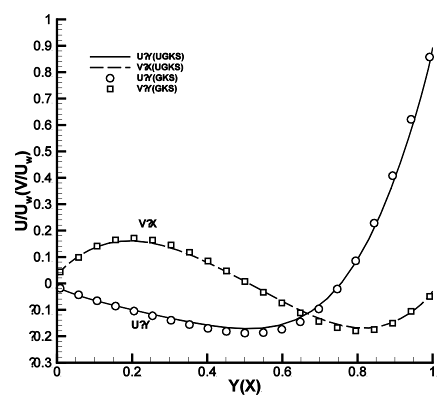
Re=10, Kn=0.0142

UGKS



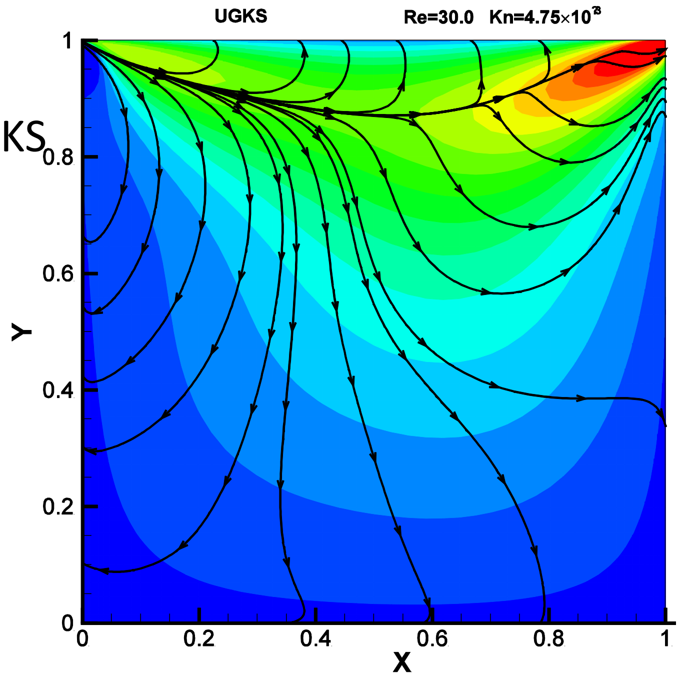


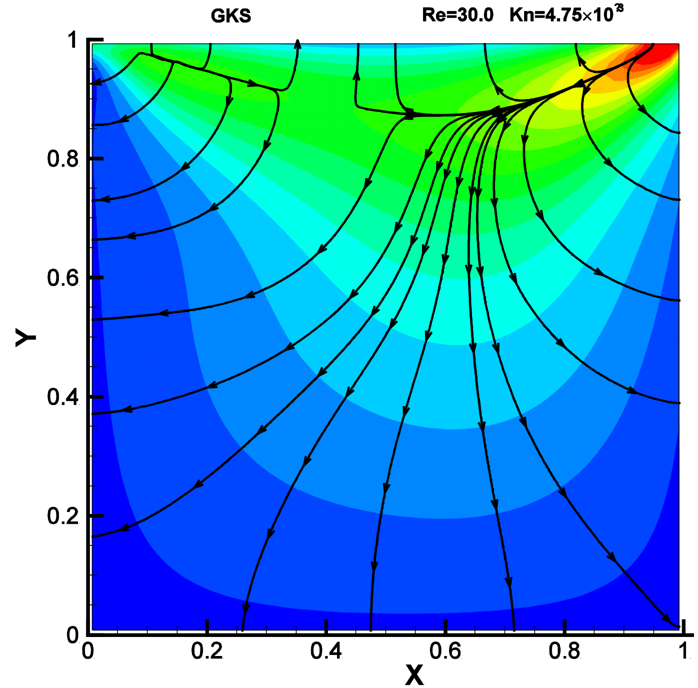
NS



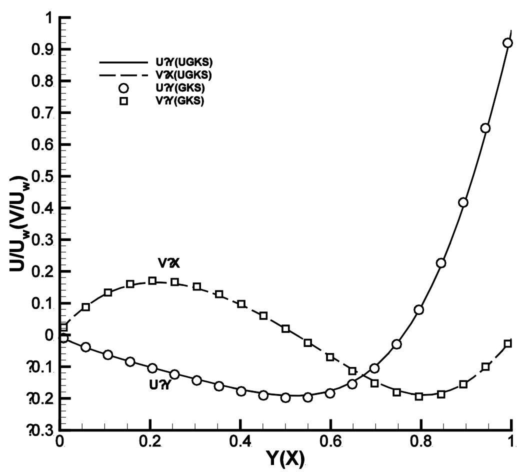
Re=30, Kn=0.00475

UGKS



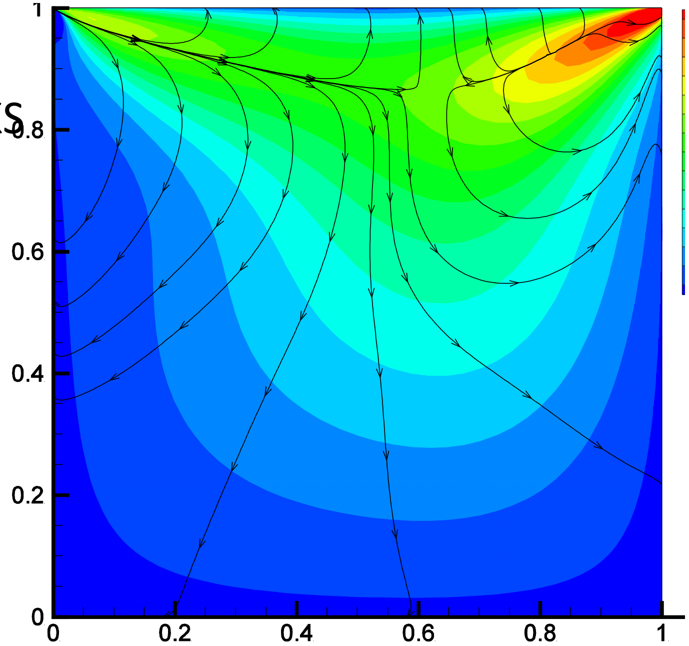


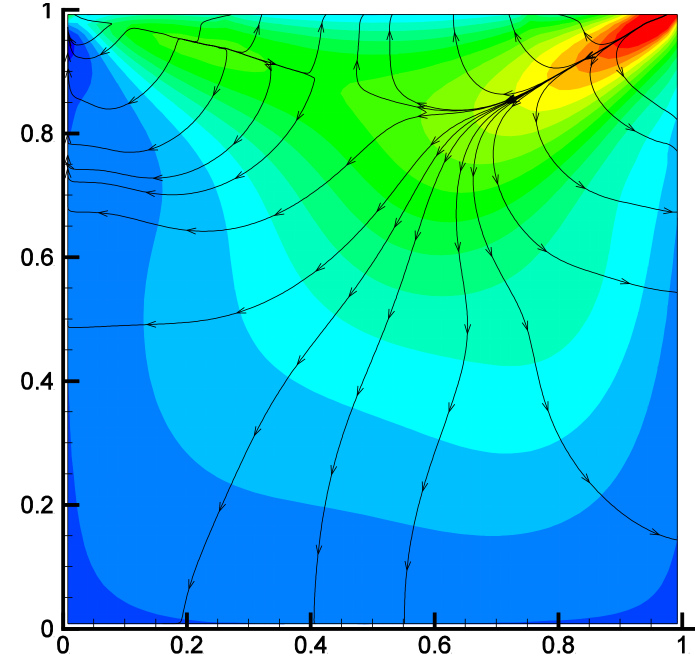
NS



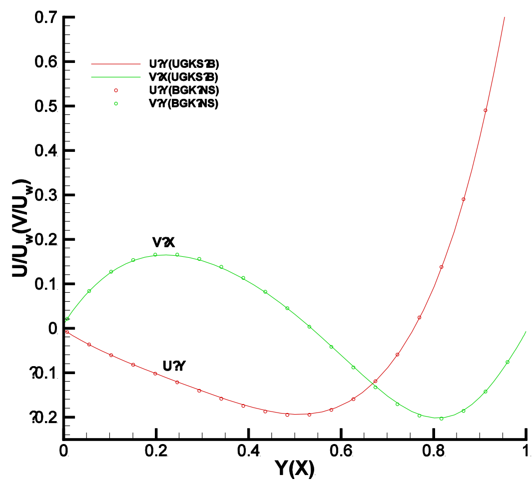
Re=50, Kn=0.00285

UGKS

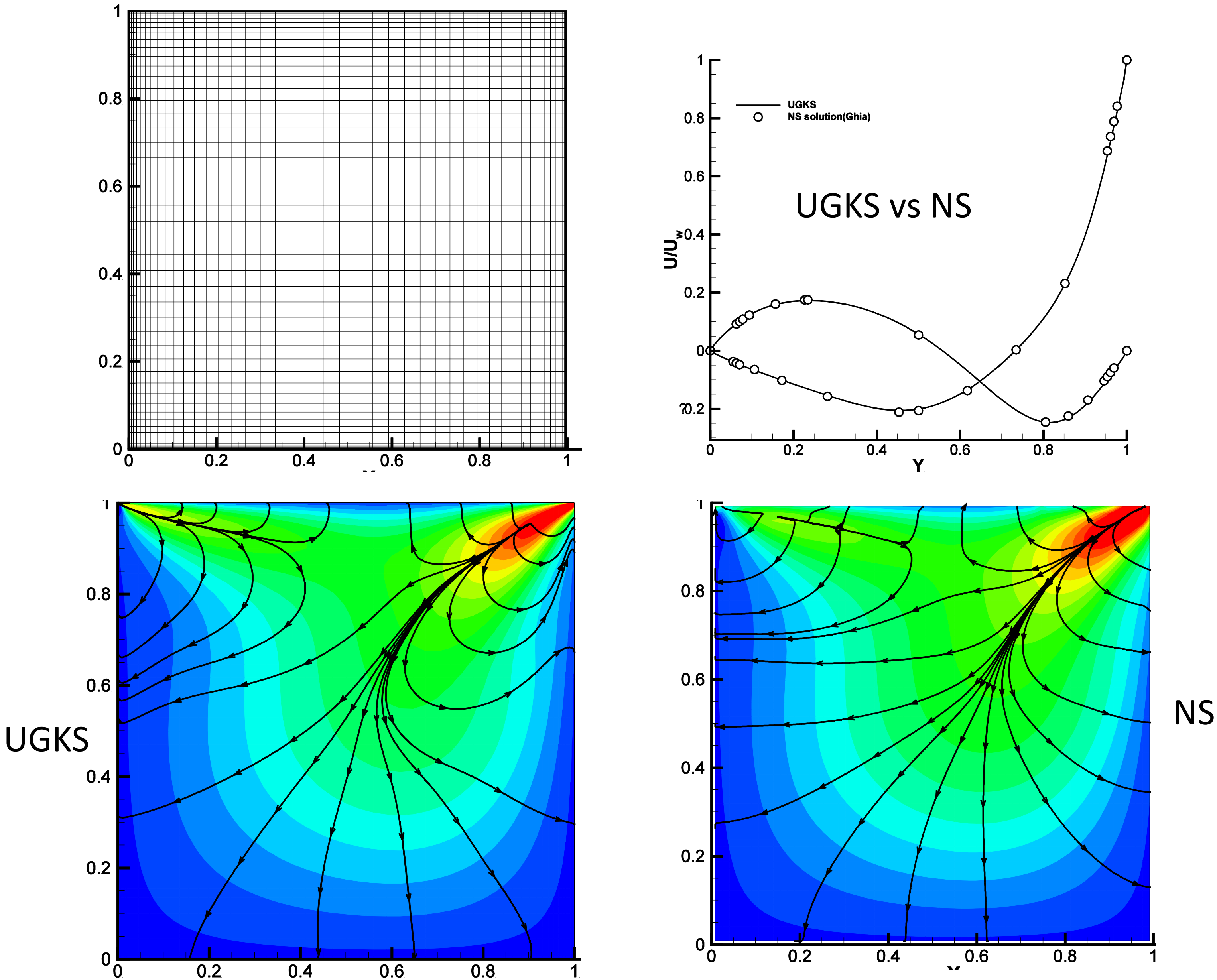




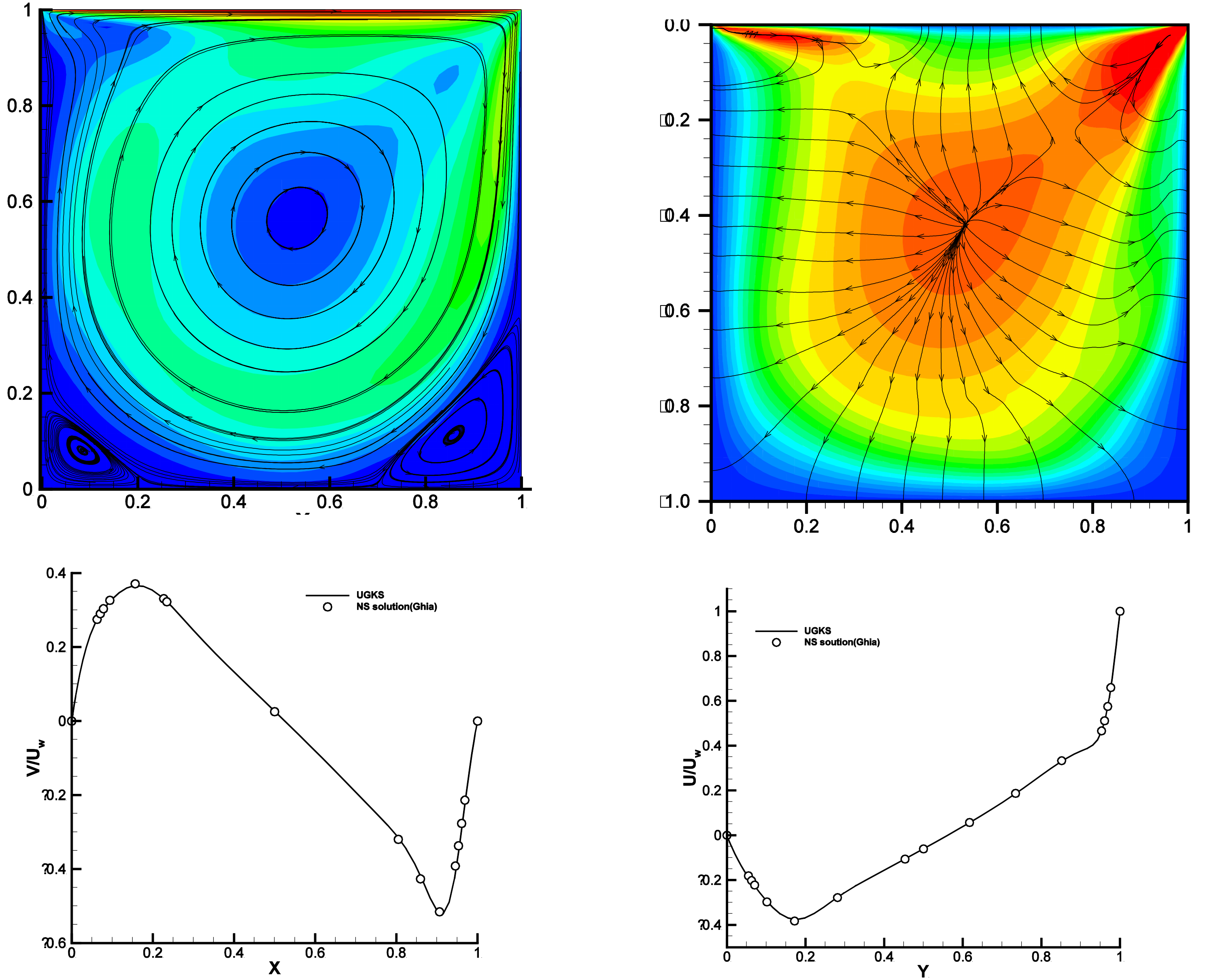
NS



Re=100, Kn=0.00142



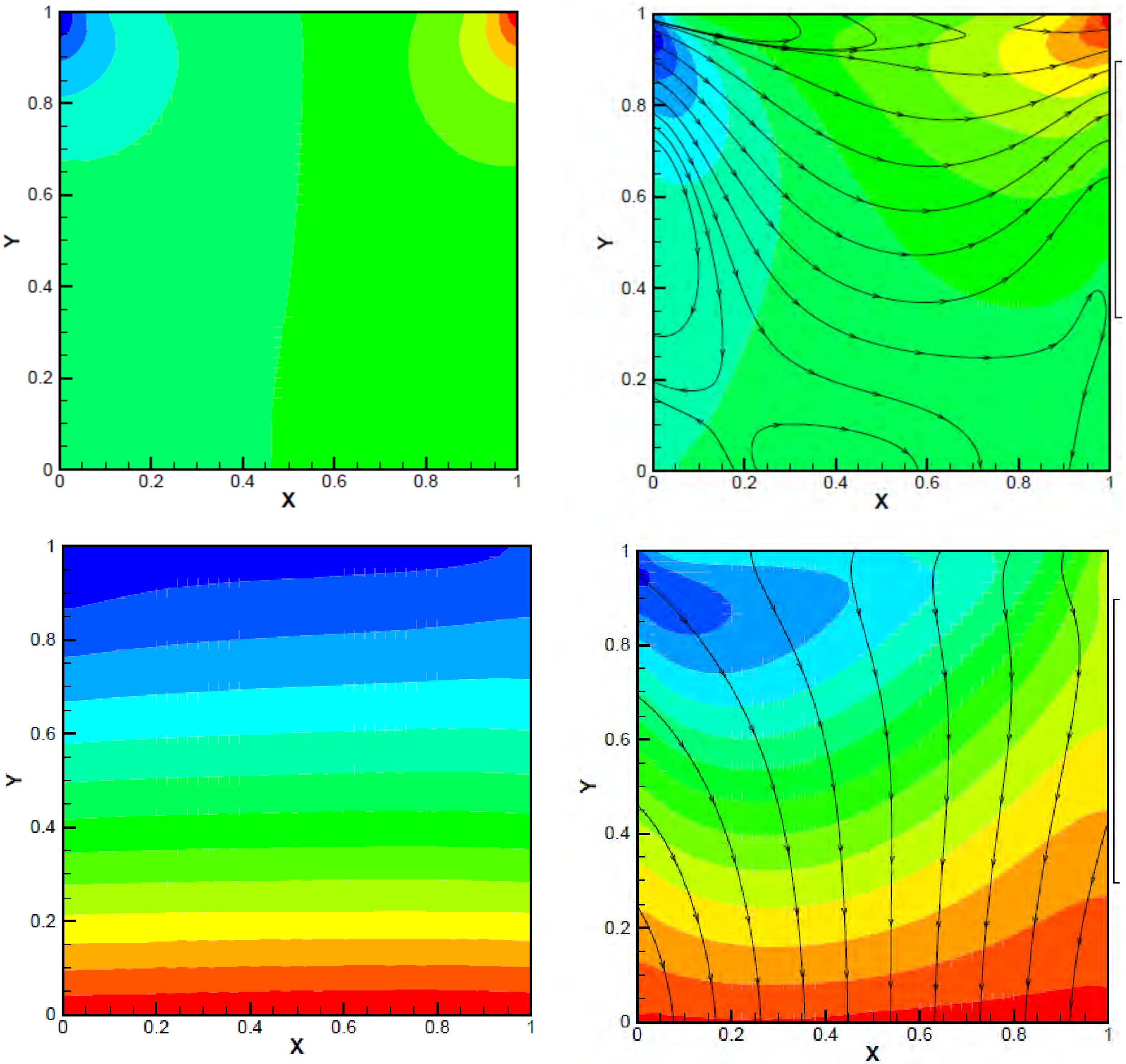
UGKS solutions: Re=1000,Kn=0.000142



UGKS with gravity: $$f_t + u_i f_{x_i} + \phi_i f_{u_i} = Q(f,f)$$

Without gravity

Gravity $\phi_y = -1.0$



UGKS vs DSMC

UGKS

1. Time step determined by the CFL condition
2. No limitation on the mesh size
3. No statistical noise
4. Suitable for all Knudsen regimes
5. Unsteady flow simulation
6. High efficiency for low speed flow
7. Massive memory requirement for hypersonic flow
8. Implicit and multigrid can accelerate the computation greatly
9. Validated in the past 5 years and the validation and development will be continuously conducted

DSMC

1. Time step should be less than the particle collision time
2. Cell size should be less than the particle mean free path
3. Significant statistical noise
4. Suitable in rarefied regime and high speed
5. Validating in the past 5 decades!

推向实战：

单位代码	90113
密　　级	公开
学　　号	

CARDC　博士学位论文

基于模型方程解析解的气体动理学算法研究

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导　　师：　邓小刚　院　士

副 导 师：　毛枚良　研究员

学科专业：　力学、流体力学

研究方向：　气体动理学方法

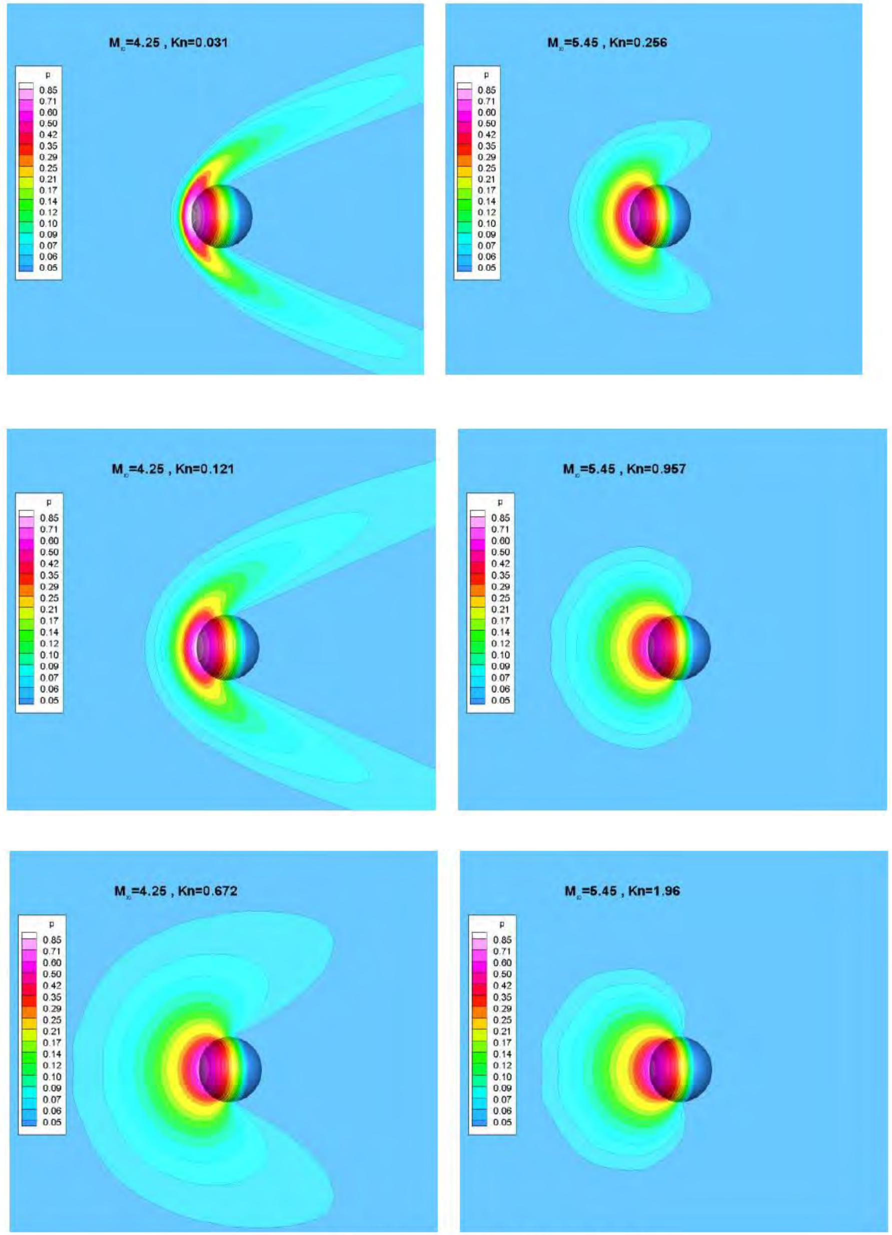


图 6.1　圆球算例，对称面及固壁压力分布

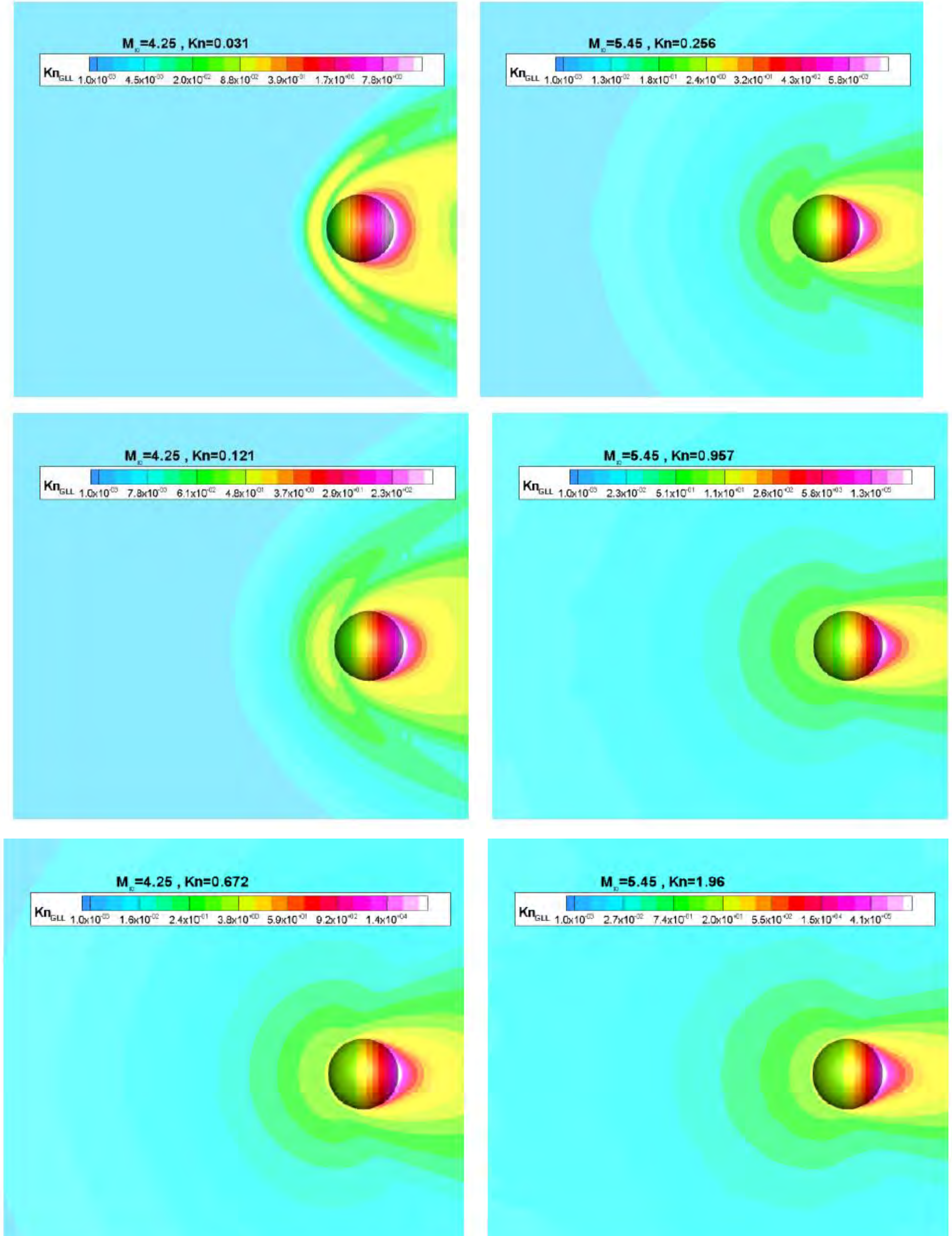


图 6.4　圆球算例，对称面及固壁局部努森数分布

表 6.1　圆球阻力系数比较（UGKS.vs.EXP）

M_∞	Re	Kn_∞	UGKS（Nitrogen）	Exp（Air）	偏差（%）
4.25	9.55	0.672	2.356	2.42	2.64%
4.25	19.0	0.338	2.101	2.12	0.87%
4.25	53.0	0.121	1.694	1.69	-0.27%
4.25	80.5	0.080	1.558	1.53	-1.80%
4.25	150.0	0.043	1.410	1.37	-2.91%
4.25	210.0	0.031	1.355	1.35	-0.39%
5.45	4.2	1.960	2.595	2.60	0.18%
5.45	8.6	0.957	2.449	2.44	-0.36%
5.45	16.8	0.490	2.248	2.28	1.41%
5.45	32.1	0.256	2.005	2.04	1.71%

表 6.2　圆球阻力系数比较（NS.vs.EXP）

M_∞	Re	Kn_∞	NS+noslip		NS+slip	
			阻力值	偏差（%）	阻力值	偏差（%）
4.25	9.55	0.672	7.400	205.77%	4.774	97.26%
4.25	19.0	0.338	4.792	126.02%	3.177	49.87%
4.25	53.0	0.121	2.826	67.23%	2.015	19.20%
4.25	80.5	0.080	2.379	55.47%	1.775	16.00%
4.25	150.0	0.043	1.921	40.21%	1.543	12.63%
4.25	210.0	0.031	1.743	29.09%	1.455	7.78%

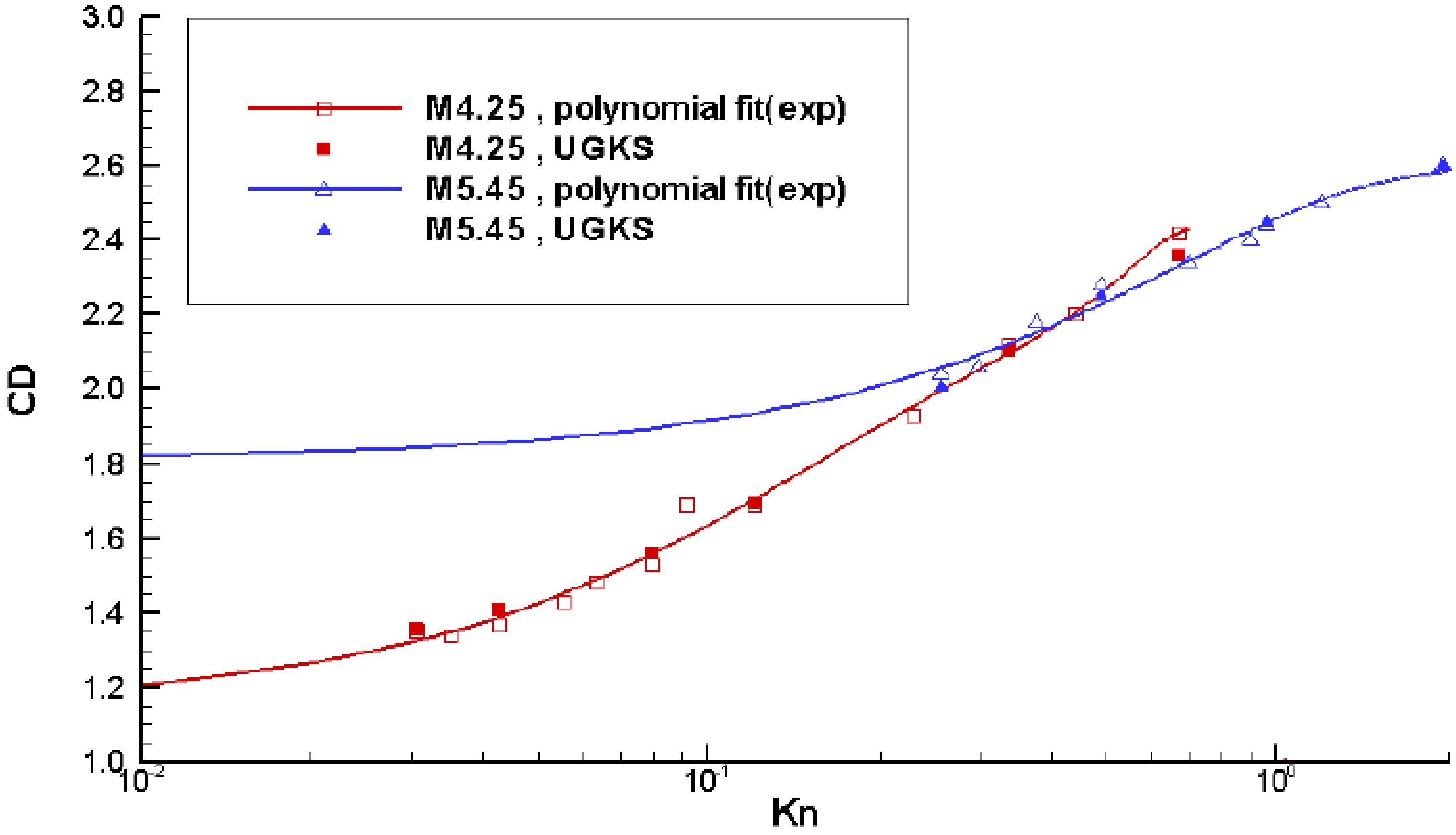

M4.25 , polynomial fit(exp)
M4.25 , UGKS
M5.45 , polynomial fit(exp)
M5.45 , UGKS
CD
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
10^{-2}
10^{-1}
10^{0}
Kn

图 6.19　圆球阻力比较

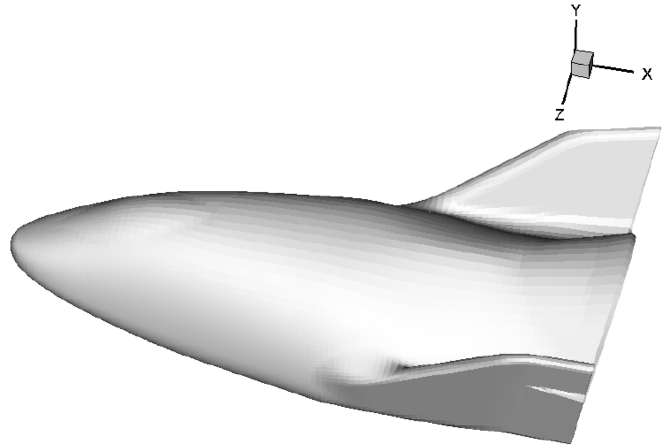
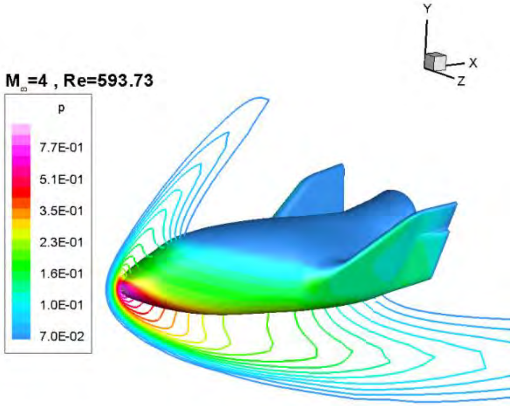
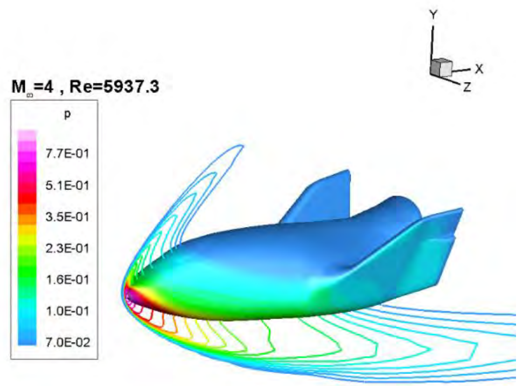
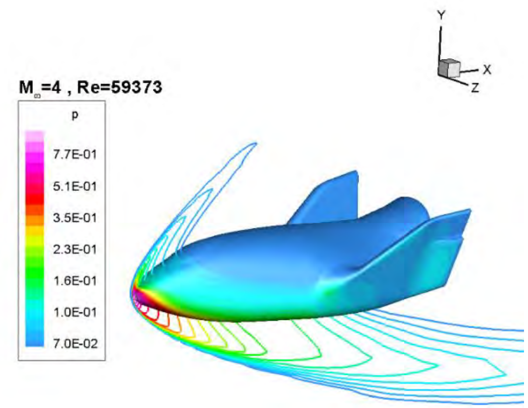


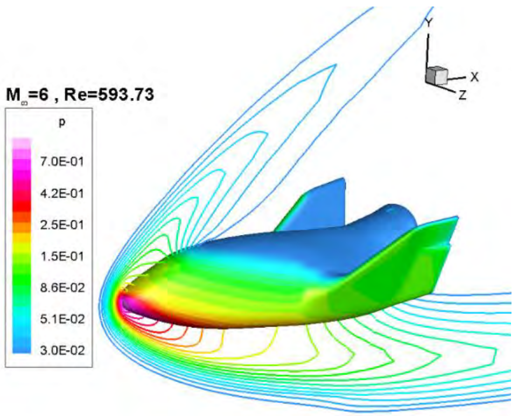
表 6.3　X38 力系数比较

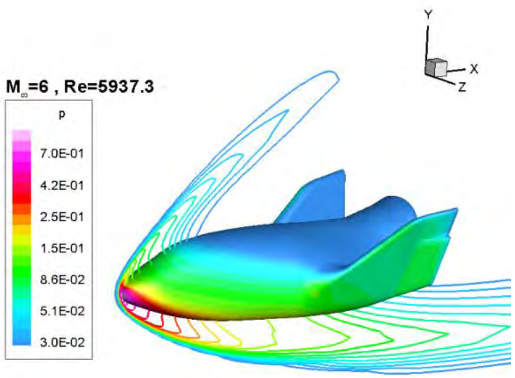
M∞	Re	Kn∞	来流介质	轴向力	法向力	俯仰力矩	升力	阻力
4	593.73	8.41E-03	Argon	0.3991	0.3929	0.1664	0.2327	0.5094
4	5937.3	8.41E-04		0.1956	0.3146	0.1419	0.2288	0.2914
4	59373	8.41E-05		0.1150	0.2821	0.1293	0.2257	0.2045
6	593.73	1.26E-02		0.4179	0.3728	0.1536	0.2074	0.5202
6	5937.3	1.26E-03		0.1955	0.2887	0.1255	0.2045	0.2825
6	59373	1.26E-04		0.1103	0.2550	0.1126	0.2019	0.1908
4	593.73	8.24E-03	Nitrogen	0.4176	0.4015	0.1695	0.2345	0.5298
6	593.73	1.24E-02		0.4330	0.3765	0.1539	0.2057	0.5357

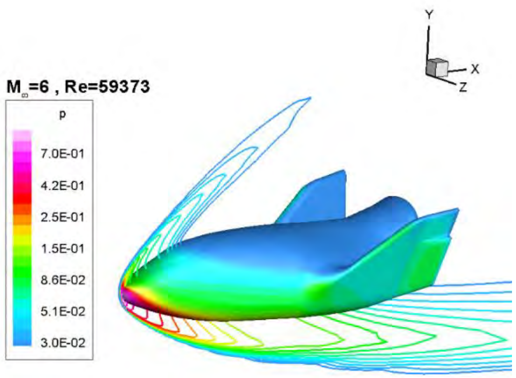

M∞=4 , Re=593.73
p
7.7E-01
5.1E-01
3.5E-01
2.3E-01
1.6E-01
1.0E-01
7.0E-02
Y
X
Z

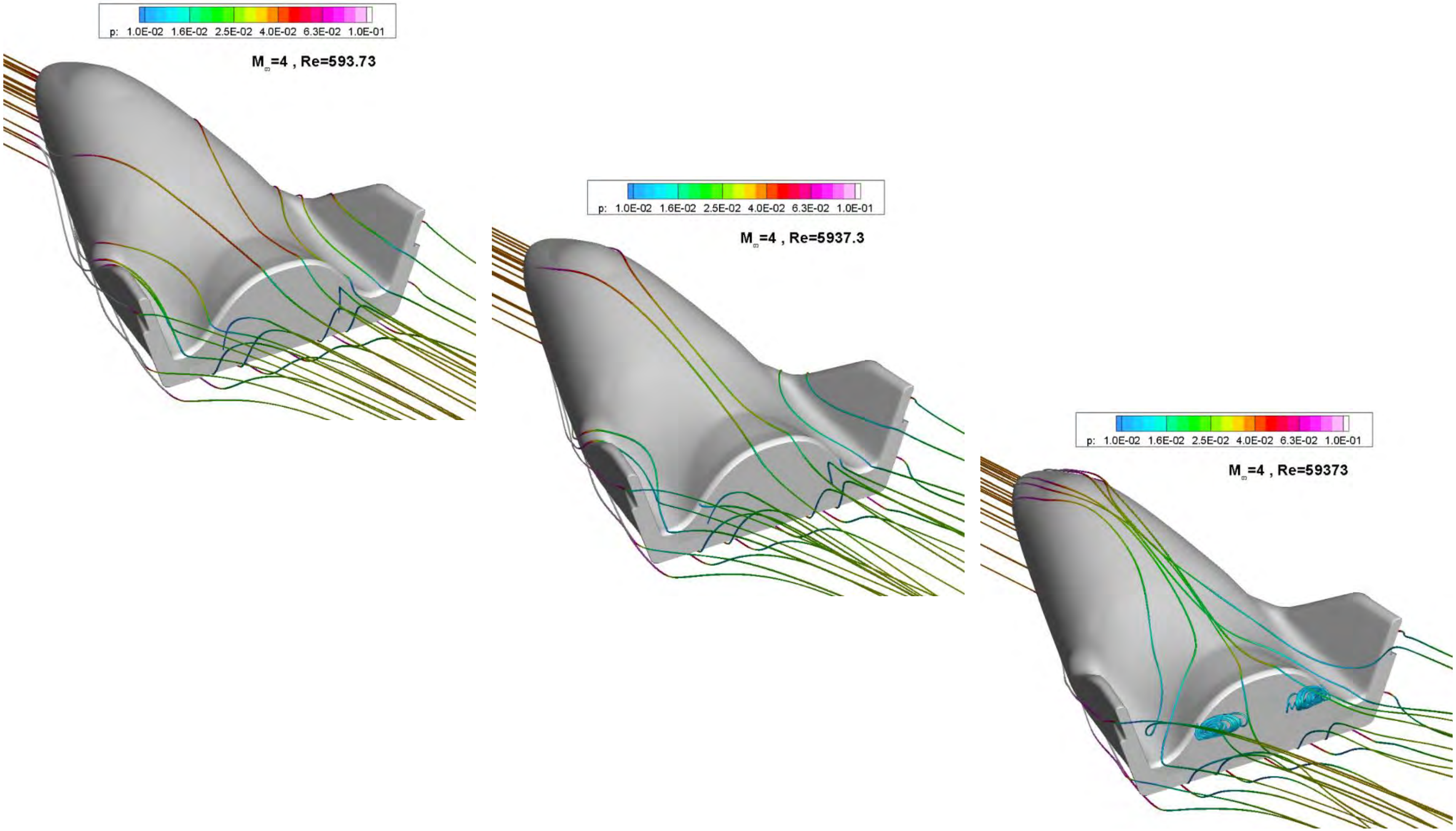

M∞=4 , Re=5937.3
p
7.7E-01
5.1E-01
3.5E-01
2.3E-01
1.6E-01
1.0E-01
7.0E-02
Y
X
Z


M∞=4 , Re=59373
p
7.7E-01
5.1E-01
3.5E-01
2.3E-01
1.6E-01
1.0E-01
7.0E-02
Y
X
Z

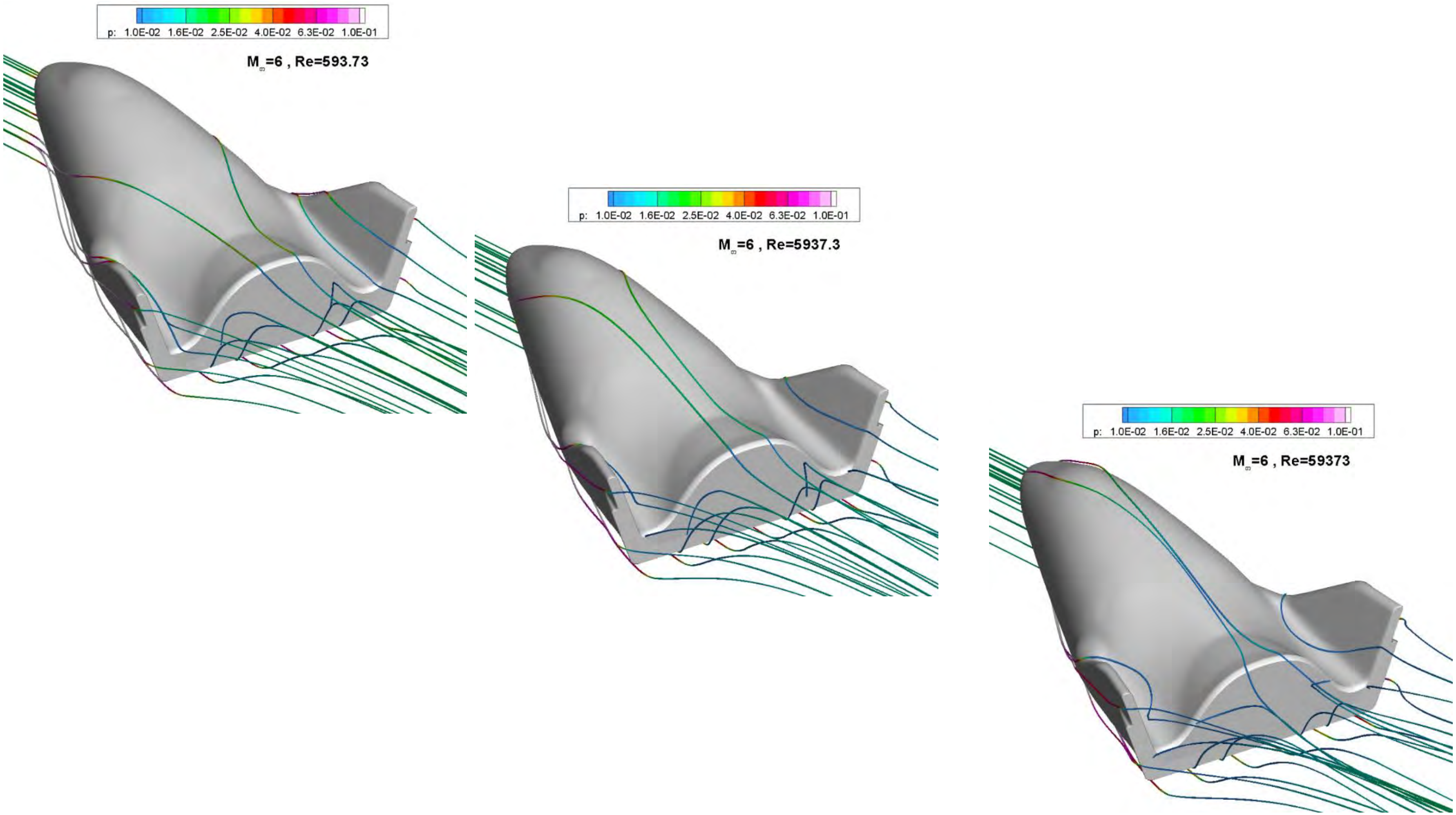

M∞=6 , Re=593.73
p
7.0E-01
4.2E-01
2.5E-01
1.5E-01
8.6E-02
5.1E-02
3.0E-02
Y
X
Z


M∞=6 , Re=5937.3
p
7.0E-01
4.2E-01
2.5E-01
1.5E-01
8.6E-02
5.1E-02
3.0E-02
Y
X
Z


M∞=6 , Re=59373
p
7.0E-01
4.2E-01
2.5E-01
1.5E-01
8.6E-02
5.1E-02
3.0E-02
Y
X
Z



p: 1.0E-02 1.6E-02 2.5E-02 4.0E-02 6.3E-02 1.0E-01
M_∞=4 , Re=593.73
p: 1.0E-02 1.6E-02 2.5E-02 4.0E-02 6.3E-02 1.0E-01
M_∞=4 , Re=5937.3
p: 1.0E-02 1.6E-02 2.5E-02 4.0E-02 6.3E-02 1.0E-01
M_∞=4 , Re=59373


p: 1.0E-02 1.6E-02 2.5E-02 4.0E-02 6.3E-02 1.0E-01
M∞=6 , Re=593.73
p: 1.0E-02 1.6E-02 2.5E-02 4.0E-02 6.3E-02 1.0E-01
M∞=6 , Re=5937.3
p: 1.0E-02 1.6E-02 2.5E-02 4.0E-02 6.3E-02 1.0E-01
M∞=6 , Re=59373

Modeling and Computation in Multiscale Transport Processes

- Rarefied Flow
- Radiative and Neutron Transport
- Plasma Physics
- Non-equilibrium Thermodynamics

Multiple frequency radiative transfer equations

$$\begin{cases} \frac{\epsilon^2}{c}\frac{\partial I}{\partial t} + \epsilon\vec{\Omega}\cdot\nabla I = \sigma(B(\nu,T) - I), \\ \epsilon^2 C_V \frac{\partial T}{\partial t} \equiv \epsilon^2 \frac{\partial U}{\partial t} = \int_{4\pi}\int_0^\infty \sigma\Big(I - B(\nu,T)\Big) d\nu d\vec{\Omega}. \end{cases}$$

$I(\vec{r}, \vec{\Omega}, t)$: Radiation intensity, $T(\vec{r}, t)$: Material temperature,
$\sigma(\vec{r}, T)$: Opacity, a: Radiation constant, c: Speed of light,
$\epsilon > 0$: Knudsen number, $C_v(\vec{r}, t)$: Specific heat,
$\vec{r}$: Spatial variable, $\vec{\Omega}$: Angular variable, t: Time variable.

$$B(\nu,T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1}$$

W.J. Sun, S. Jiang, S. Li

Macroscopic equations

$$\begin{cases} \frac{\epsilon^2}{c}\frac{\partial \rho}{\partial t} + \epsilon \nabla \cdot < \vec{\Omega} I > = \sigma(\phi - \rho), \qquad < \vec{\Omega} I > := \int \vec{\Omega} I d\vec{\Omega}, \\ \epsilon^2 \frac{\partial \phi}{\partial t} = \beta\sigma(\rho - \phi). \end{cases}$$

Evaluate macroscopic variables first

$$\rho_{i,j}^{n+1} = \rho_{i,j}^{n} + \frac{\Delta t}{\Delta x}(\Phi_{i-1/2,j} - \Phi_{i+1/2,j}) + \frac{\Delta t}{\Delta y}(\Psi_{i,j-1/2} - \Psi_{i,j+1/2}) + \frac{\sigma c \Delta t}{\epsilon^2}(\phi_{i,j}^{n+1} - \rho_{i,j}^{n+1}),$$

$$\phi_{i,j}^{n+1} = \phi_{i,j}^{n} + \frac{\beta\sigma\Delta t}{\epsilon^2}(\rho_{i,j}^{n+1} - \phi_{i,j}^{n+1}),$$

Interface radiation intensity

$$I_m(t, x_{i-1/2}, y_j, \mu_m, \xi_m) = e^{-\nu_{i-1/2,j}(t-t^n)} I_{m,0}(x_{i-1/2} - \tfrac{c\mu_m}{\epsilon}(t - t^n)) + \int_{t^n}^{t} e^{-\nu_{i-1/2,j}(t-s)} \tfrac{c\sigma_{i-1/2,j}}{2\pi\epsilon^2} \phi(s, x_{i-1/2} - \tfrac{c\mu_m}{\epsilon}(t-s)) ds,$$

Microscopic equation for the update of radiation intensity

$$I_{i,j,m,g}^{n+1} = I_{i,j,m,g}^{n} + \frac{\Delta t}{\Delta x}(F_{i-1/2,j,m,g} - F_{i+1/2,j,m,g}) + \frac{\Delta t}{\Delta y}(H_{i,j-1/2,m,g} - H_{i,j+1/2,m,g})$$
$$+ \frac{c\Delta t}{\epsilon^2}\big((\sigma_g^e)_{i,j}^{n+1}\phi_{g,i,j}^{n+1} - (\sigma_g^a)_{i,j}^{n+1} I_{i,j,m,g}^{n+1}\big),$$

Homogeneous problems

$$\sigma(x,\nu,T)=\frac{\sigma_0(x)}{(h\nu)^3\sqrt{kT}}$$

$\sigma_0 = 10\ \mathrm{keV}^{7/2}/\mathrm{cm},$

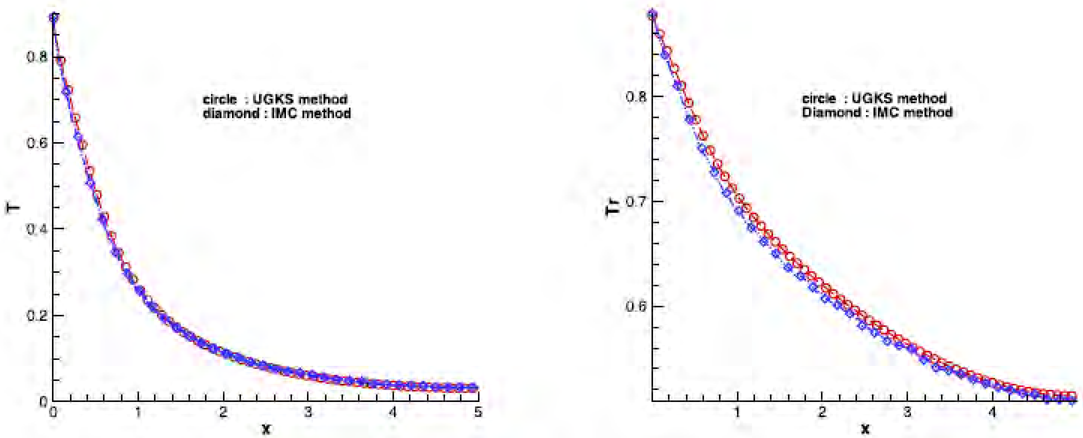


Fig. 2. Results at 1 ns for Case 1 of Example 1 with homogeneous opacity $\sigma_0 = 10\ \mathrm{keV}^{7/2}/\mathrm{cm}$.

$\sigma_0 = 100\ \mathrm{keV}^{7/2}/\mathrm{cm}$

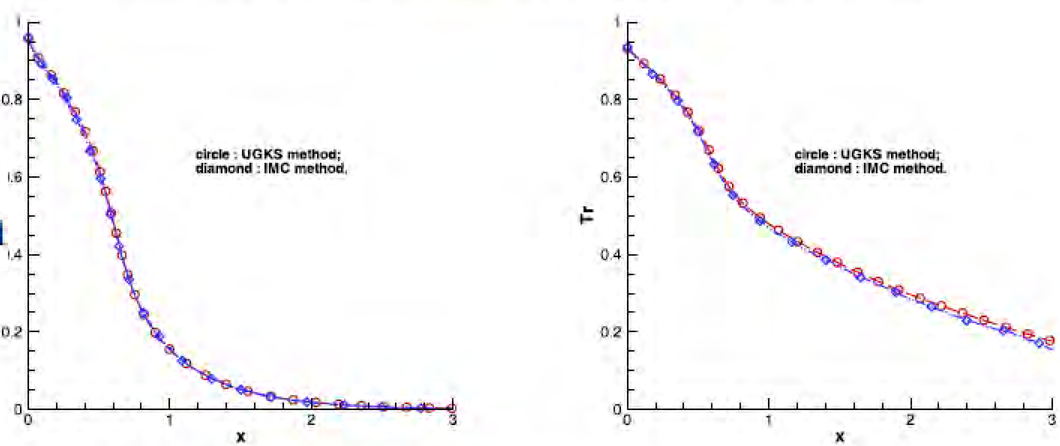


Fig. 3. Results at 1 ns for Case 1 of Example 1 with homogeneous opacity $\sigma_0 = 100\ \mathrm{keV}^{7/2}/\mathrm{cm}$.

$$\sigma_0 = 1000 \ \text{keV}^{7/2}/\text{cm}$$

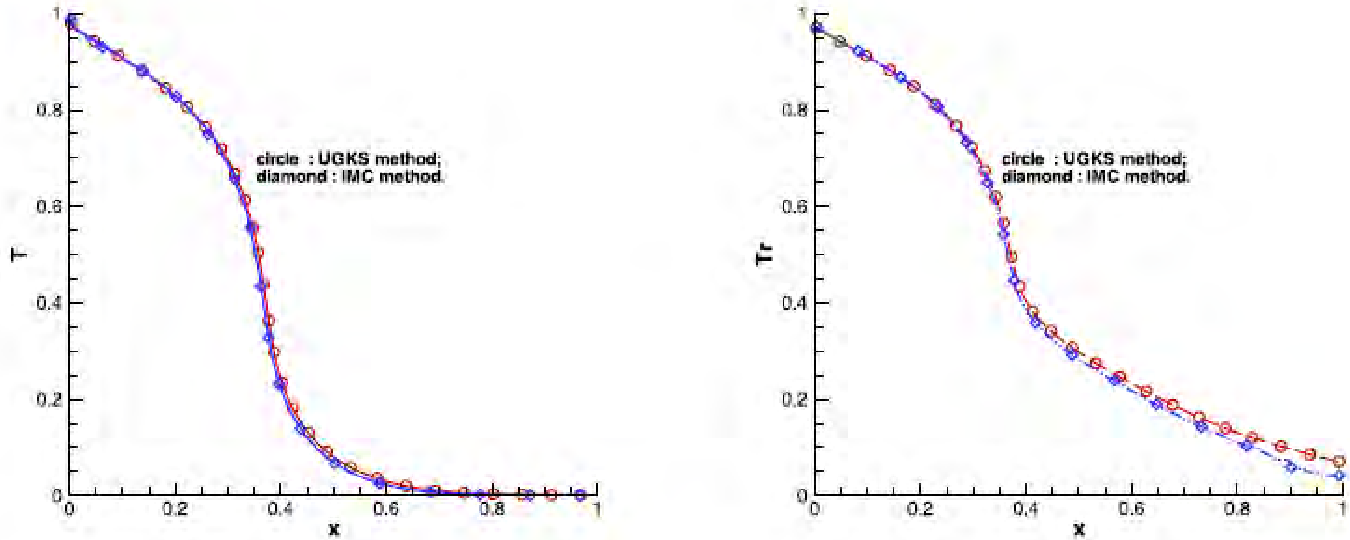
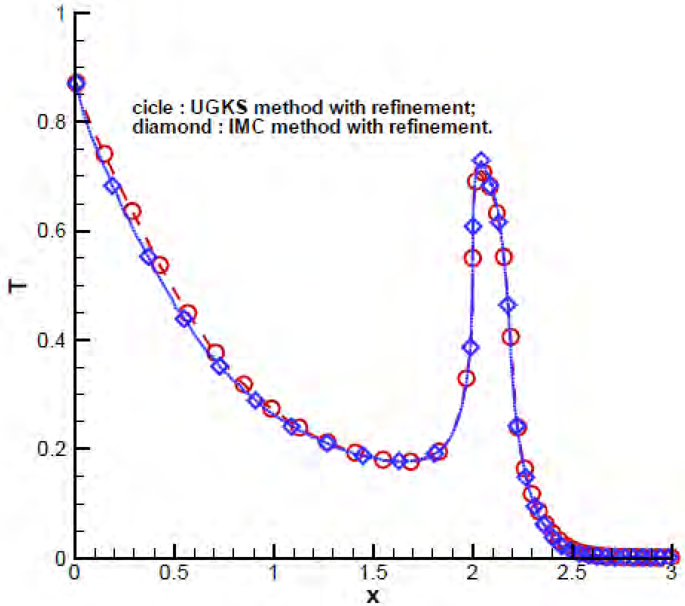
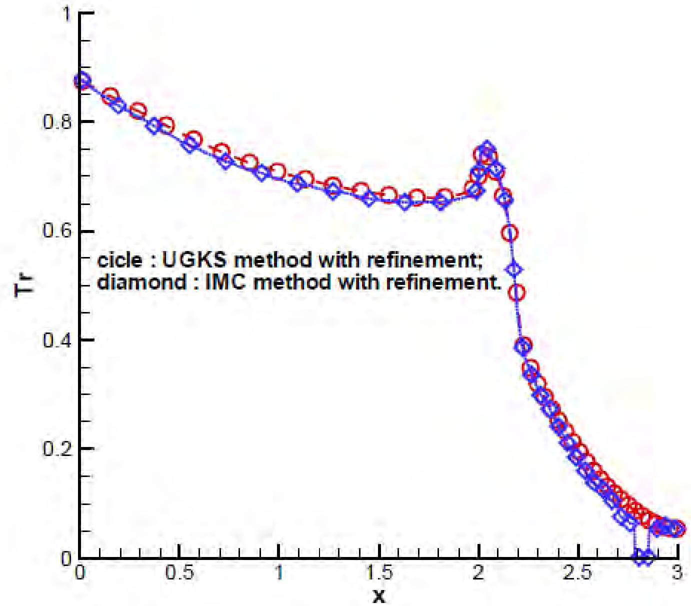


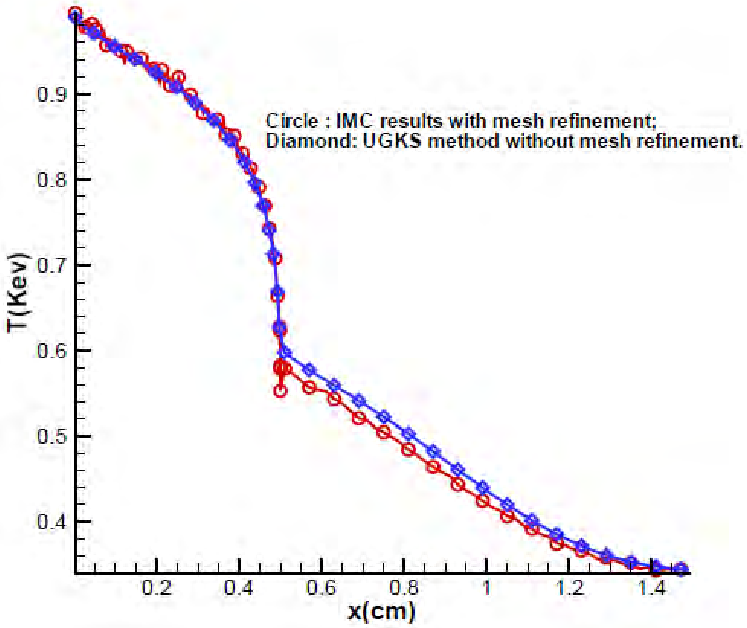
Fig. 4. Results at 1 ns for Case 1 of Example 1 with homogeneous opacity $\sigma_0 = 1000 \ \text{keV}^{7/2}/\text{cm}$.

$$\sigma_0(x) = \begin{cases} 10\,\mathrm{keV}^{7/2}/\mathrm{cm}, & 0\mathrm{cm} < x < 2\mathrm{cm}, \\ 1000\,\mathrm{keV}^{7/2}/\mathrm{cm}, & 2\mathrm{cm} < x < 3\mathrm{cm}. \end{cases}$$





$$\sigma_0(x) = \begin{cases} 1000\,\mathrm{keV}^{7/2}/\mathrm{cm}, & 0\mathrm{cm} < x < 0.5\mathrm{cm}, \\ 10\,\mathrm{keV}^{7/2}/\mathrm{cm}, & 0.5\mathrm{cm} < x < 1.5\mathrm{cm}. \end{cases}$$



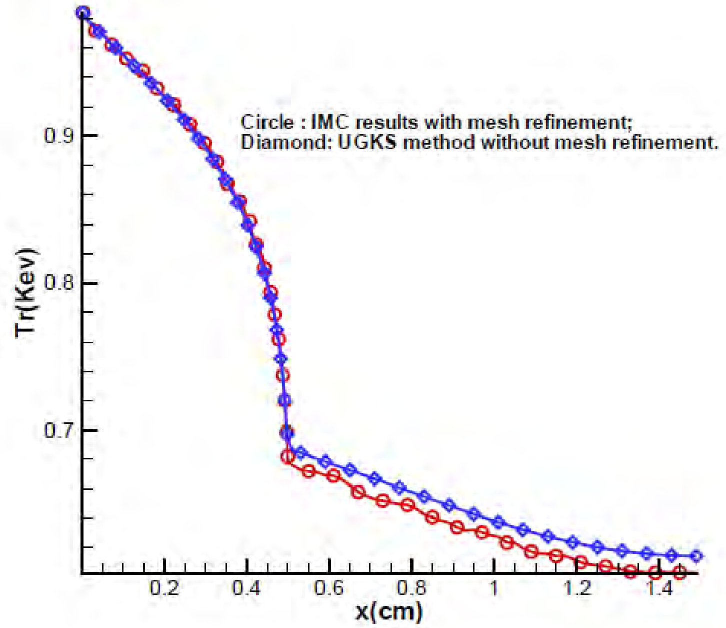


Table 1
The computation time of UGKS and IMC for Example 1.

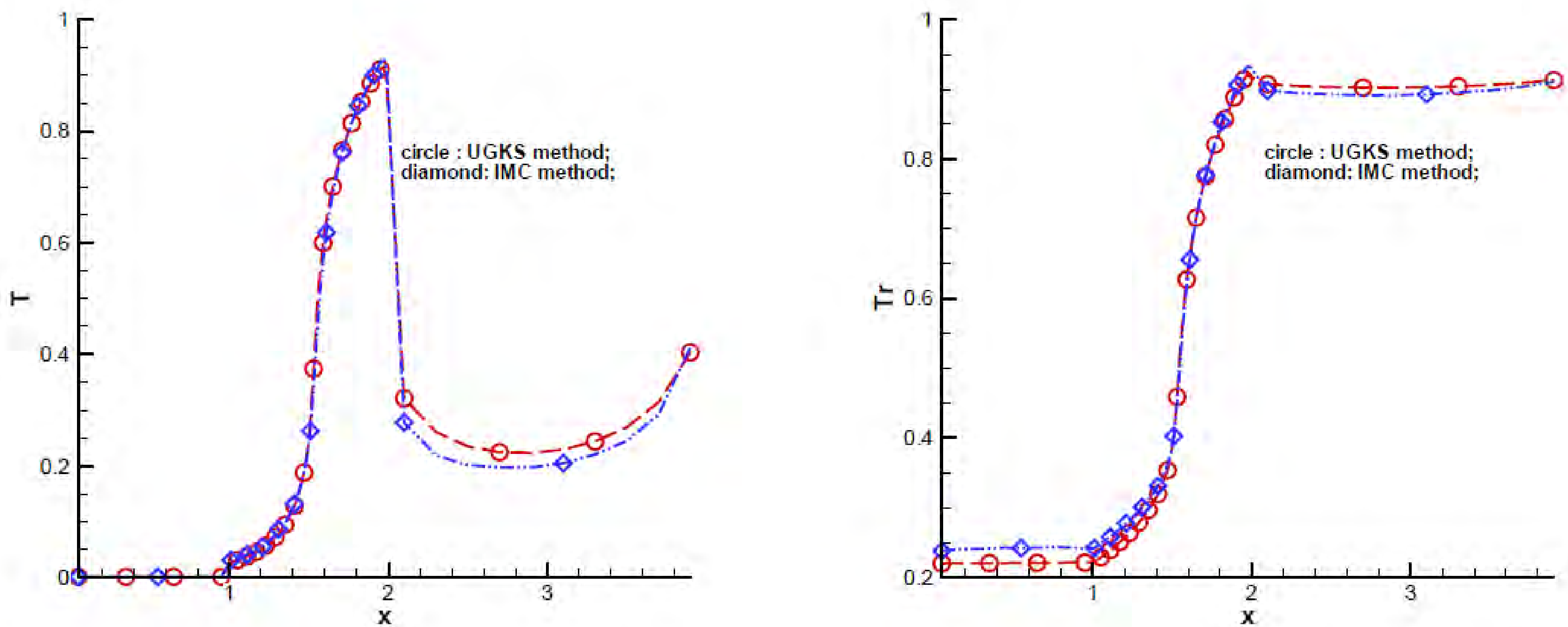
Example 1		UGKS	IMC
Case 1	$\sigma_0 = 10$	21 minutes	96 minutes
	$\sigma_0 = 100$	22 minutes	173 minutes
	$\sigma_0 = 1000$	40 minutes	1344 minutes
Case 2	1st heterogeneous problem	8 minutes	363 minutes
Case 3	2nd heterogeneous problem	34 minutes	5184 minutes

$$\sigma(\nu, T, x) = \gamma(x)\frac{1 - e^{-h\nu/kT}}{(h\nu)^3},$$

$$\gamma(x) = \begin{cases} 1\,\mathrm{keV}^3/cm, & 0cm < x < 1cm, \\ 1000\,\mathrm{keV}^3/cm, & 1cm < x < 2cm, \\ 1\,\mathrm{keV}^3/cm, & 2cm < x < 4cm. \end{cases}$$

Table 2
The computation time of UGKS and IMC for Example 2.

UGKS	IMC
2 minutes	63 minutes



Plasma Simulation

Vlasov-BGK system for ion and electron:

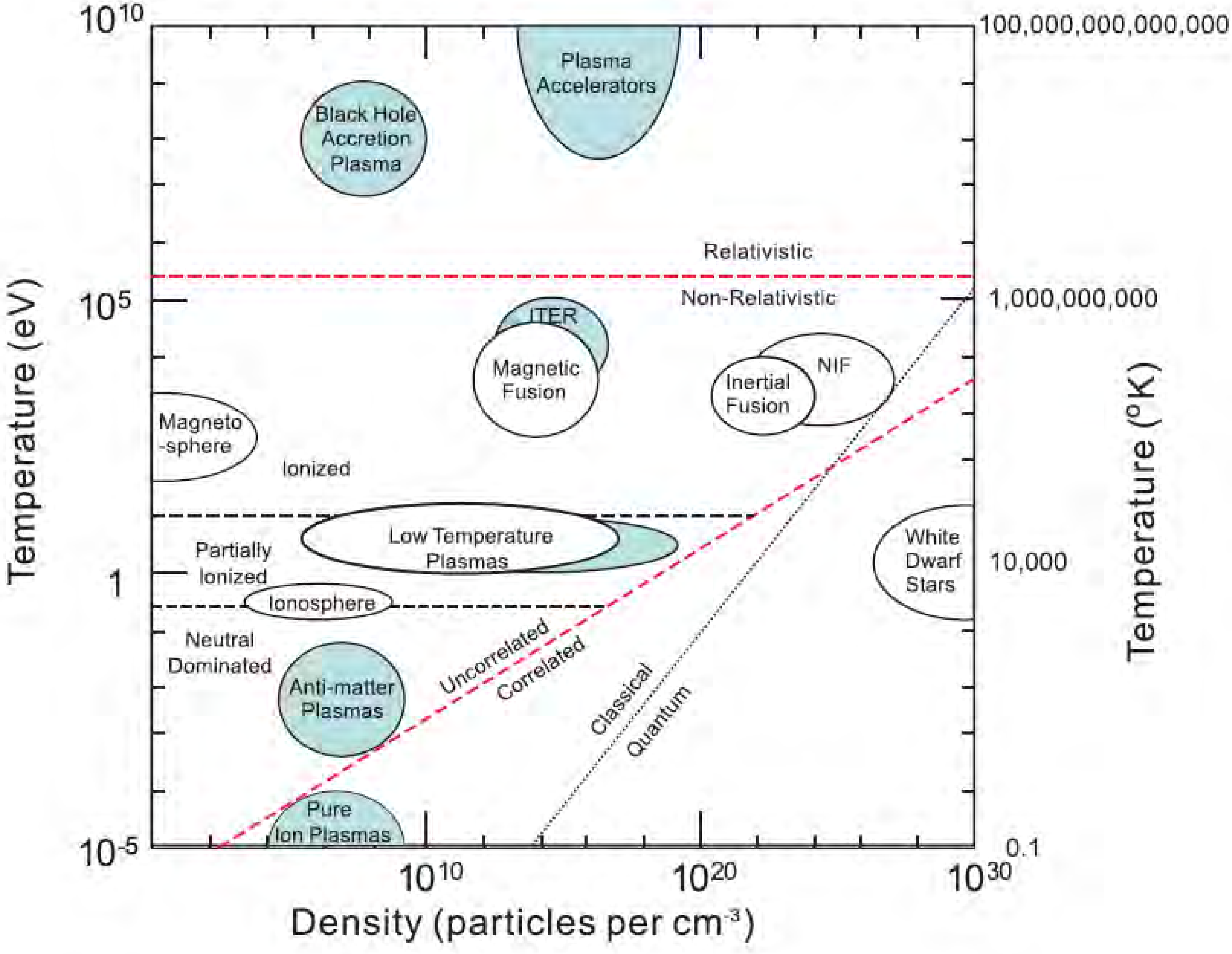
$$\frac{\partial f_\alpha}{\partial t} + v \cdot \nabla_x f_\alpha + \frac{q_\alpha}{m_\alpha}(E + v_\alpha \times B) \cdot \nabla_v f_\alpha = \sum_\beta Q_{\alpha,\beta}(f_\alpha, f_\beta)$$

Maxwell equations for electro-magnetic fields

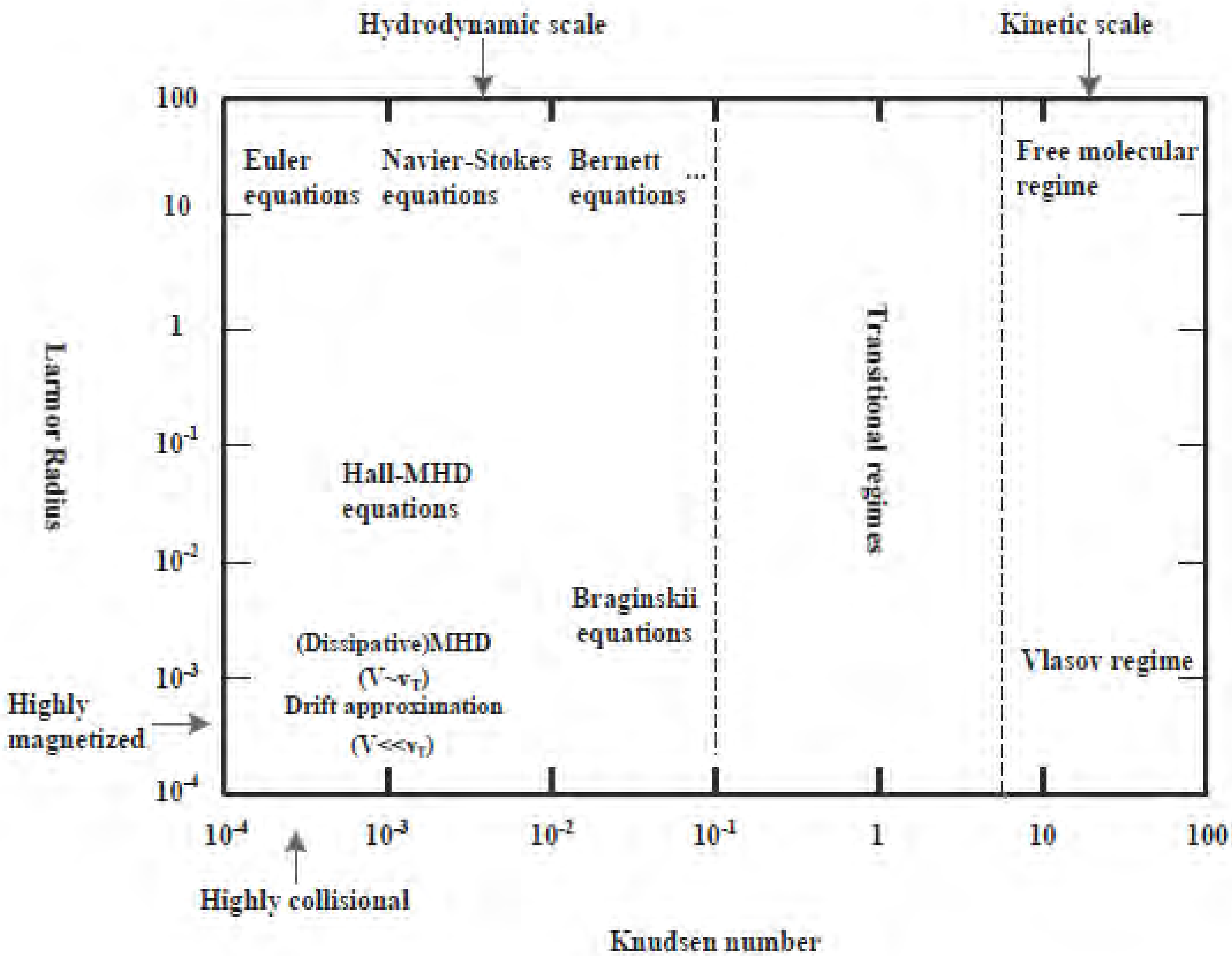
$$\frac{\partial B}{\partial t} = -\nabla \times E, \qquad \frac{\partial E}{\partial t} = c^2 \nabla \times B - \frac{1}{\mu_0} J$$

$$\nabla \cdot E = \frac{1}{\epsilon_0}(\rho_e - \rho_i), \qquad \nabla \cdot B = 0$$

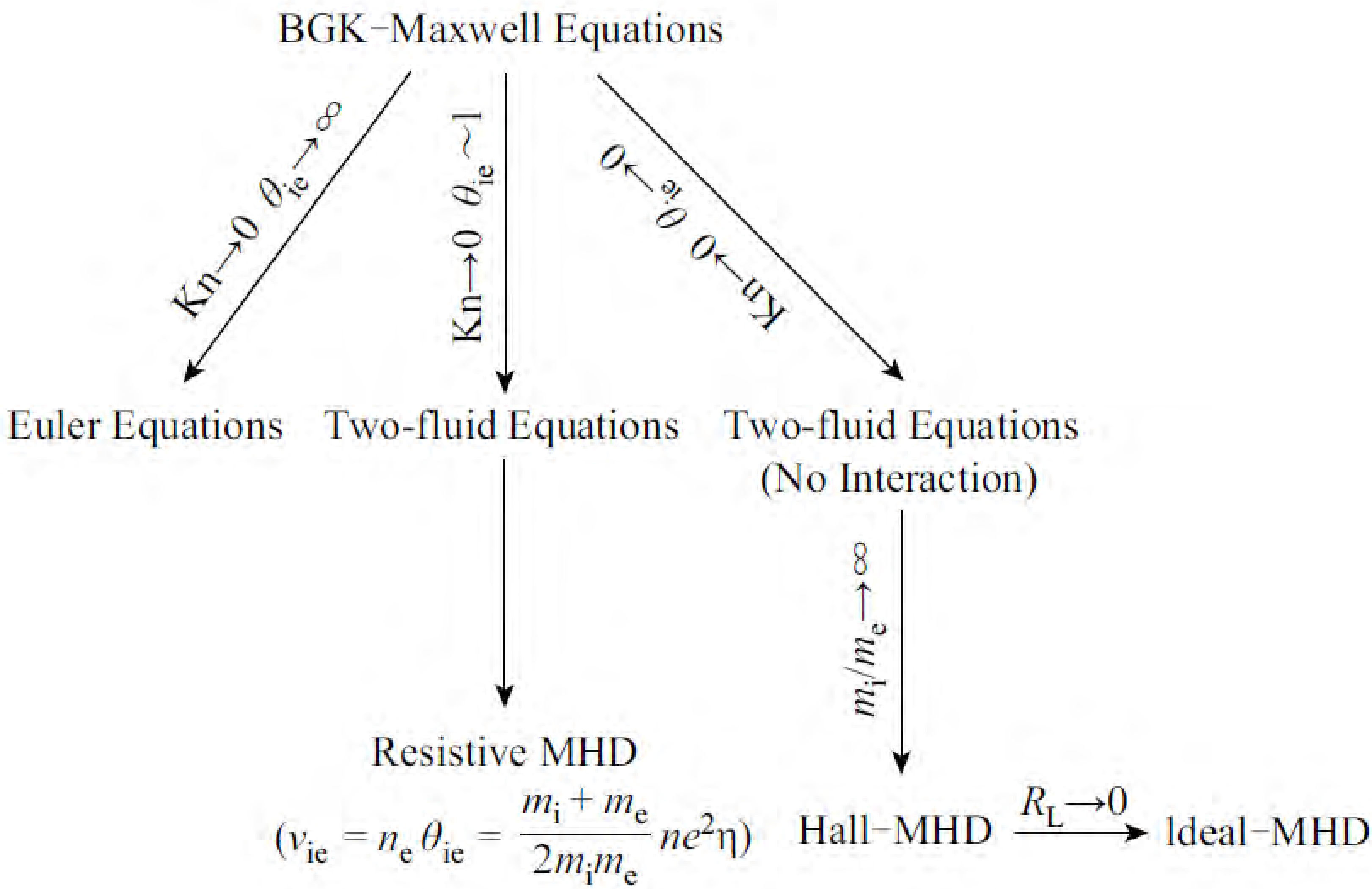
C. Liu



Plasma Accelerators
Black Hole Accretion Plasma
Relativistic
Non-Relativistic
ITER
Magnetic Fusion
NIF
Inertial Fusion
Magneto -sphere
Ionized
Low Temperature Plasmas
Partially Ionized
Ionosphere
White Dwarf Stars
Neutral Dominated
Uncorrelated
Correlated
Anti-matter Plasmas
Classical
Quantum
Pure Ion Plasmas
Temperature (eV)
10^10
10^5
1
10^-5
Temperature (°K)
100,000,000,000,000
1,000,000,000
10,000
0.1
10^10
10^20
10^30
Density (particles per cm^-3)



Hydrodynamic scale
Kinetic scale
100
10
1
10-1
10-2
10-3
10-4
Larmor Radius
Euler equations
Navier-Stokes equations
Bernett equations
...
Free molecular regime
Transitional regimes
Hall-MHD equations
Braginskii equations
(Dissipative)MHD
(V~vT)
Drift approximation
(V<<vT)
Vlasov regime
Highly magnetized
10-4
10-3
10-2
10-1
1
10
100
Highly collisional
Knudsen number


BGK−Maxwell Equations
Kn→0 θie→∞
Kn→0 θie~1
Kn→0 θie→0
Euler Equations
Two-fluid Equations
Two-fluid Equations
(No Interaction)
mi/me→∞
Resistive MHD
(vie = ne θie = (mi + me)/(2mime) ne²η)
Hall−MHD
RL→0
Ideal−MHD

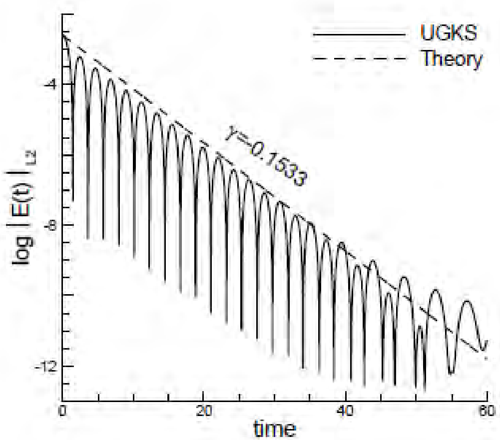
Linear Landau damping

Consider Landau damping for the Vlasov Poisson system with initial condition:

$$f_0(x,v) = \frac{1}{\sqrt{2\pi}}\left(1+\alpha\cos(kx)\right)\mathrm{e}^{-\frac{v^2}{2}}$$

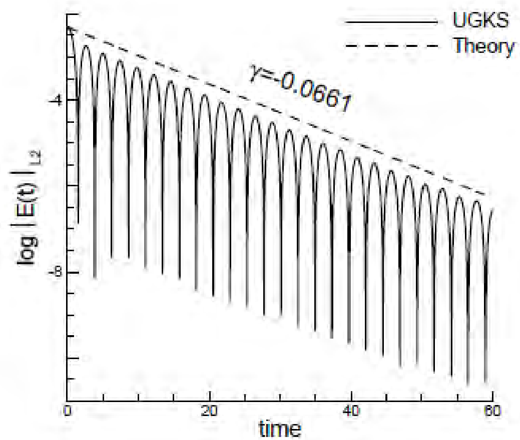
with $\alpha = 0.01$ for linear case. The length of the domain in the x direction is $L = \frac{2\pi}{k}$

$k = 0.5$



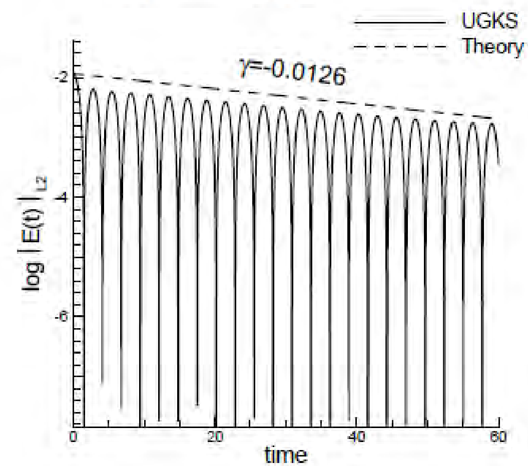
$\omega = 1.41$

$k = 0.4$



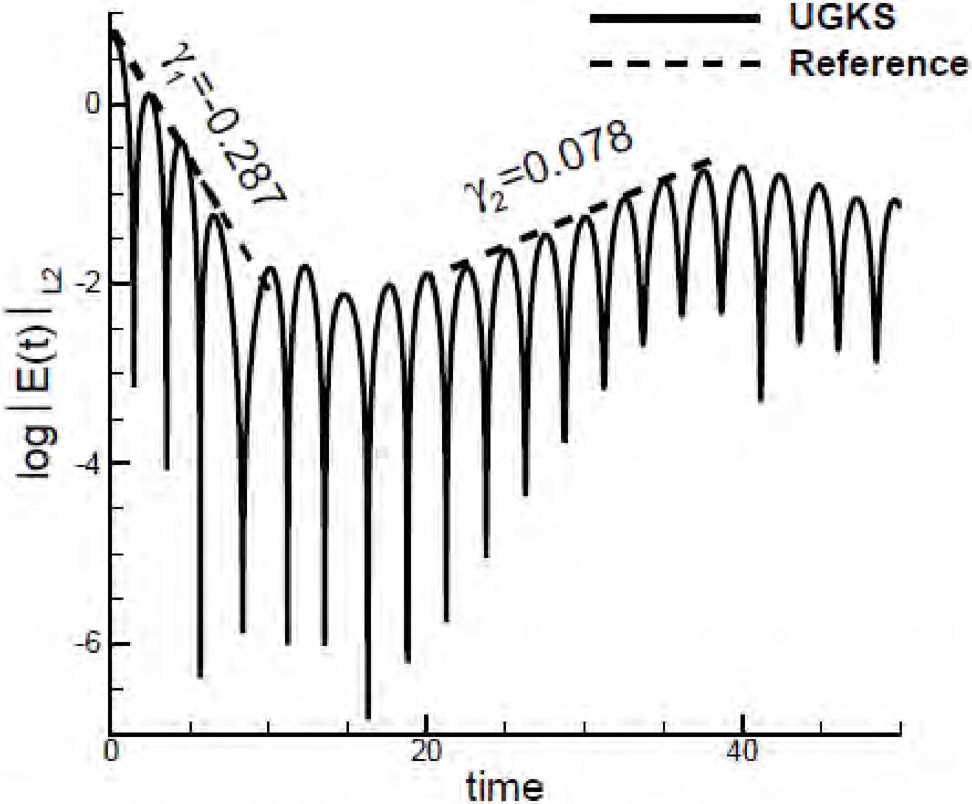
$\omega = 1.29$

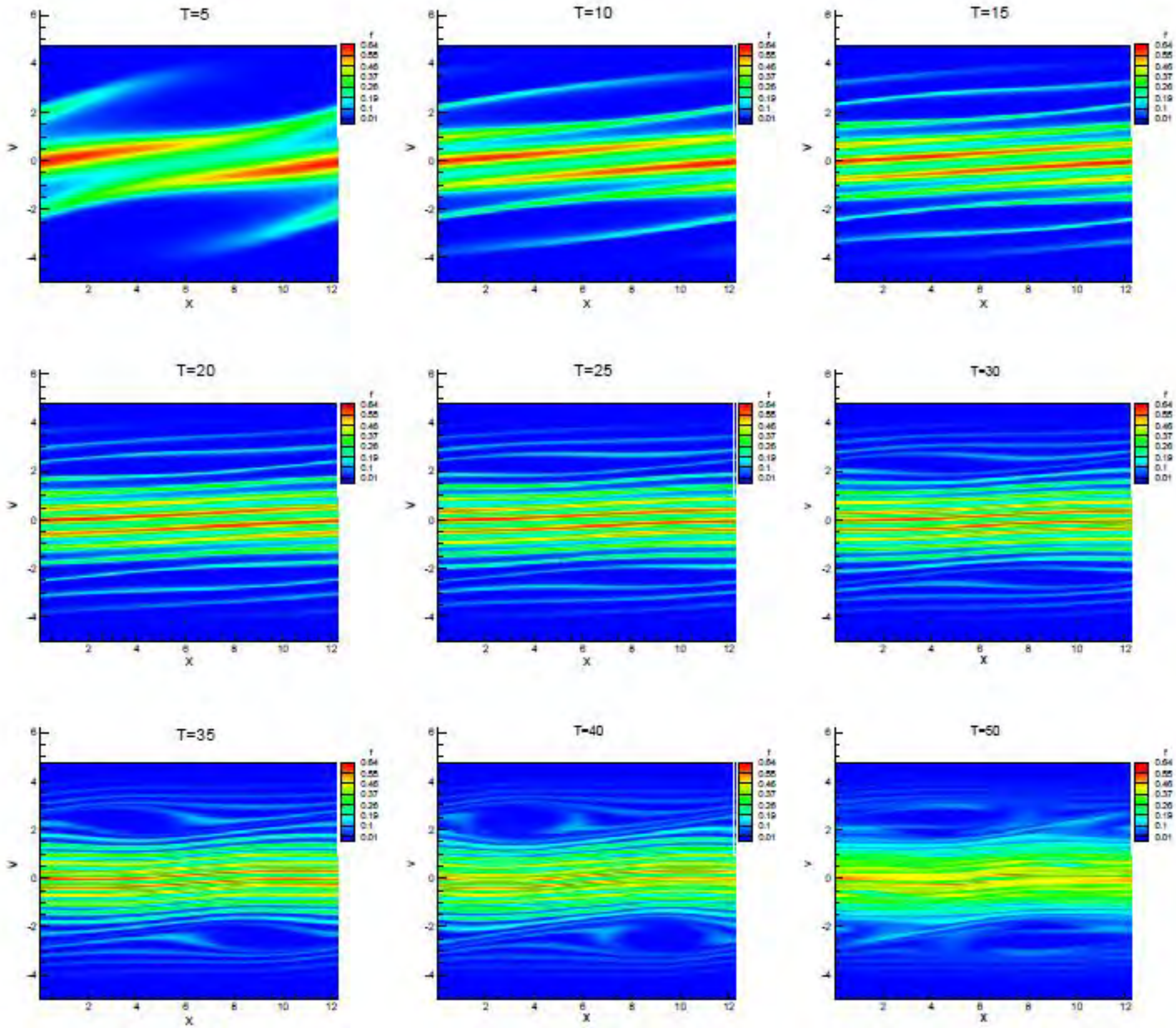
$k = 0.3$



$\omega = 1.16$

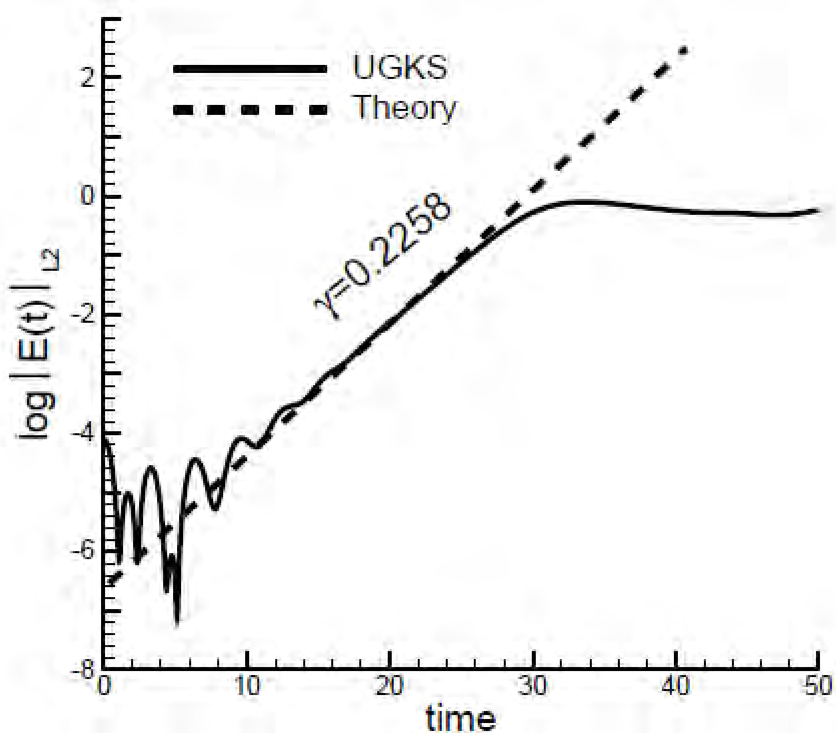
Nonlinear Landau Damping $\alpha = 0.5$ and $k = 0.5$




T=5
T=10
T=15
T=20
T=25
T=30
T=35
T=40
T=50
x
v
f

Linear two stream instability

$$f_0(x,v)=\frac{2}{7\sqrt{2\pi}}(1+5v^2)(1+\alpha((\cos(2kx)+\cos(3kx))/1.2+\cos(kx))\mathbf{e}^{-\frac{v^2}{2}}$$



$$\alpha=0.001,\ k=0.2$$

Velocity distribution contours at t=70

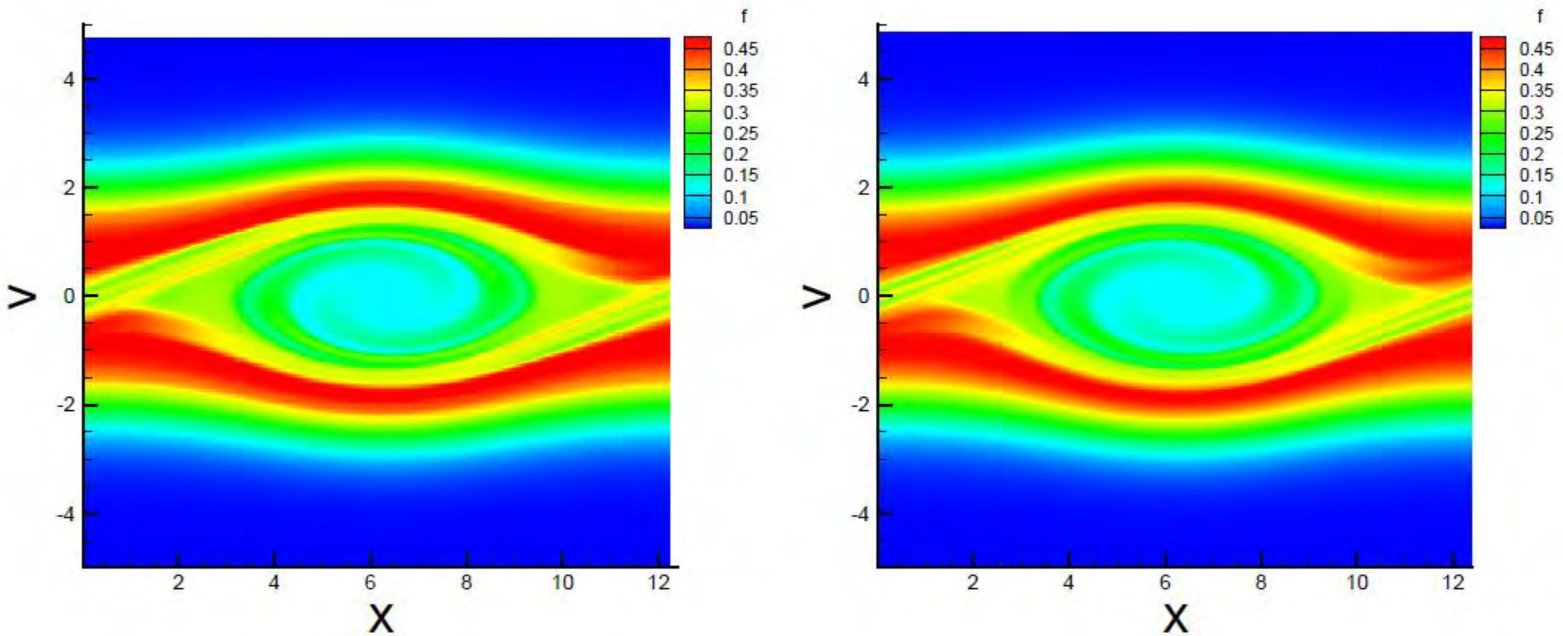


Figure: Linear two stream instability. Velocity distribution contours at $t = 70$. $N_x \times N_v = 256 \times 256$ (left); $N_x \times N_v = 512 \times 512$ (right).

Nonlinear two stream instability

$$f(x,u,t=0) = \frac{1}{2v_{th}\sqrt{2\pi}}\left[\exp\left(-\frac{(u-U)^2}{2u_t^2}\right) + \exp\left(-\frac{(u+U)^2}{2u_t^2}\right)\right](1+\alpha\cos(kx))$$

$$\alpha = 0.05, U = 0.99, u_t = 0.3, k = \frac{2}{13}$$

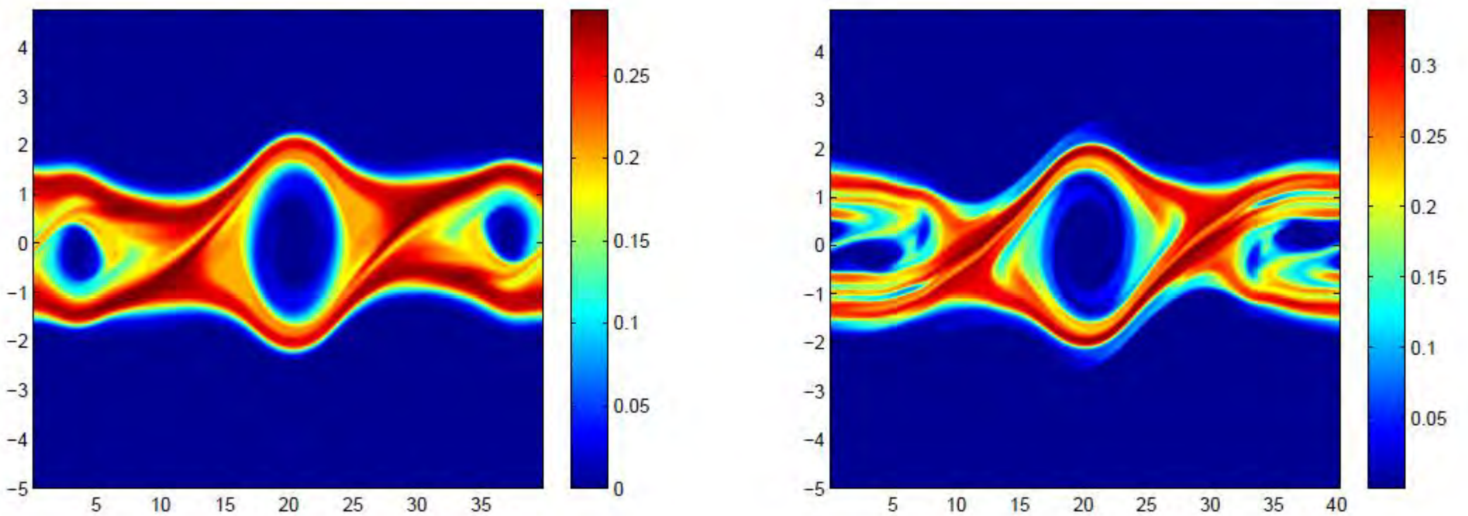


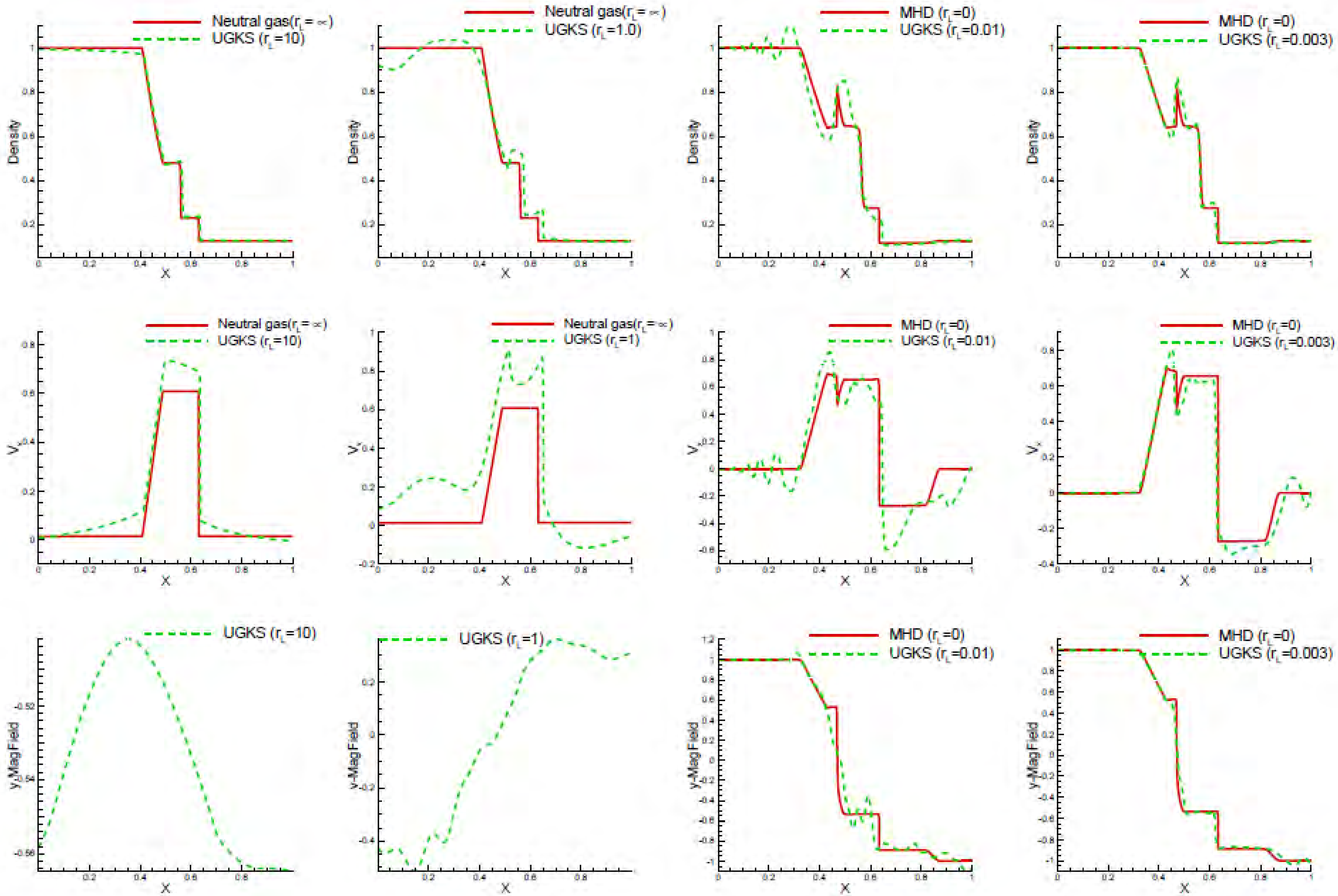
Figure 12: Two stream instability (nonlinear). Velocity distribution contours at $t = 70$. $N_x \times N_u = 256 \times 256$ (left); $N_x \times N_u = 512 \times 512$ (right)

The Brio-Wu shock tube is a standard test case for MHD solvers in continuum regime with initial condition shown in following figure.

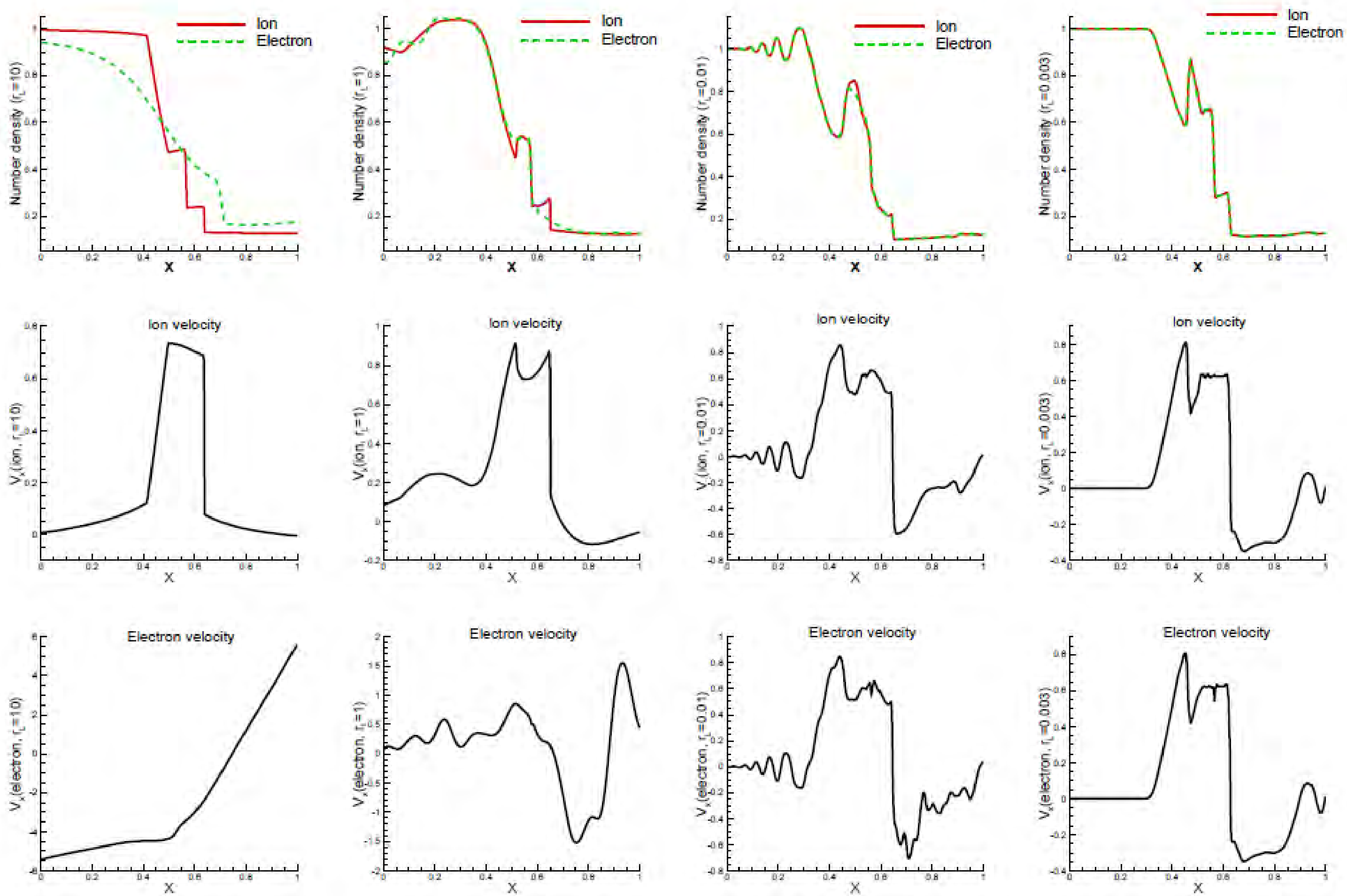
The ion to electron mass ration is set to 1836 (∞ in MHD). The normalized Debye length is set to 0.01. The normalized speed of light is set to 100. The ion Larmor radius is varied as $r_L = 10, 1, 0.01, 0.003$. The Knudsen number is set to 10^{-4}. The grid points in physical space are 1000. The grid points in velocity spaces are 32 in each direction.

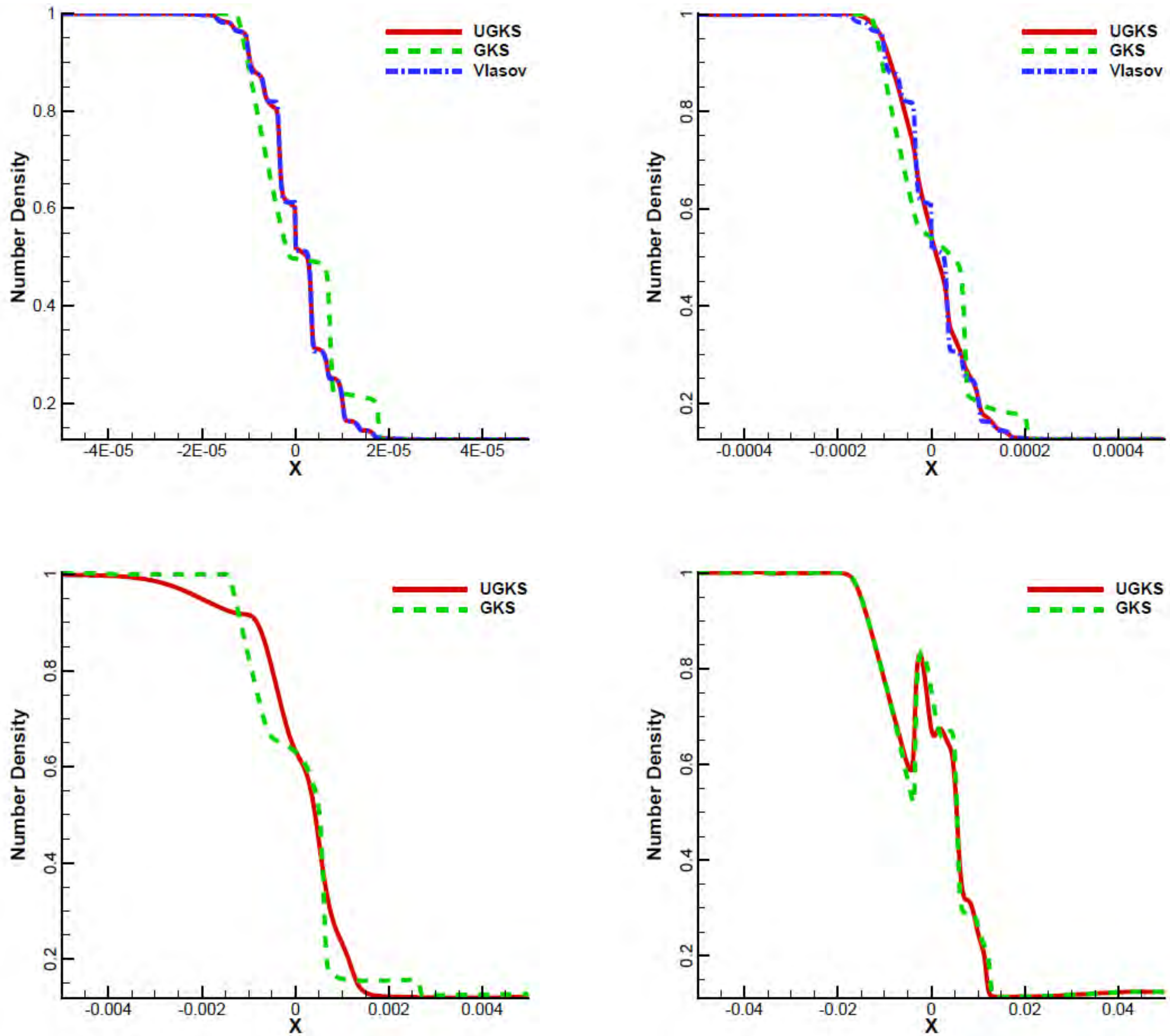
$\rho_i = 1.0, P_i = 5.0\times10^{-5}, \vec{V_i} = 0;$	$\rho_i = 0.125, P_i = 5.0\times10^{-6}, \vec{V_i} = 0;$
$\rho_e = 1.0\frac{m_e}{m_i}, P_e = 5.0\times10^{-5}, \vec{V_e} = 0;$	$\rho_e = 0.125\frac{m_e}{m_i}, P_e = 5.0\times10^{-6}, \vec{V_e} = 0;$
$\vec{B} = (0.75, 1.0, 0);$	$\vec{B} = (0.75, -1.0, 0);$
$\vec{E} = 0.$	$\vec{E} = 0.$

Results of the density (top), velocity (middle) and magnetic field (bottom) profiles are shown in the following figure with $r_L = 10, 1, 0.01, 0.003$ from left to right.

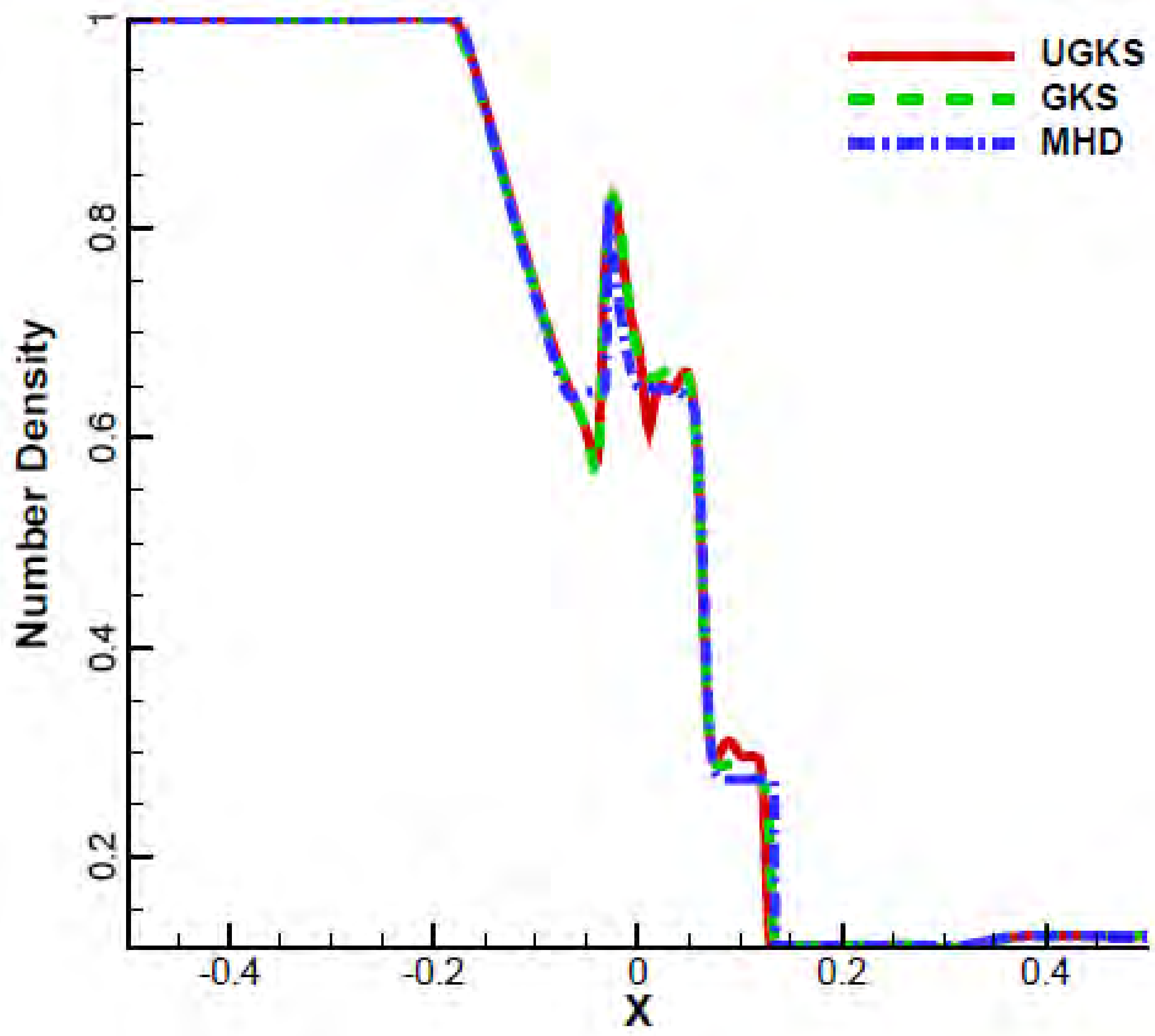


The following figures show the density and velocity profiles for ions and electrons with $r_L = 10, 1, 0.01, 0.003$ from left to right.

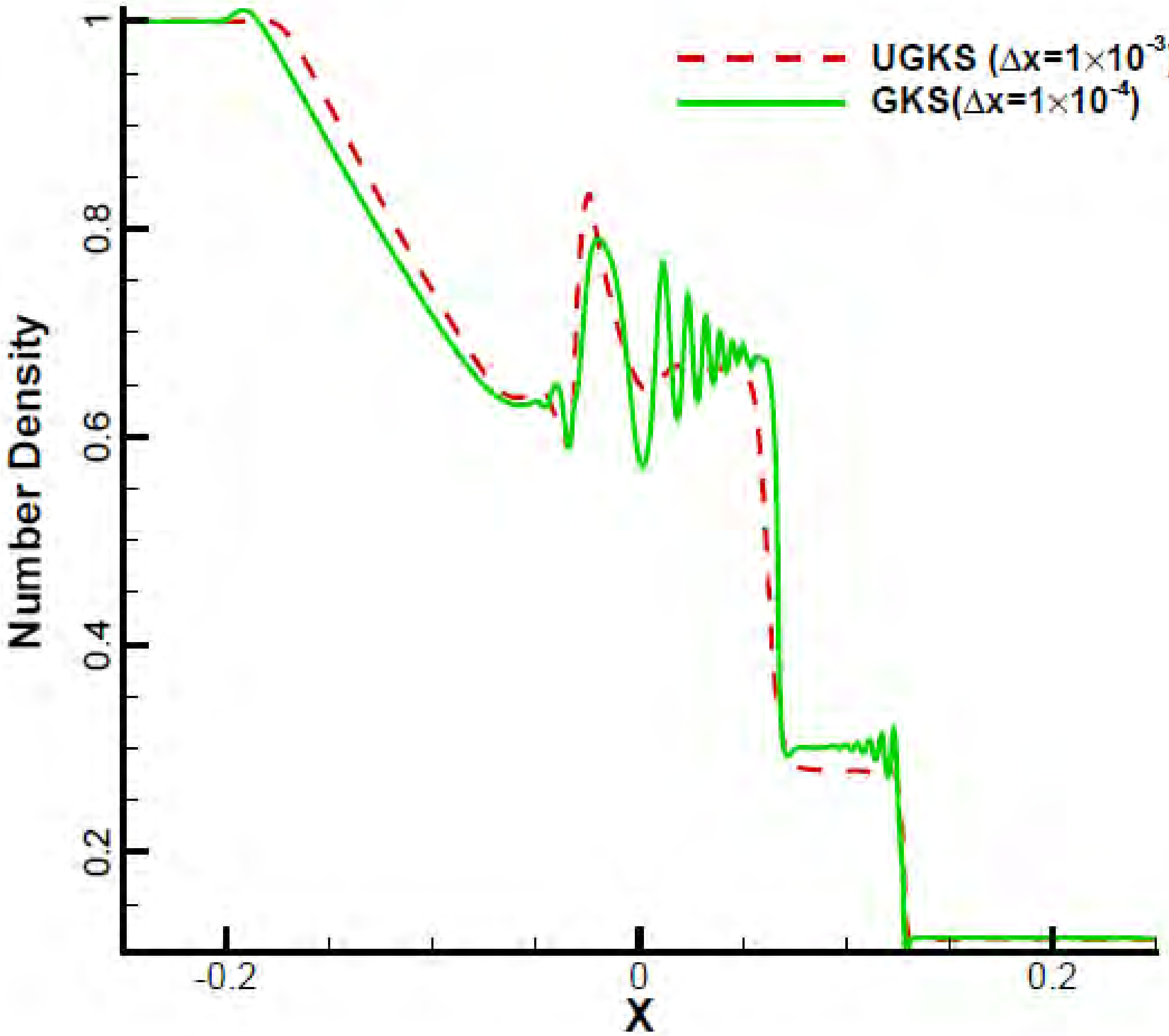




$$t = 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}.$$

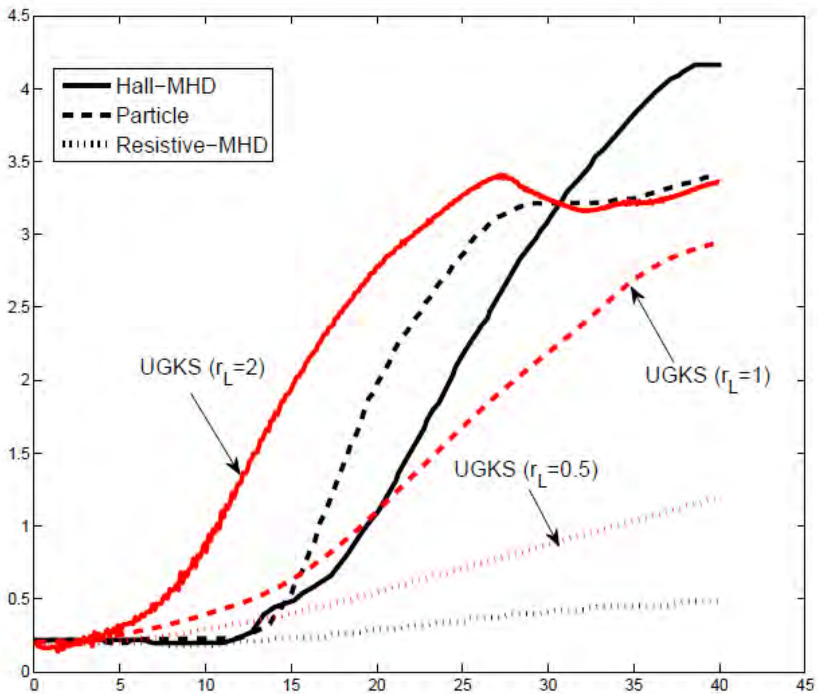


t=0.1

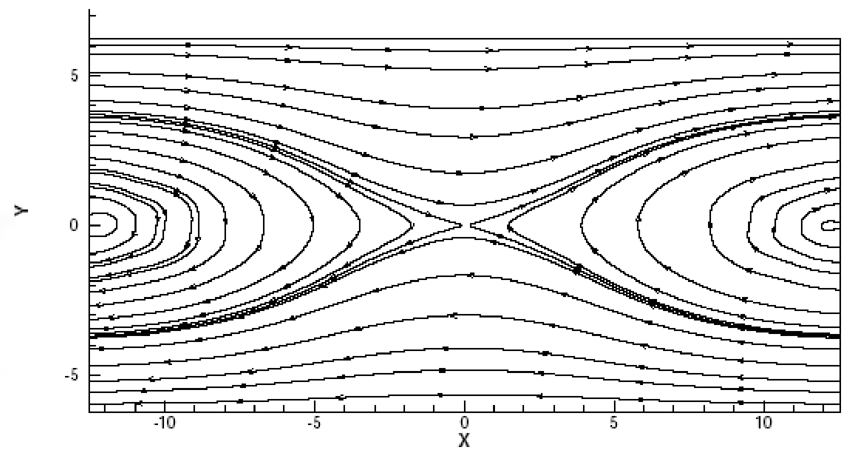


UGKS (Δx=1×10^-3)
GKS(Δx=1×10^-4)
Number Density
1
0.8
0.6
0.4
0.2
-0.2
0
0.2
X

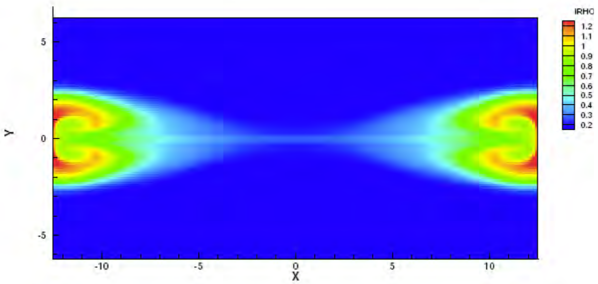
Magnetic Reconnection



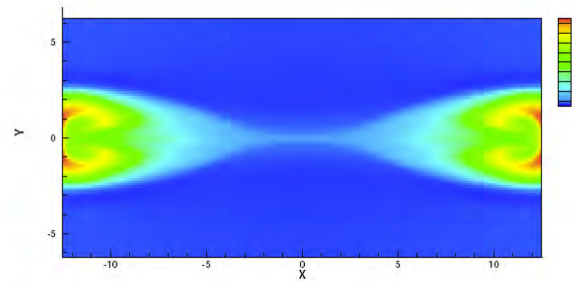
y-component absolute magnetic field



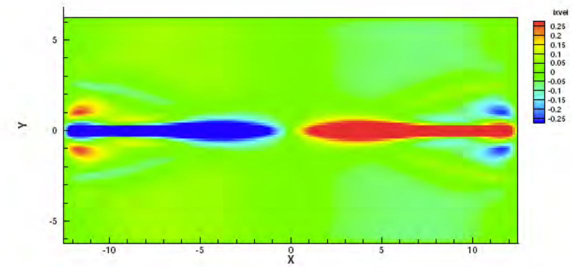
magnetic field distribution



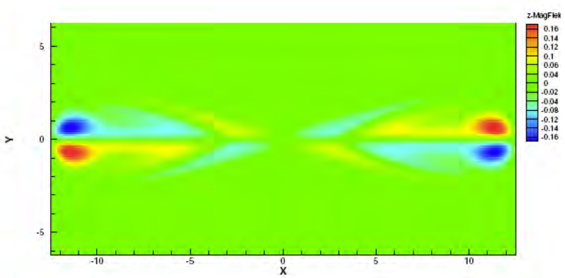
ion density



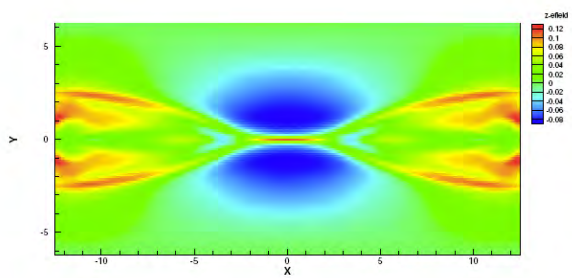
electron density



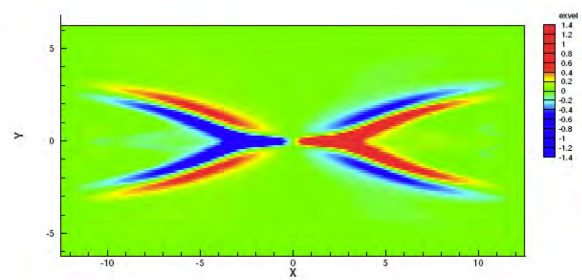
x-momentum of ion



z-direction magnetic field

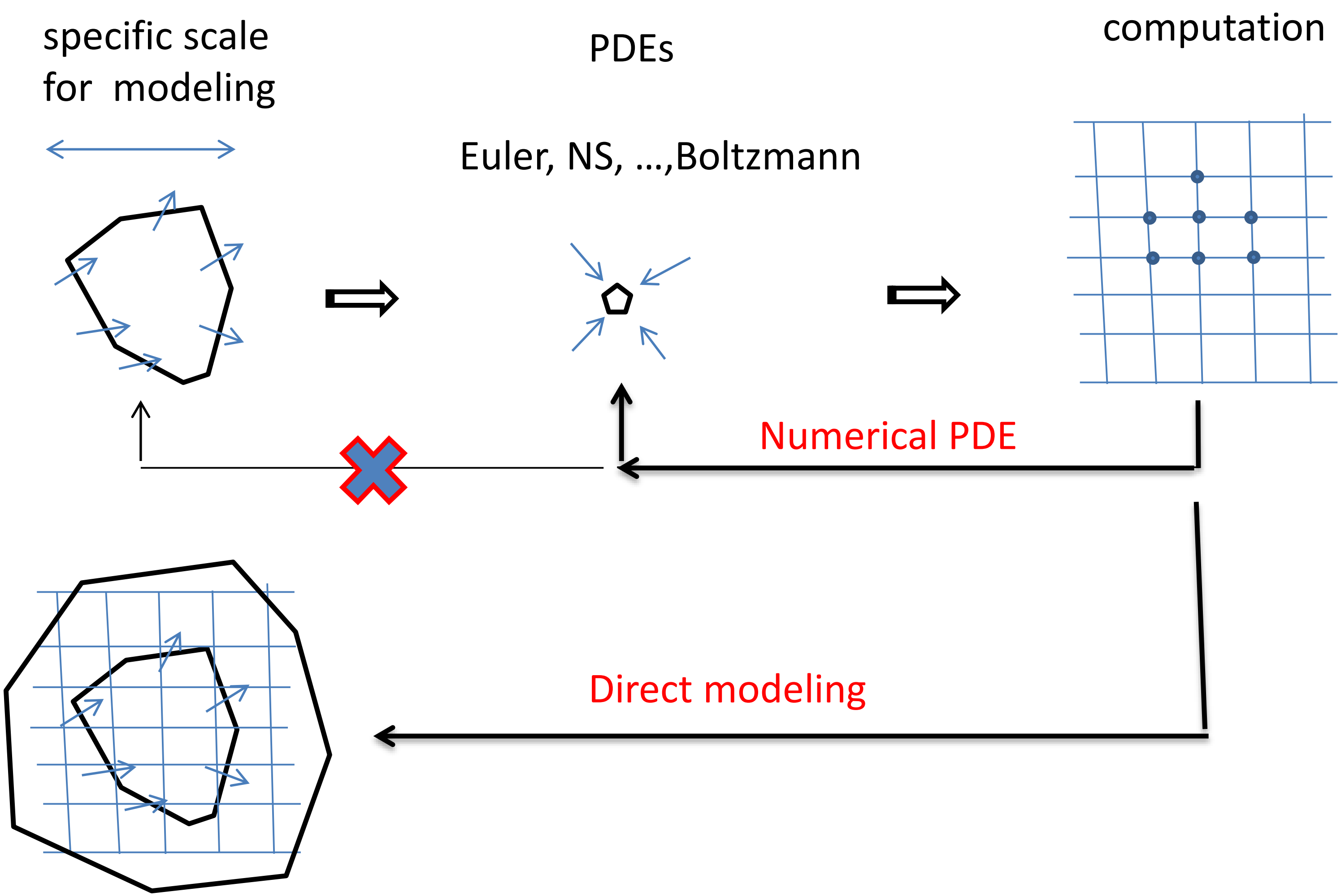


z-direction electric field

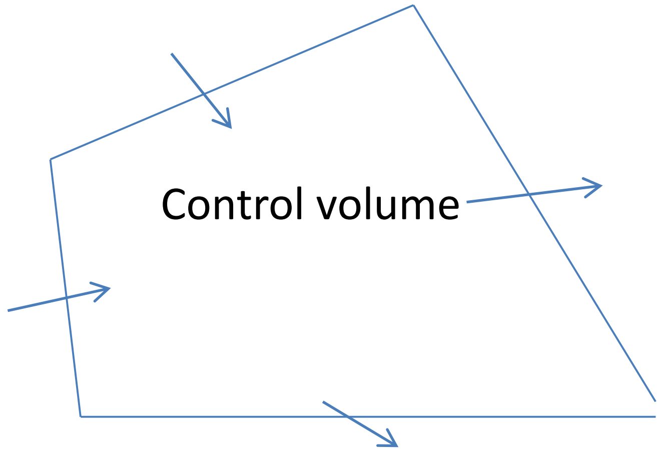


x-momentum of electron

DIRECTION OF COMPUTATIONAL FLUID DYNAMICS



Direct Modeling for CFD



$\Delta x \not\to 0, \quad \Delta t \not\to 0$

Consistency ?
Convergence ?
Order of accuracy (numerical, physical) ?
…

Direct Modeling for Computations

Direct description of transport process in a discretized space; Construction of the evolution model; Development of the algorithm

Recover a Multiple Scale Transporting Process

UGKS:

- **Non-equilibrium Gas Dynamics**: *Boltzmann* ⟷ *Navier-Stokes equations*
- **Radiative transfer**: *ray transport* ⟷ *diffusion equations*
- **Plasma**: *Valsov equation* ⟷ *Magneto-hydrodynamics*
- **turbulence ?**