International Conference on Flow Physics and its Simulation
–In memory of Prof. Jaw-Yen Yang

New aspects of the friction of finite dry granular mass

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Prevalence of granular flows (flows of discrete solid objects)

Nature hazards

Industrial/engineering processes

Prediction / control: effective continuum model

constitutive relation + boundary condition

Granular material & its complex phasic features

Granular gas:
short **collision** dominated
"Rapid granular flow"
Kinetic Theory

Granular solid:
enduring contacts (force chains)
friction dominated
"Mohr-Coulomb continuum"
"Soil mechanics"

Granular liquid or 'dense' granular flow: **collision + friction**
-Superposition as a complex fluid (Bingham, viscoplastic ..)
-statistical treatment (glassy material)
-**phenomenological relation**

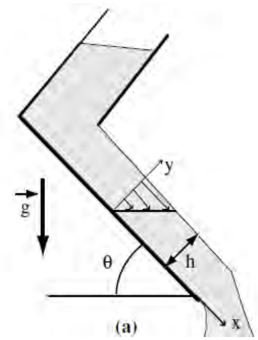
Effective friction coefficient

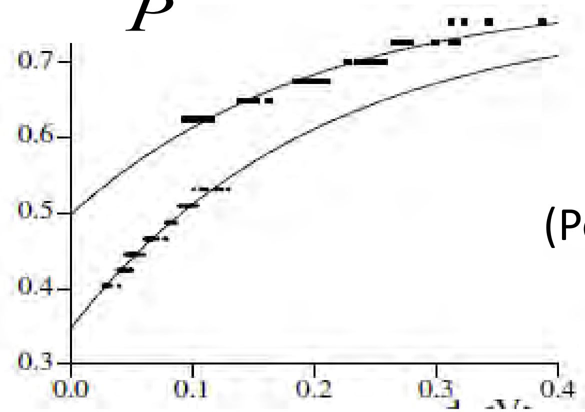
Treated as a solid (dense nature)

$$\mu = \tan\theta = \frac{\tau}{P}$$

Free surface flow experiment

Internal (basal) μ from force balance at steady state

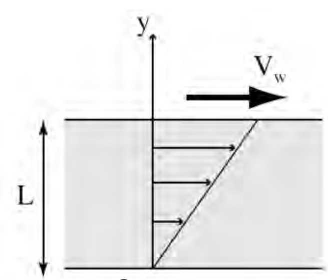




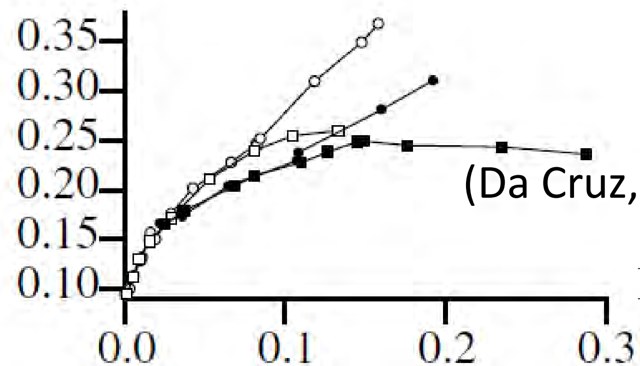
(Pouliquen 1999)

$$\frac{\|U\|}{\sqrt{gh}}\frac{d}{h} - \alpha\frac{d}{h}$$

2D Confined flow DE simulation



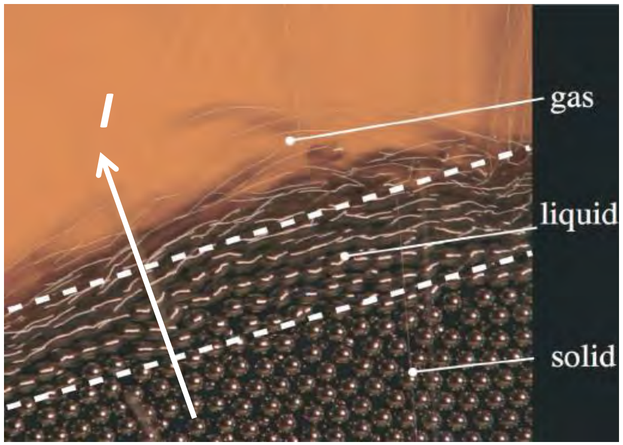
$$\dot{\gamma} = V/\mathrm{H}$$



(Da Cruz, GDR Midi 2004)

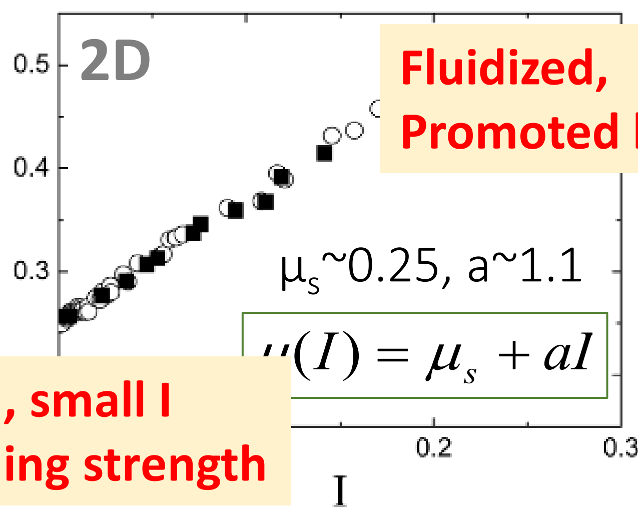
$$\mathrm{I} = \frac{\dot{\gamma}\, d}{\sqrt{P/\rho_s}}$$

Inertial number & μ(I) rheology law



$$I = \frac{\dot{\gamma}\, d}{\sqrt{P/\rho_s}} \sim \frac{\text{Shear-induced \textbf{streamwise} motion}}{\text{\textbf{transverse} settling motion due to confinement}}$$

$$\left(\sqrt{gh}\right)$$



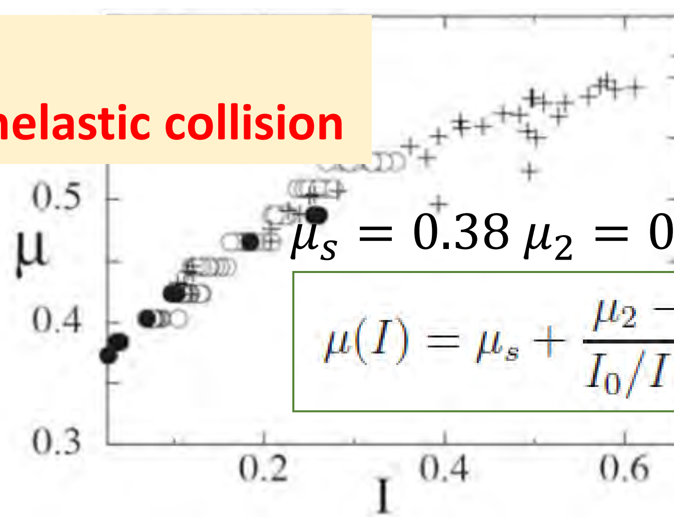
Fluidized,
Promoted by inelastic collision

μ_s~0.25, a~1.1

$$\mu(I) = \mu_s + aI$$

Solid, small I
Yielding strength

(Da Cruz 2004, GFR Midi 2004)



$\mu_s = 0.38$ $\mu_2 = 0.64$ $I_0 = 0.279$

$$\mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{I_0/I + 1}$$

(Pouliquen 2006)

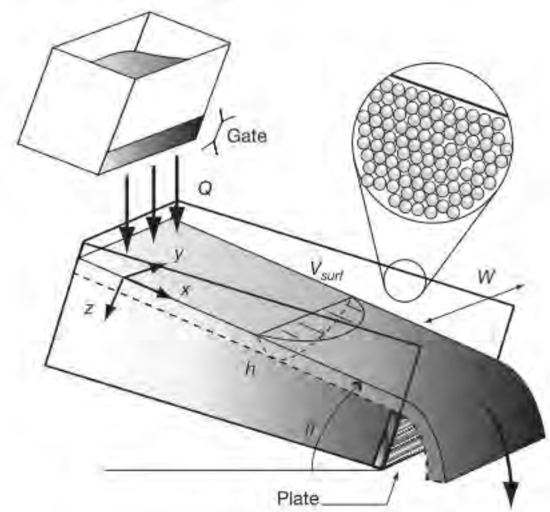
Remarks: boundary condition

Experiments are 3D: Lateral friction ?

-- neglected when H/W is low

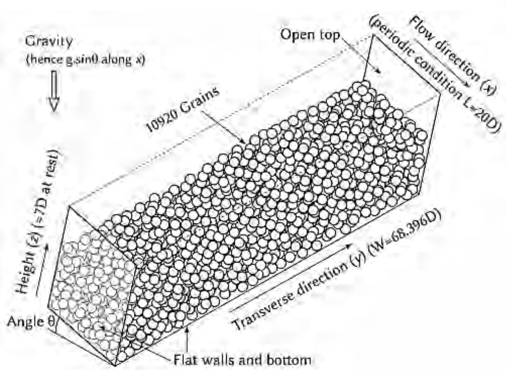
-- included as Coulomb friction:

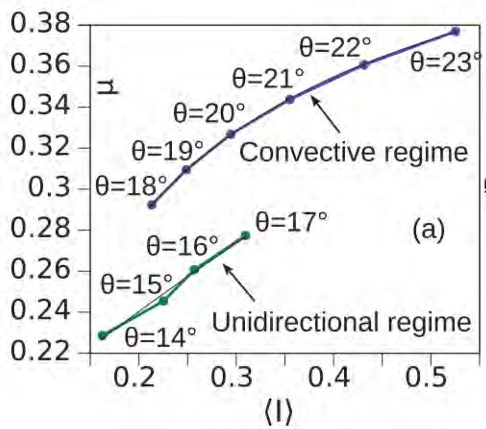
hydro P × constant μ_w



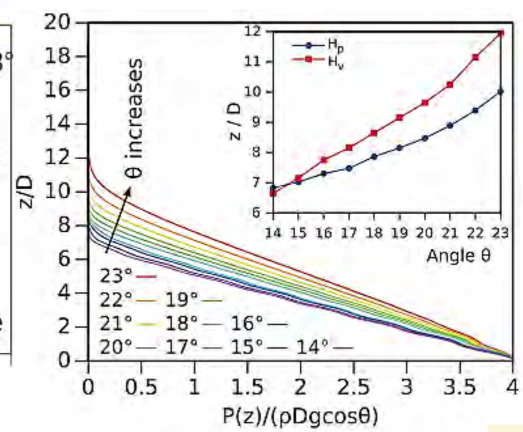
2D Numerical simulations → 3D

(Brodu, Richard, & Delannay 2013)

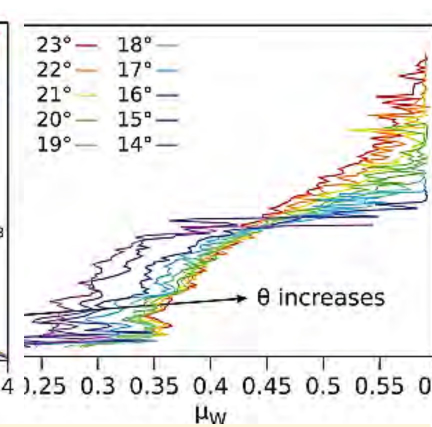




confirm internal μ-I



hydro P



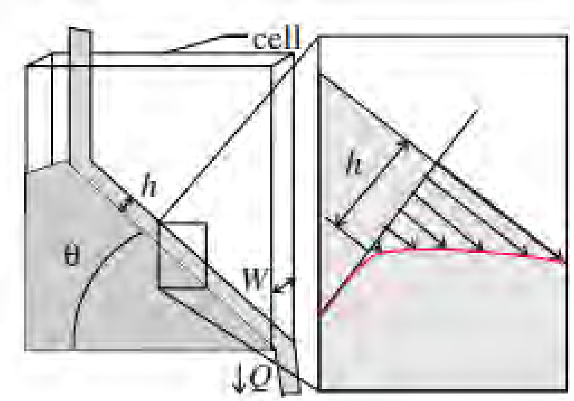
Depth-weakening μ_w

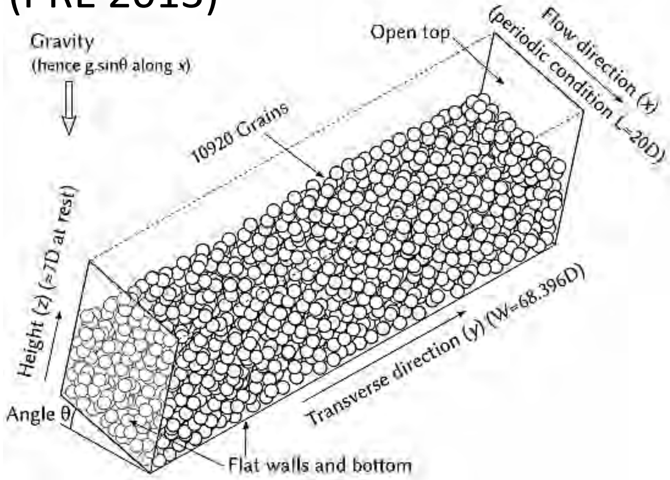
Non-constant wall friction coefficient (simulation based)

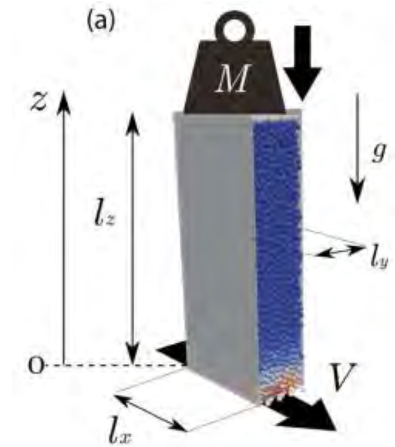
Richard et al. (PRL 2008)

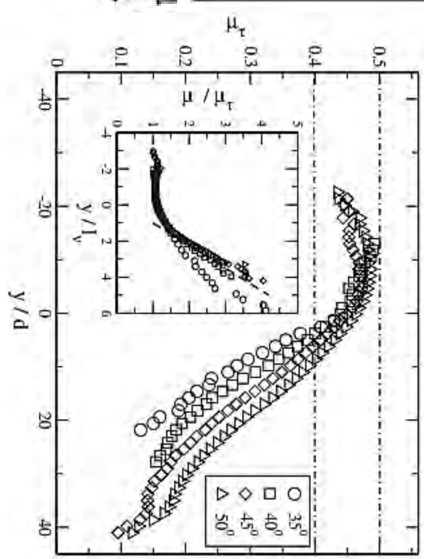
Brodu, Richard, Delannay (PRE 2013)

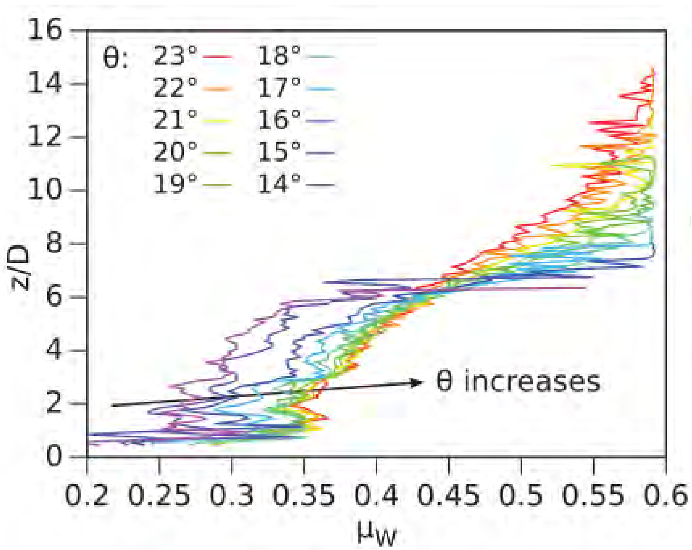
Artoni & Richard (PRL 2015)

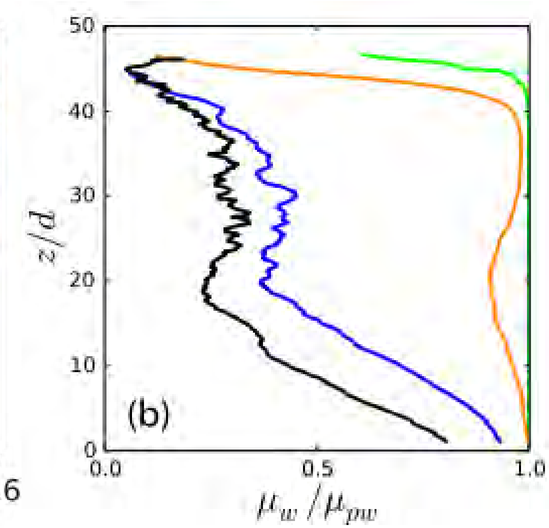












Tall, narrow

Wide, shallow (loose)

Tall, narrow (compact)

Questions:

Constitutive model: in unsteady/finite flows?

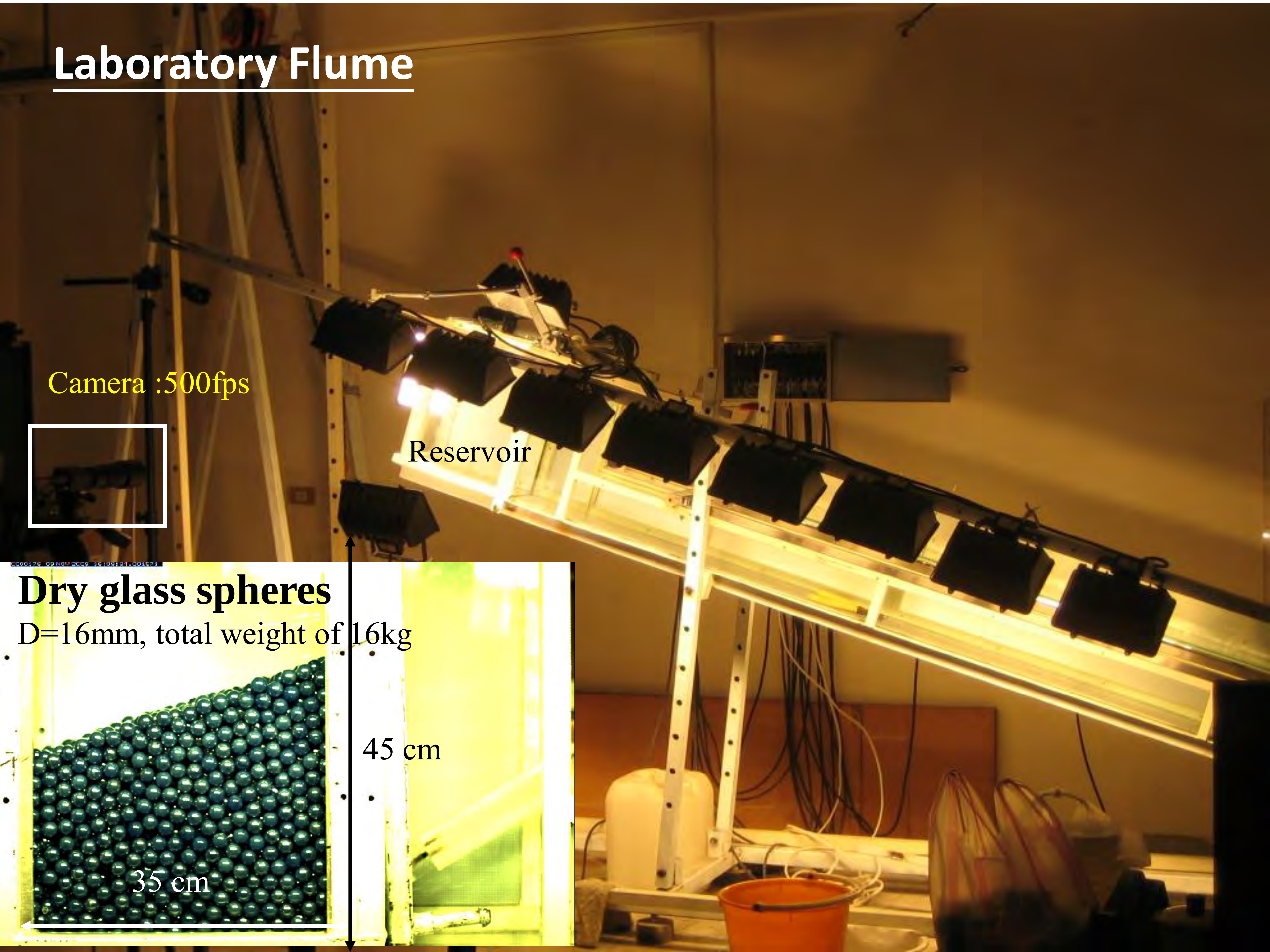
Boundary condition: experimental evidence?
Mechanism/model?

Internal μ

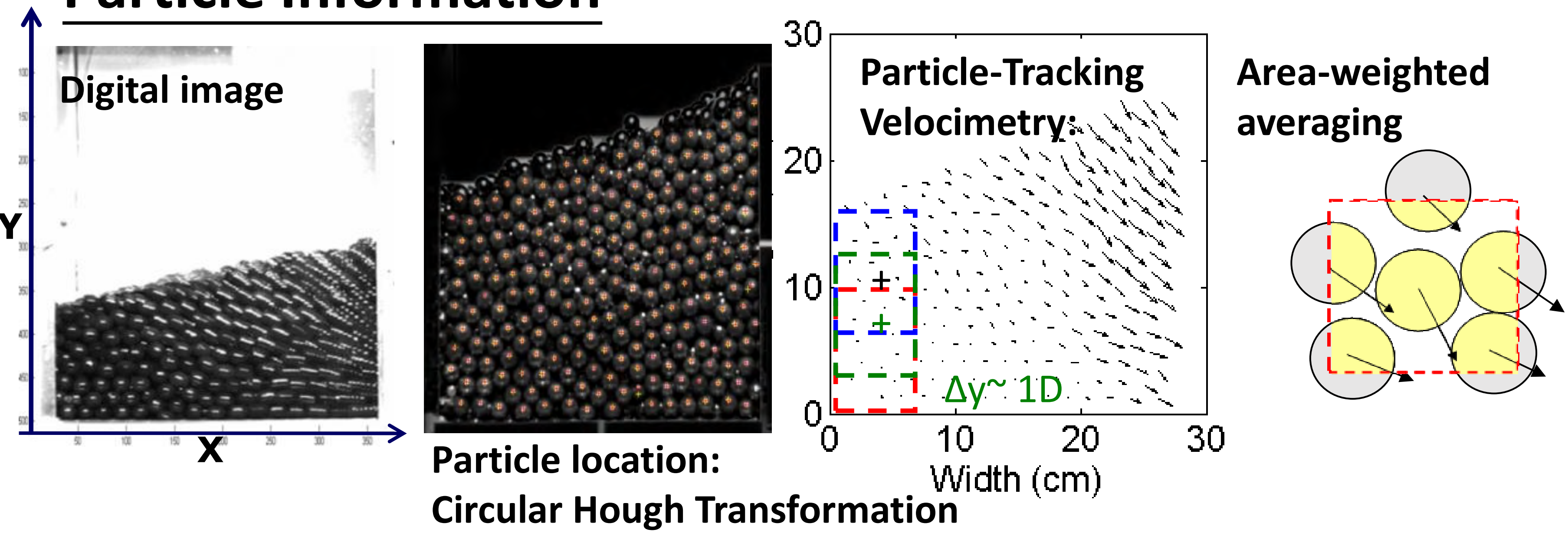
in unsteady non-uniform avalanche

Inclined Flume Experiments

(indirect) image-based control-vol analysis

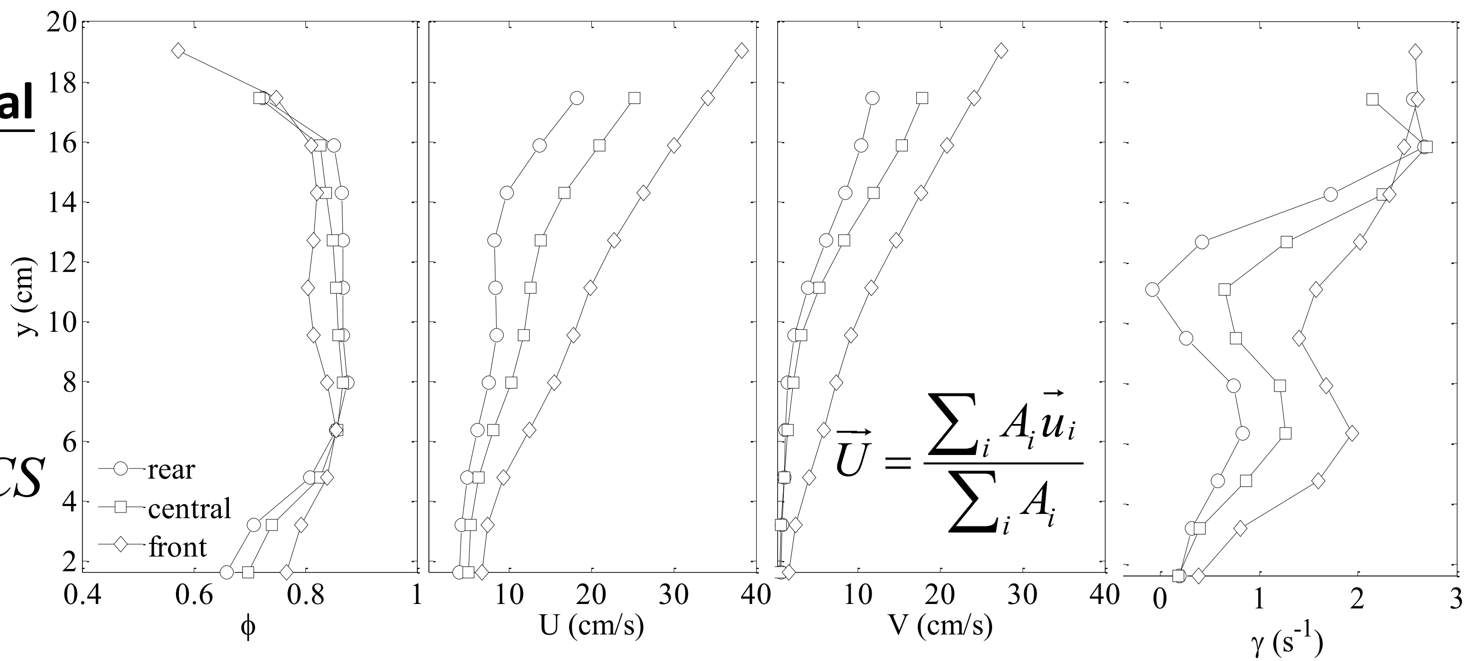

Laboratory Flume
Camera :500fps
Reservoir
Dry glass spheres
D=16mm, total weight of 16kg
45 cm
35 cm

Particle Information



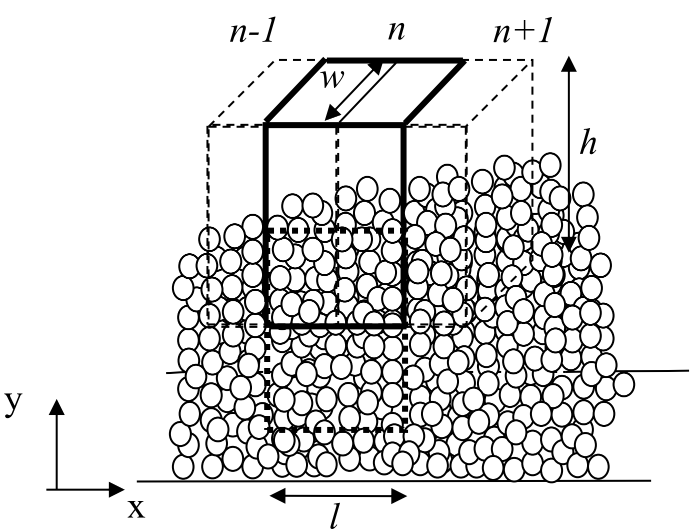
Instantaneous local bulk properties

$$\phi_n(x,y) = \sum A_i / CS$$



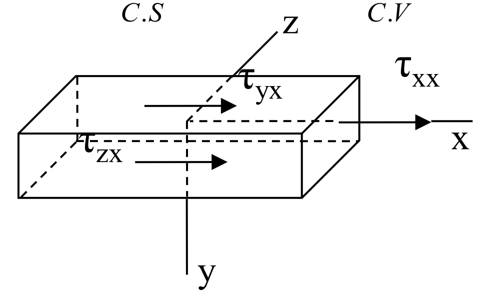
$$\vec{U} = \frac{\sum_i A_i \vec{u}_i}{\sum_i A_i}$$

Quasi-2D Control Volume Analysis

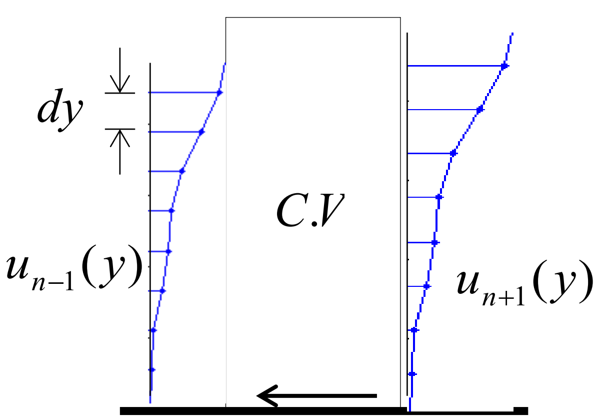


x-momentum conservation :

$$\iiint_{C.V} \frac{\partial}{\partial t}(\rho u_i)dV + \oiint_{C.S} \rho u_i u_j n_j dA_j = \oiint_{C.S} \tau_{ji} n_j dA_j + \iiint_{C.V} \rho f_i dV$$



$$\frac{\partial}{\partial t} wl\int_0^h \rho U dy + w\left[\int_0^h \rho U^2 dy\right]_{n-1}^{n+1} = -wg_y\left[\int_0^h \rho(h-y)dy\right]_{n-1}^{n+1} - 2F_W - F_B - wlg_y\int_0^h \rho dy$$



*Coulomb friction

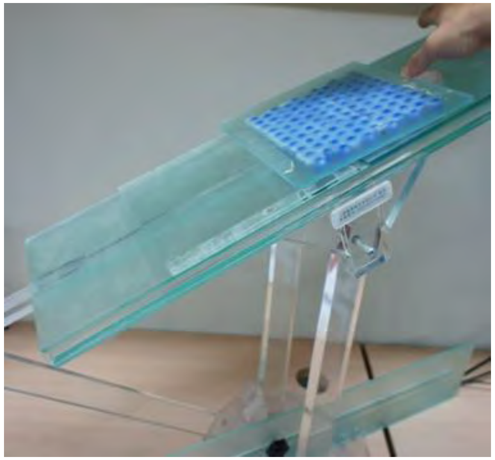
$$\int \tau_{zx}\, dA_z = \int \mu_w \tau_{zz}\, dA_z$$

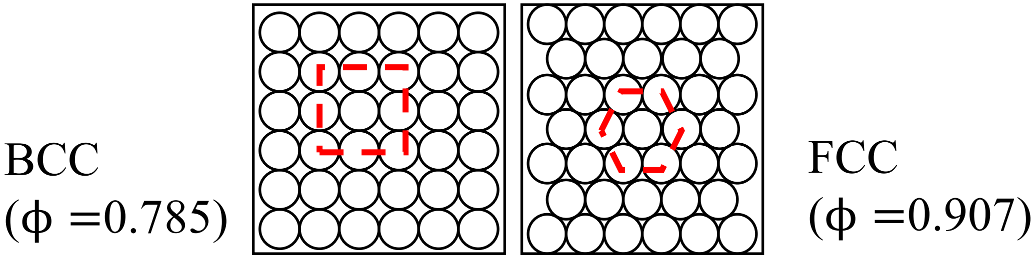
*Hydrostatic normal stress, hence isotropic

$$\tau_{zz} \approx \tau_{yy} \approx \tau_{zz} = \rho g_y h$$

Results

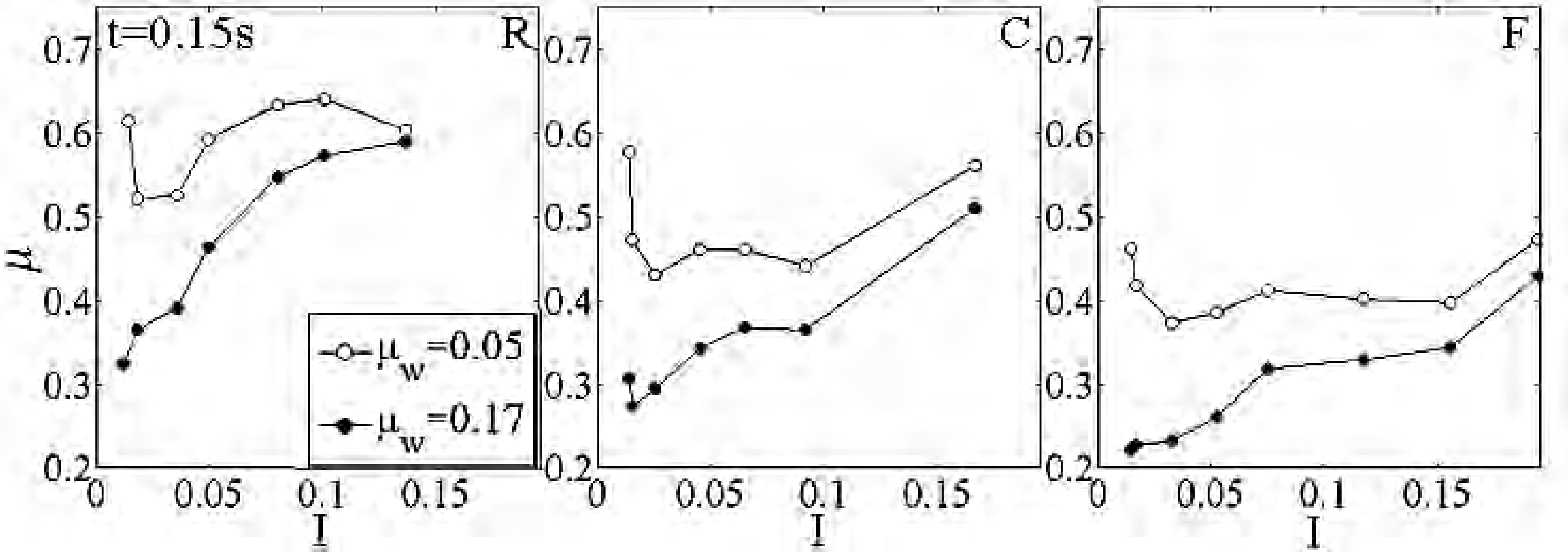
Sliding-table experiment using a layer of glued particles yields μ_w~0.17



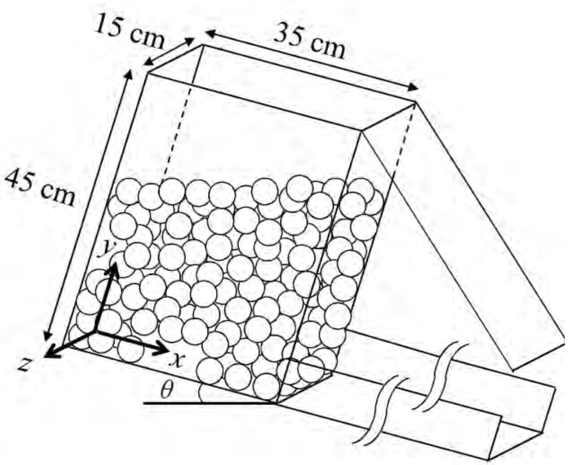


$$\alpha_{exp} = \alpha\ \phi_B + (1-\alpha)\ \phi_F$$

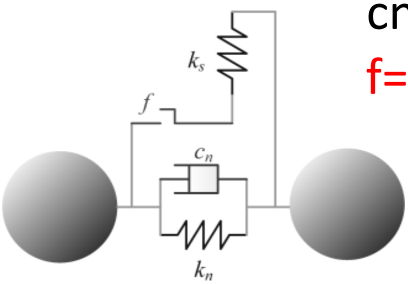
$$\mu_w = \alpha\ \mu_{wB} + (1-\alpha)\mu_{wF} \cong 0.171$$



Huang & Yang (GM, 2016)



$$\vec{F} = \vec{F}_n + \vec{F}_t$$



Classical soft-sphere contact model

kn, kt (elasticity, Hertzian contact time)

cn (coefficient of restitution)

f=0.2 by 'bulk' discharge flow rate

Internal μ in unsteady non-uniform avalanche

Validated Discrete element simulation

Yang et al. (PoF, 2013)

for direct force calculation

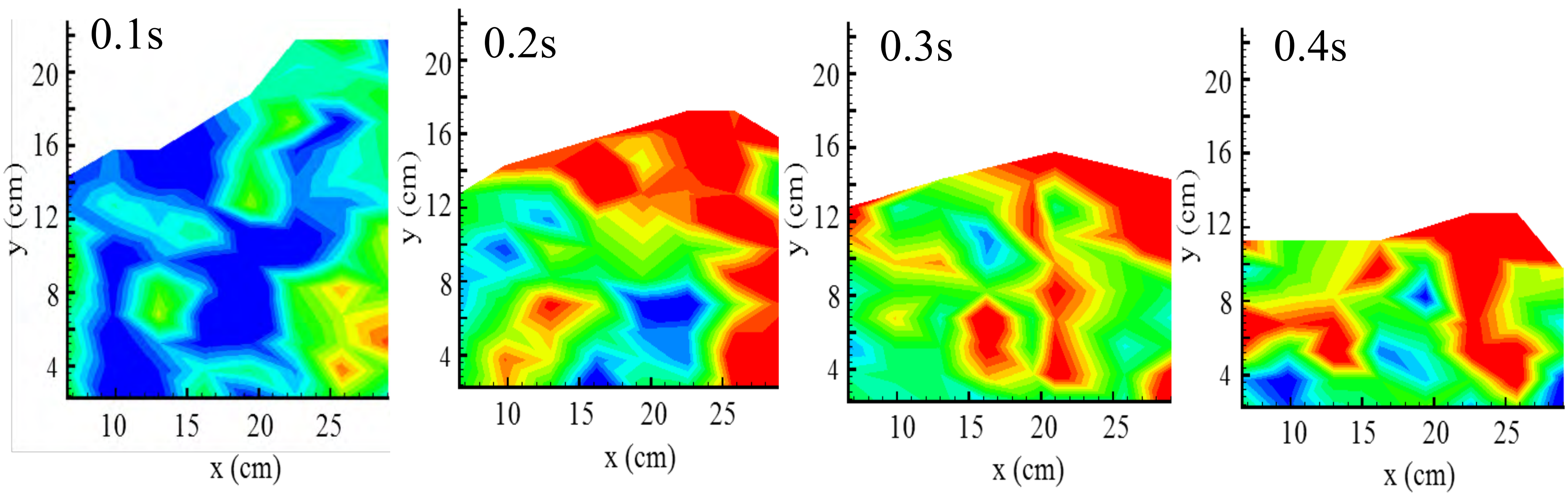
Virial Stress tensor for $\boldsymbol{\mu}$

$$\boldsymbol{\sigma} = \frac{1}{\Omega}\left[\frac{1}{2}\sum_{\substack{\alpha,\beta\in\Omega\\ \alpha\neq\beta}} \vec{F}^{\alpha\beta}\otimes\vec{r}^{\alpha\beta} - \sum_{\alpha} m^{\alpha}\delta v^{\alpha}\otimes\delta v^{\alpha}\right] \Rightarrow \mu = |\tau|/P$$

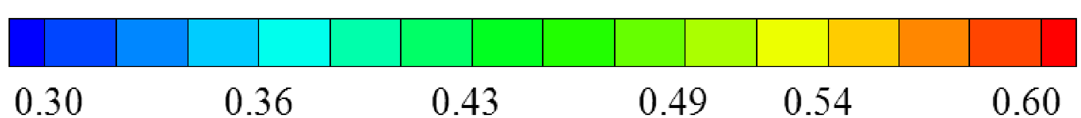
contact contribution

kinetic contribution (negligible for dense flow)

Second/first invariant of stress tensor

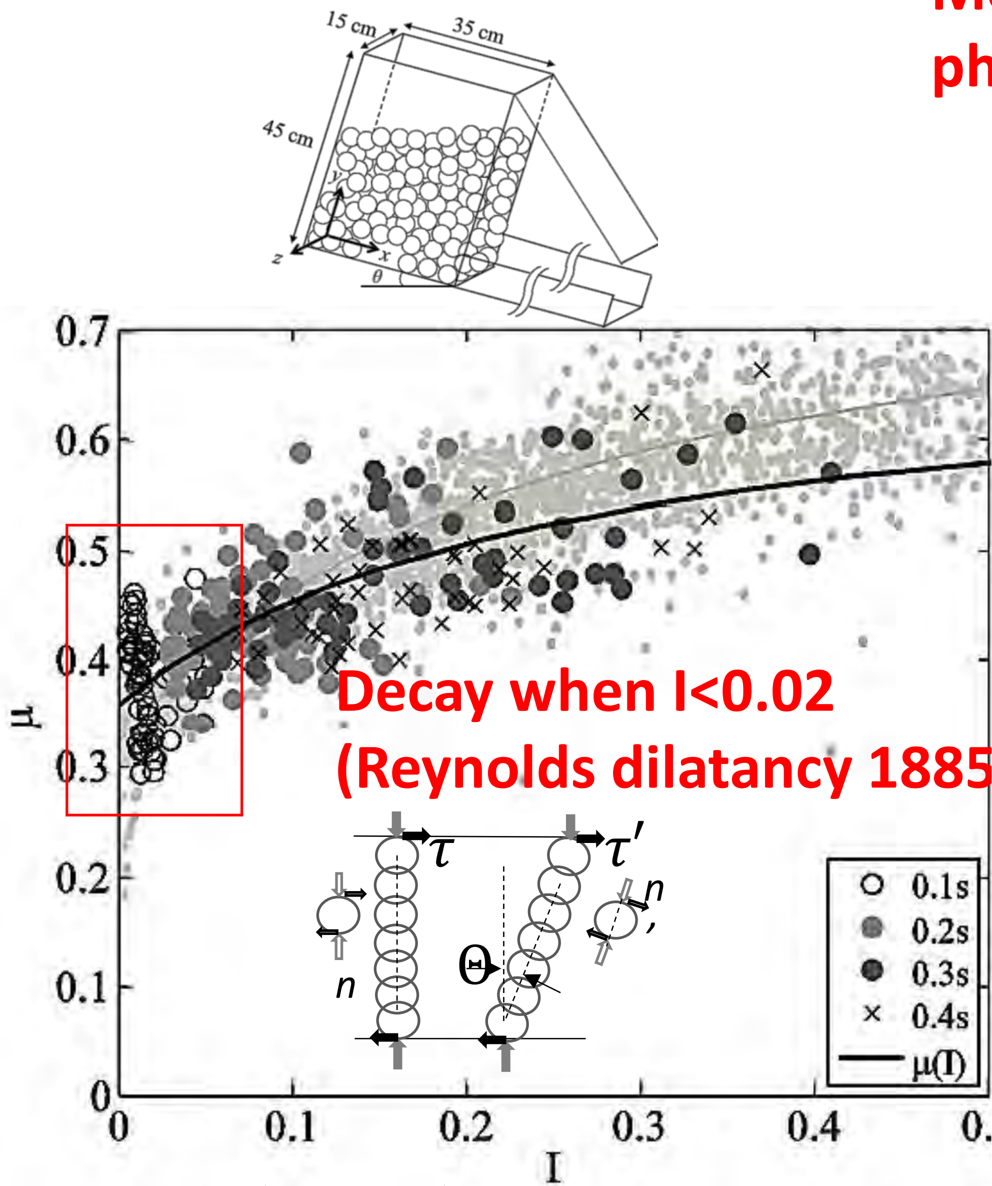


≈ tan 19° (= 0.34)



Yang & Huang (GM, 2016)

Instantaneous μ –I relation

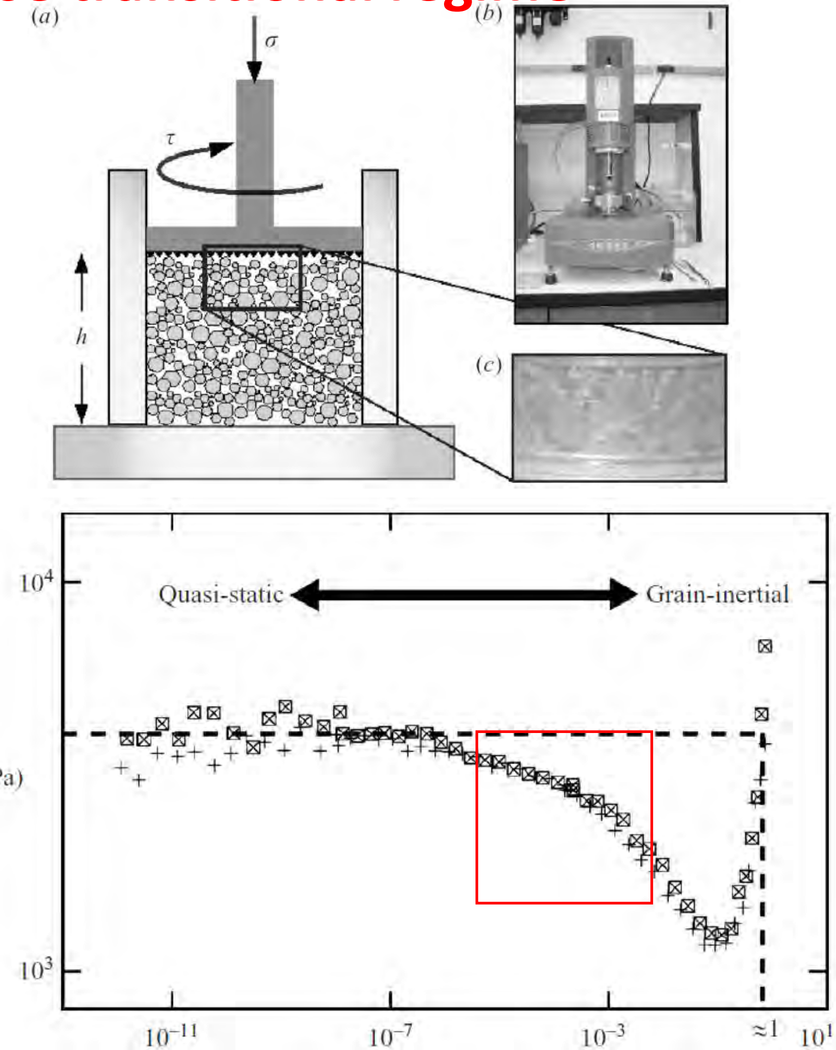


Decay when I<0.02

(Reynolds dilatancy 1885

Yang & Huang (GM, 2016)

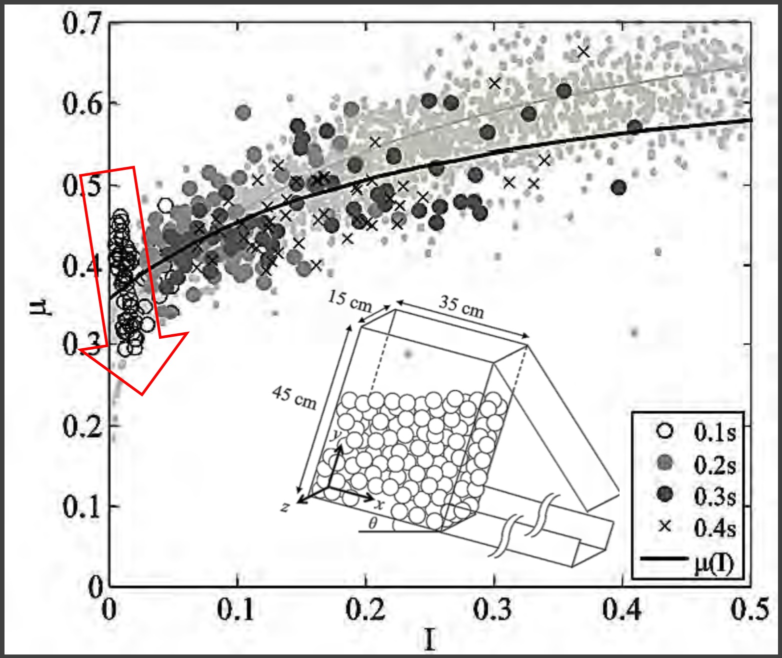
Measured shear-weakening at s-l phase transitional regime

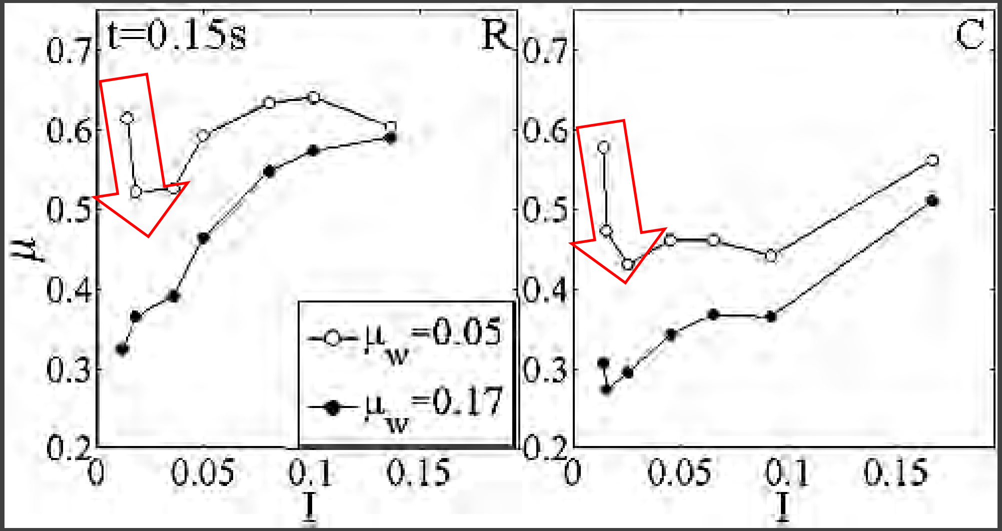


Savage number ~ I^2

Summary I

- Monotonically rising μ –I confirmed above I_c~0.02





- Decay trend below I_c (with μ_w reduced from pure-sliding value)
- Transition as a bifurcation phenomenon

Boundary condition: μ_w

Direct measurement in steady uniform flows

Confirmation in real steady uniform flows

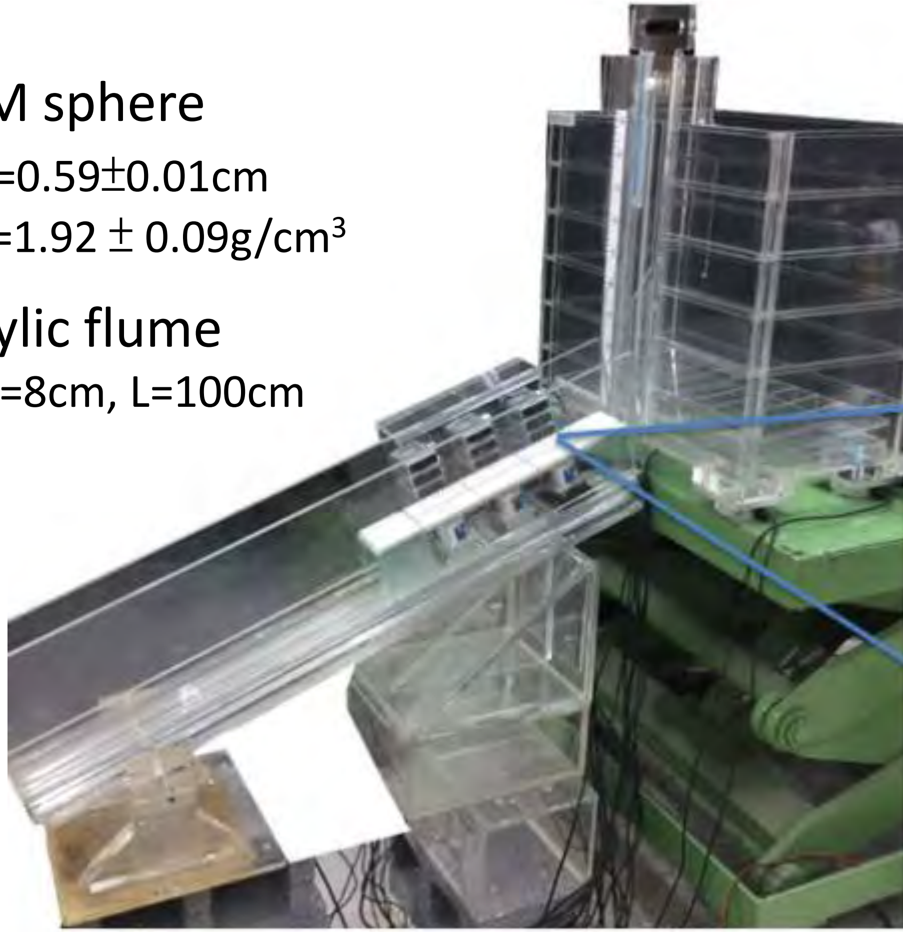
POM sphere

D=0.59±0.01cm

ρ=1.92 ± 0.09g/cm^3

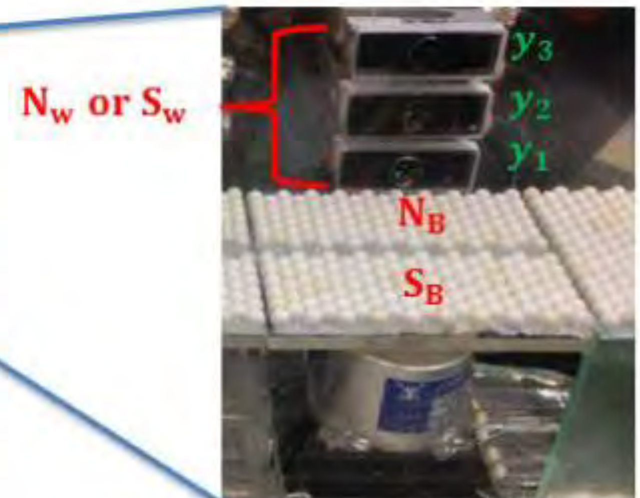
Acrylic flume

W=8cm, L=100cm



Direct force measurement via load cell (LC) array

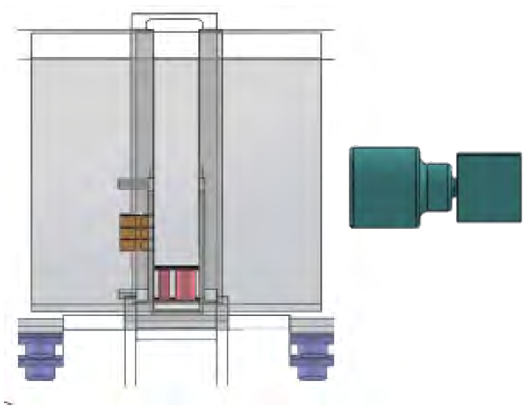
N or S in depth on side wall



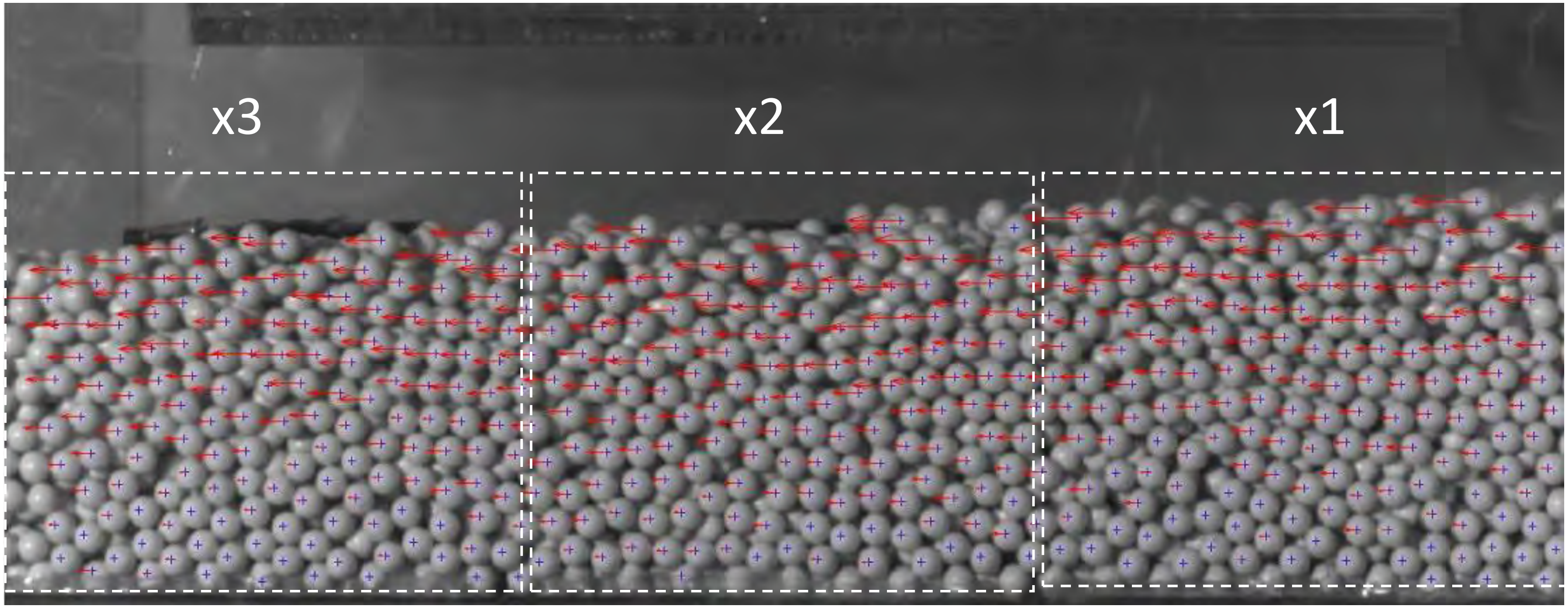
Concurrent N and S @ base

Roughened by spheres

Concurrent high-speed imaging

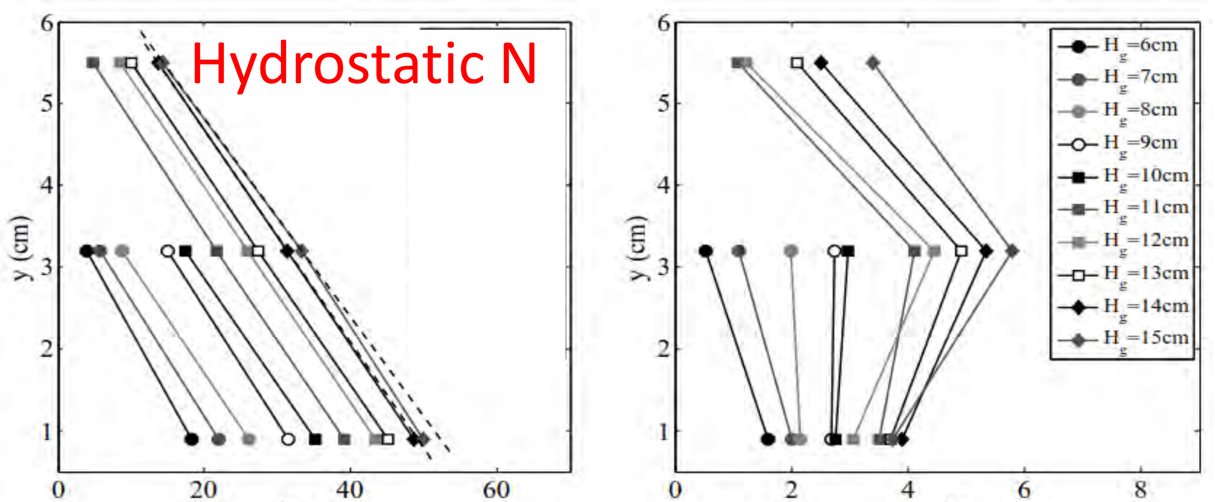


Indirect measurement [500pfs]
+ PTV (particle tracking velocimetry)
@ side: flow height h(x)
: solid volume fraction ϕ(x,y)
: velocity u(x,y)



Wall μ_w

Steady mean
LC data



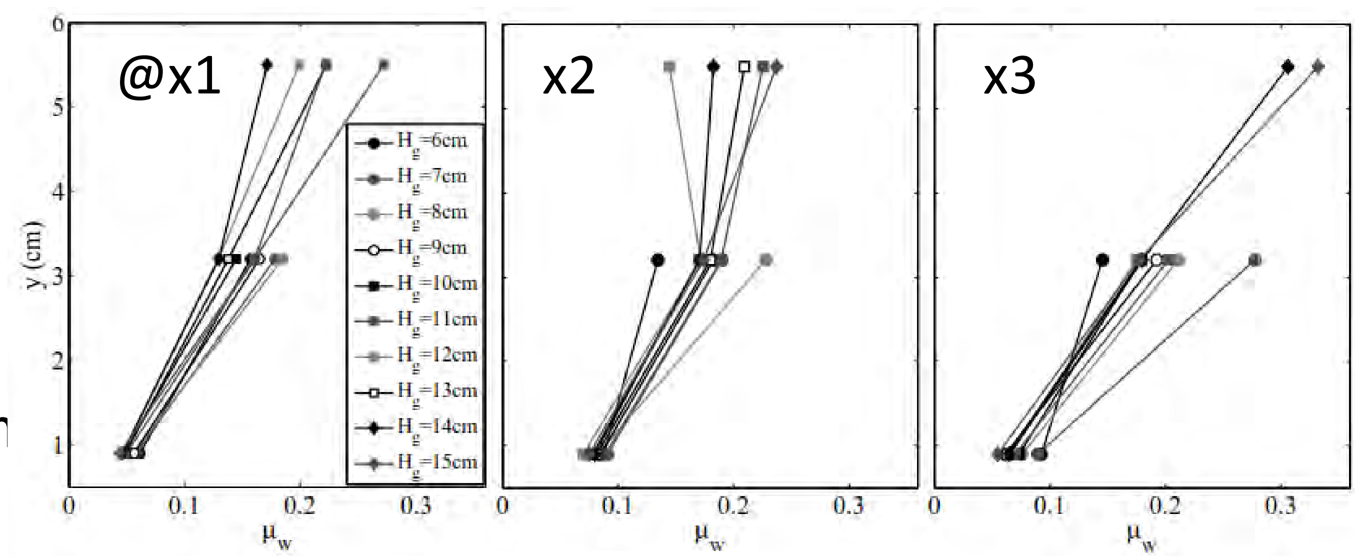
Normal force

Shear force

Non-constant

Decays in depth

Grows downstream



$$\mu_w = S_w / N_w$$

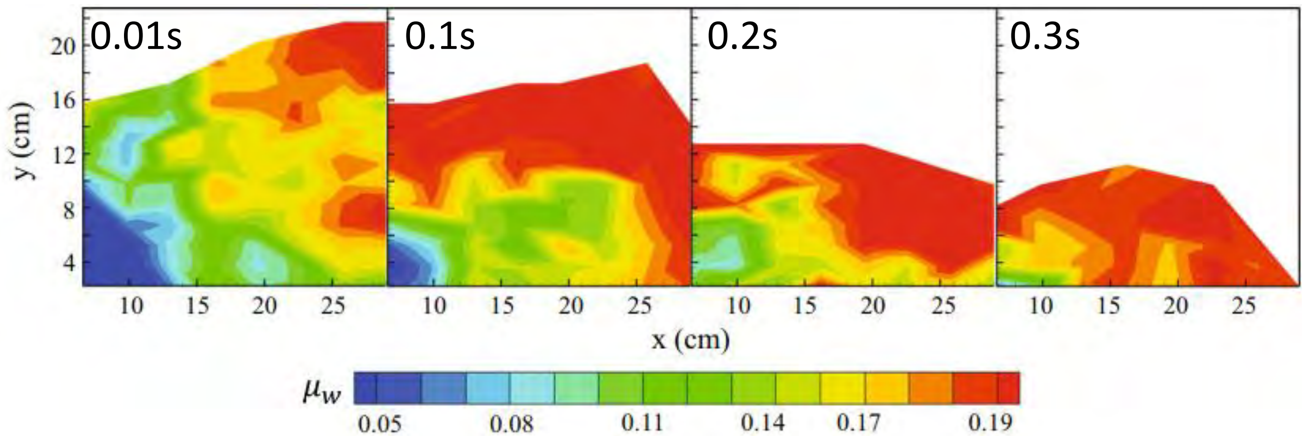
Boundary condition: μ_w

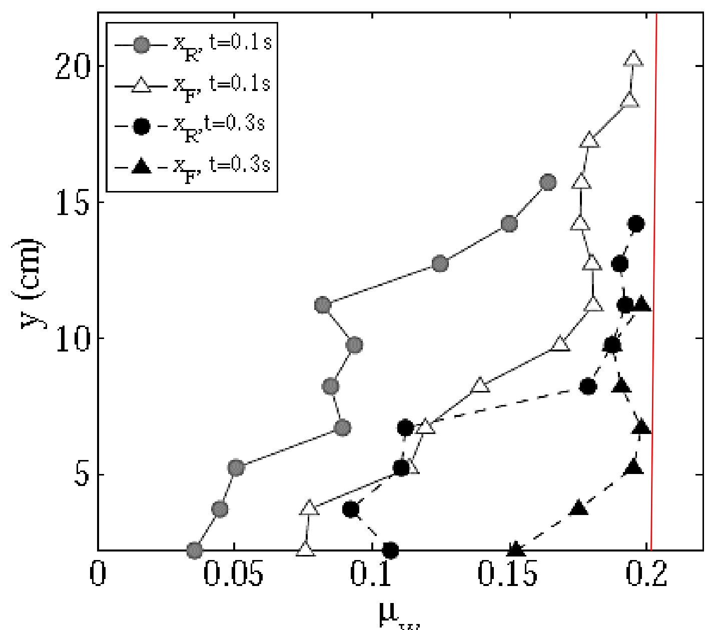
DEM simulated avalanche

DEM-rendered μ_w

$$\sigma_w = \sum_{\alpha \in A} \vec{F}^{\alpha w} / \Lambda$$

Bulk friction coefficient is non-constant: depth-weakening & flow-dependent
Always smaller than the microscopic sphere-wall f=0.2

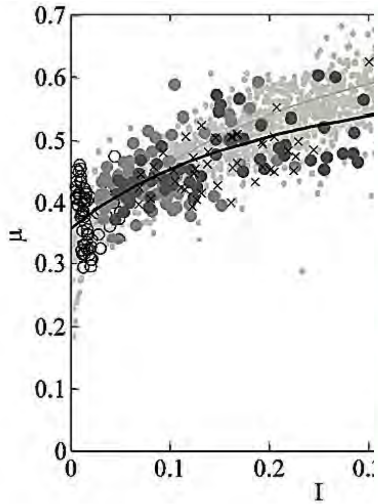




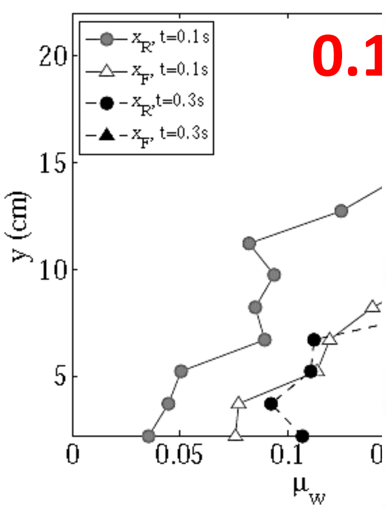
Depth-decaying
Growing towards 'microscopic' f

Inference of reduced μ_w

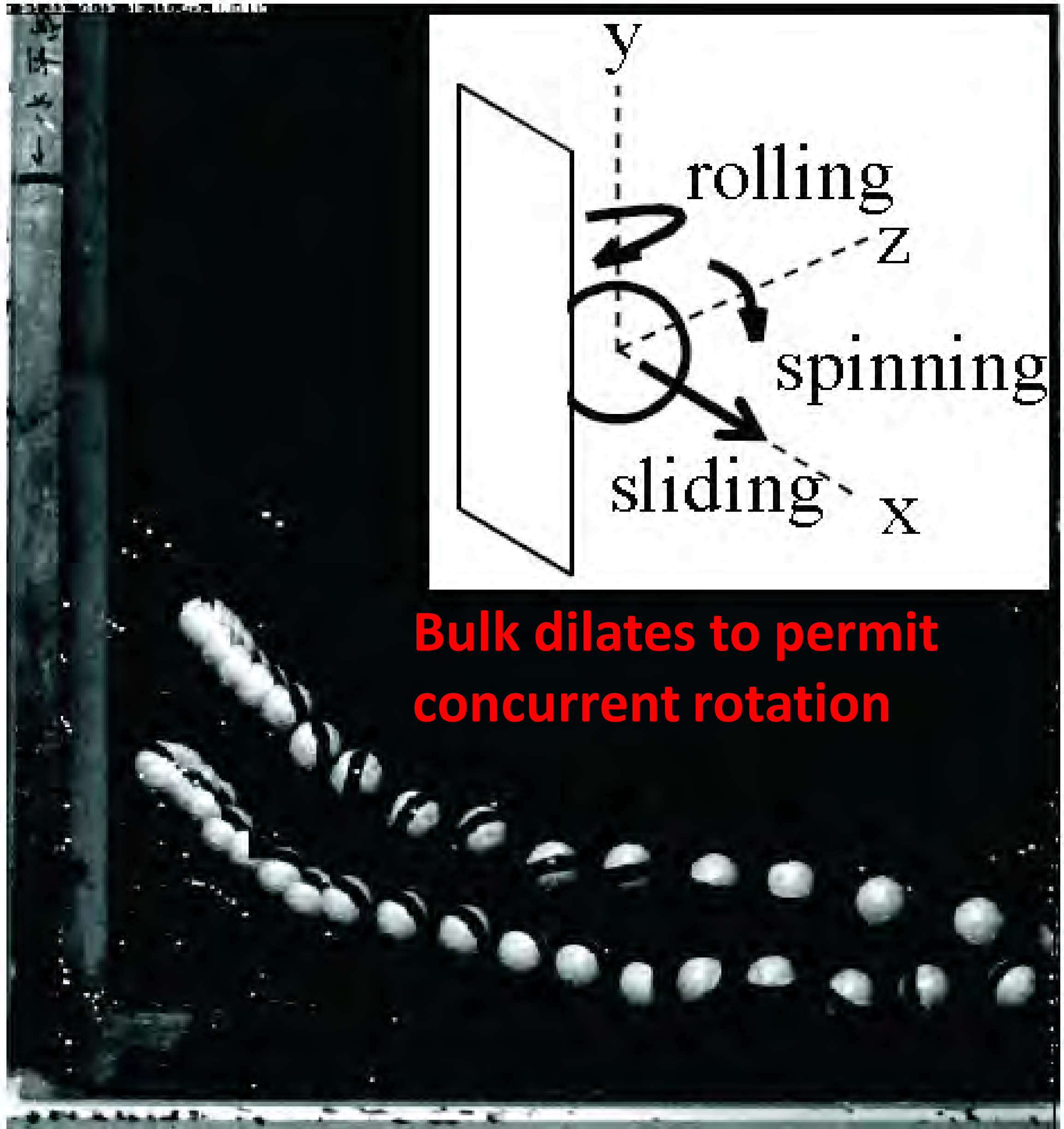
1. **Decay of internal μ is phy**



2. **Simulated μ_w lower than sl**



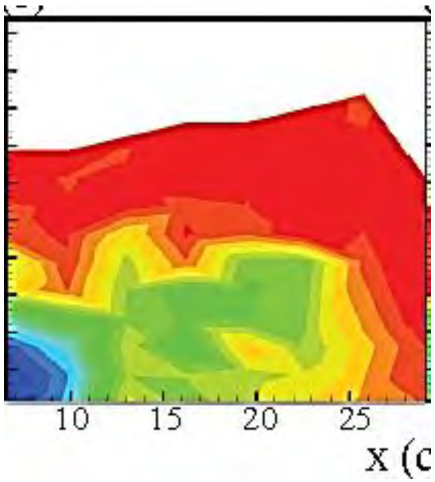
0.1



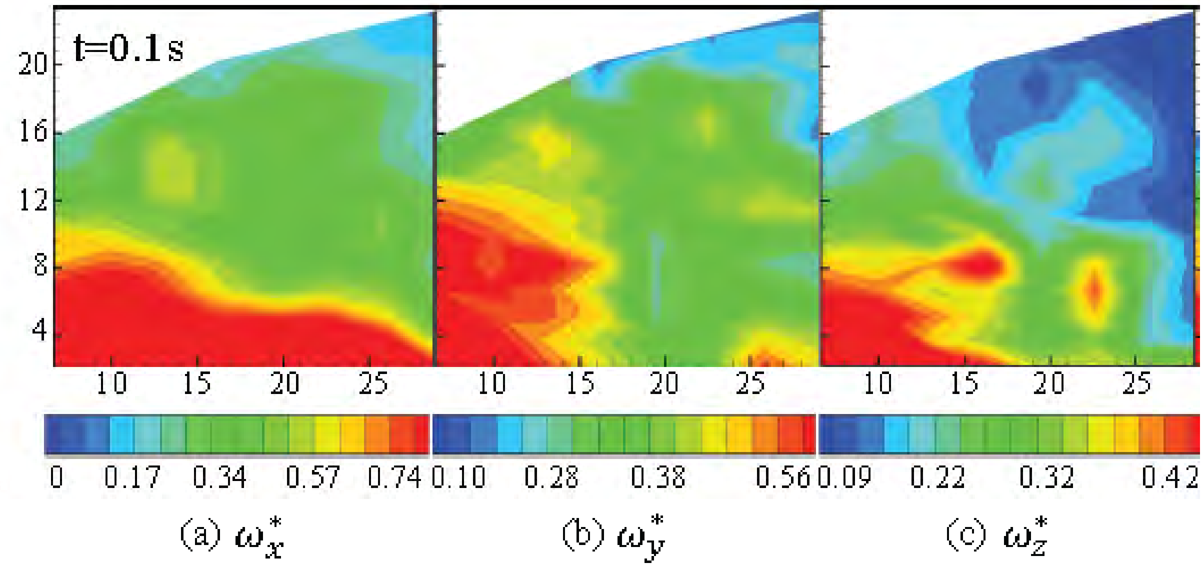
Bulk dilates to permit concurrent rotation

Rotation as a weakening mechanism

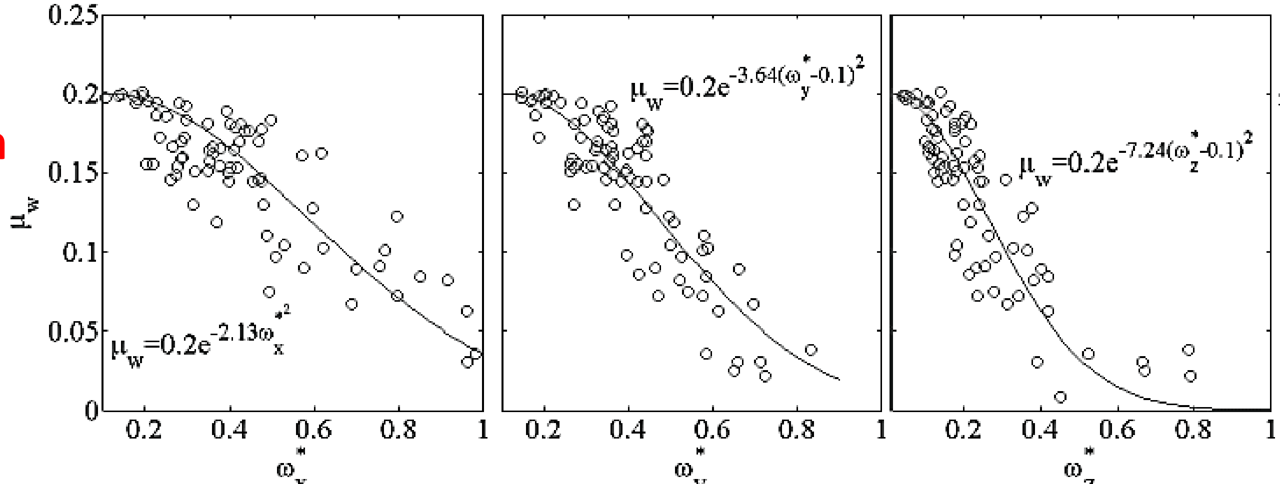
μ_w



Normalized angular speeds

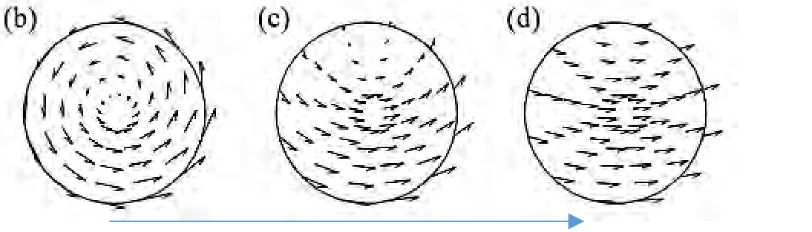


Strong correlation between μ_w & scaled angular speeds

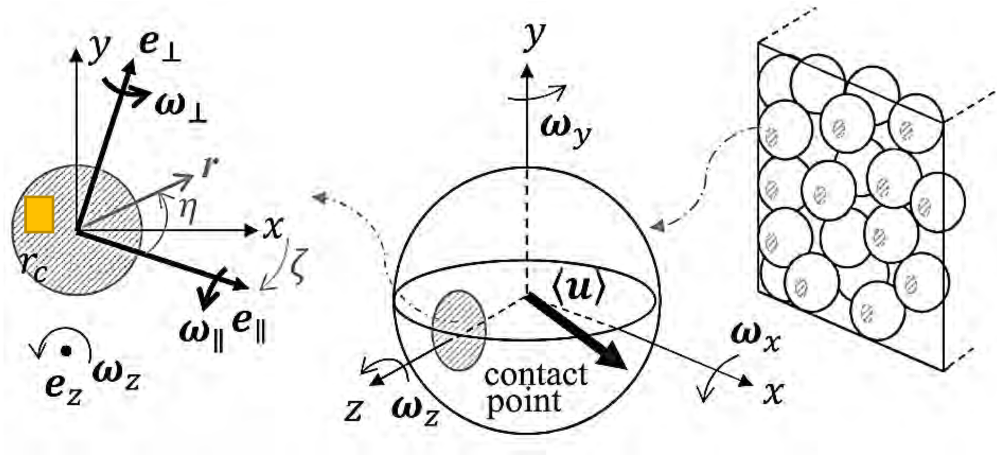


Microscopic interaction model

Farkas et al. (PRL 2002)



Pure spin → Sliding, minor spin



Coulomb friction is 'more' valid over dA along $\mathbf{u}_{tot}$ modified by grain rotation

$$\mathbf{F} = -\frac{f^{sw} P}{A_c} \int \mathbf{u}_{tot} / |\mathbf{u}_{tot}| dA$$

$$\tilde{\mathbf{u}}_{tot} = (\tilde{u}_{\parallel} + R\tilde{\omega}_{\perp} - r \sin \eta \tilde{\omega}_z)\mathbf{e}_{\parallel} + (-R\tilde{\omega}_{\parallel} + r \cos \eta \tilde{\omega}_z)\mathbf{e}_{\perp} + (\tilde{u}_z + r \sin \eta \tilde{\omega}_{\parallel} - r \cos \eta \tilde{\omega}_{\perp})\mathbf{e}_z$$

$$\mu_w = f^{sw} K(\Omega, \tilde{\boldsymbol{\omega}}^*),$$

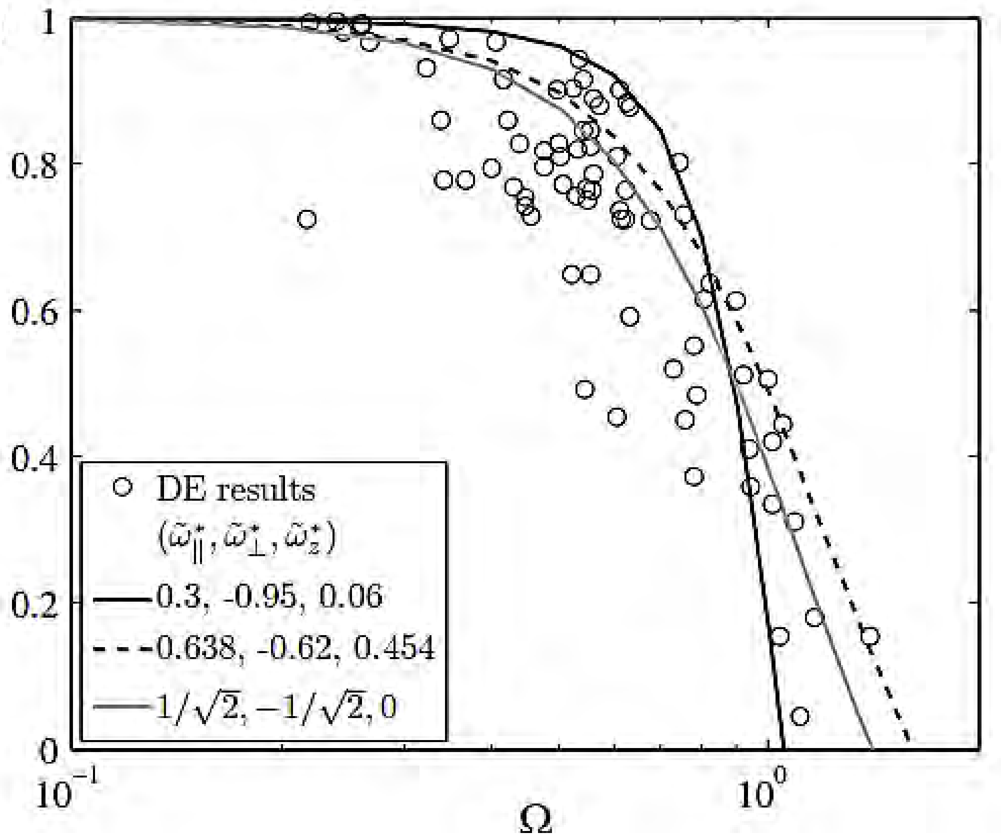
$$K(\Omega, \tilde{\boldsymbol{\omega}}^*) = (\pi r_c^{*2})^{-1} \int_0^{2\pi} \int_0^{r_c^*} \frac{\tilde{\mathbf{u}}_{2D} \cdot \mathbf{e}_{\parallel}}{|\tilde{\mathbf{u}}_{2D}|} r^* dr^* d\eta$$

Rotation index

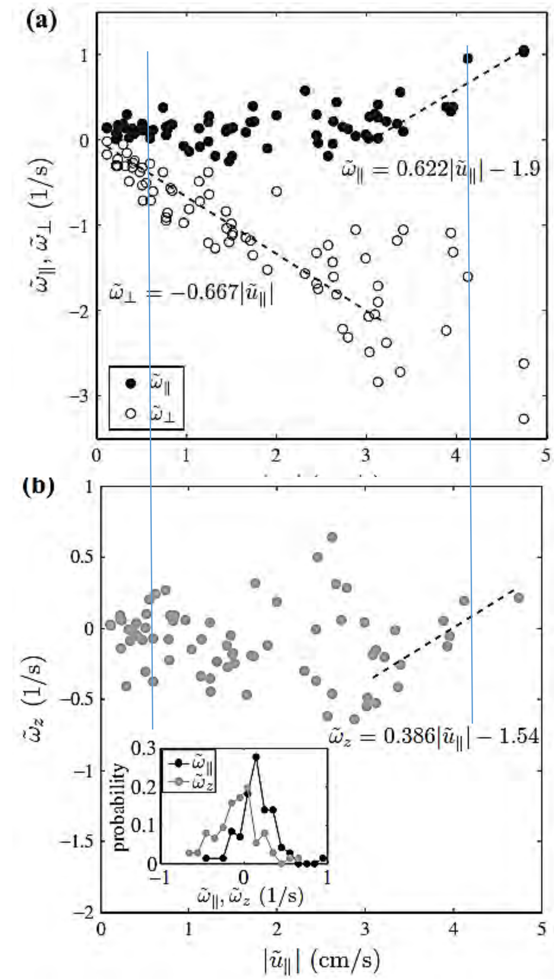
$$\Omega = \frac{R|\omega|}{|u_{\parallel}|}$$

Numerical evidence

$$K = \frac{\mu_w}{f_{sw}}$$



(bulk averaged)

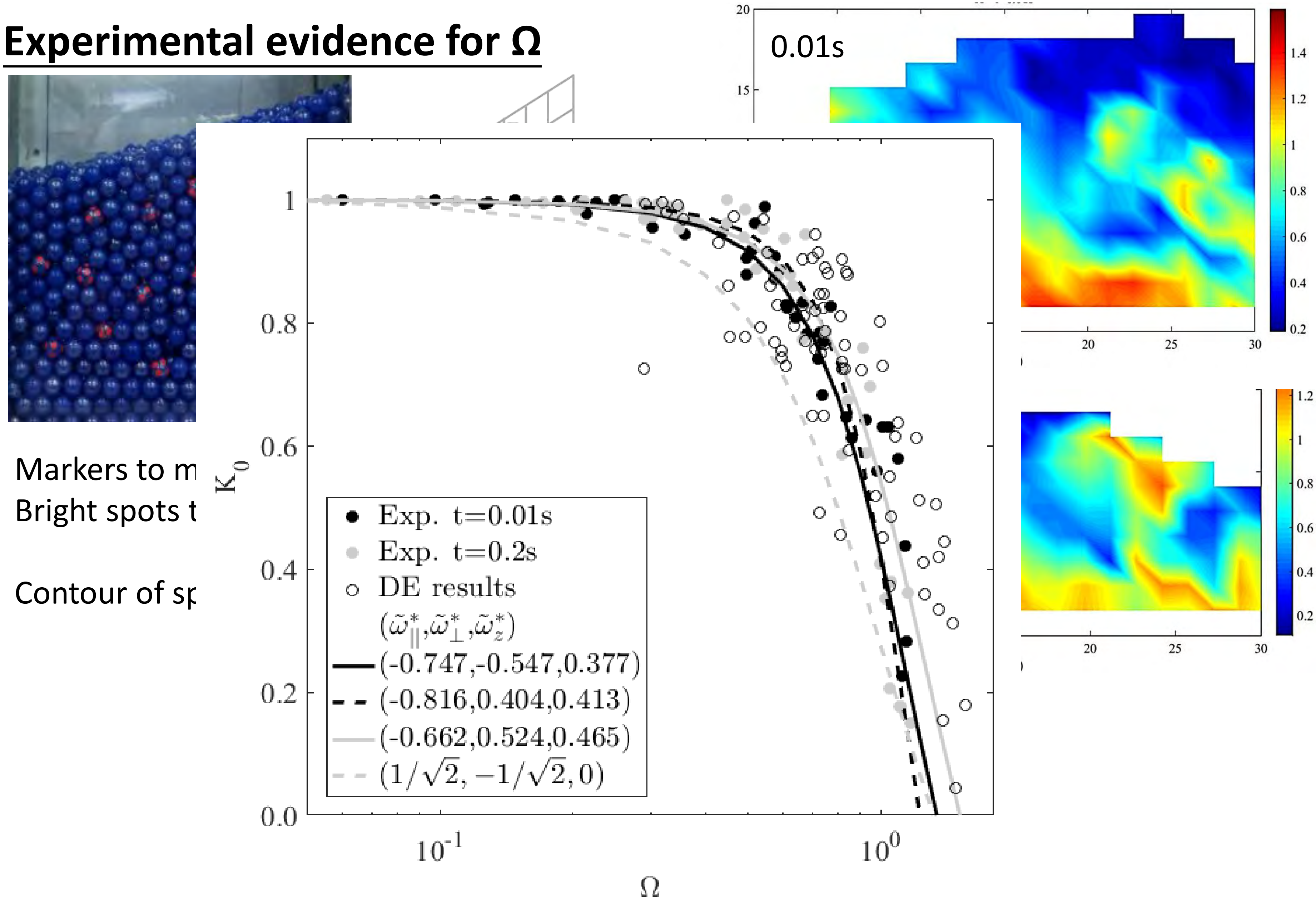


$$K(\Omega, \tilde{\omega}^*) = (\pi r_c^{*2})^{-1} \int_0^{2\pi} \int_0^{r_c^*} \frac{\tilde{\mathbf{u}}_{2D} \cdot \mathbf{e}_{\|}}{|\tilde{\mathbf{u}}_{2D}|} r^* dr^* d\eta$$

$$K_o(\Omega, \tilde{\omega}_{\|}^*, \tilde{\omega}_{\perp}^*) = \frac{1 + \Omega \tilde{\omega}_{\perp}^*}{[(1 + \Omega \tilde{\omega}_{\perp}^*)^2 + (\Omega \tilde{\omega}_{\|}^*)^2]^{1/2}}.$$

Explicit approximation

Experimental evidence for Ω



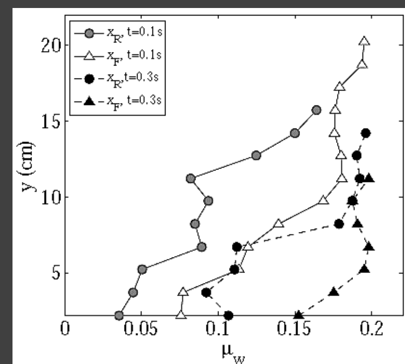
Markers to m
Bright spots t

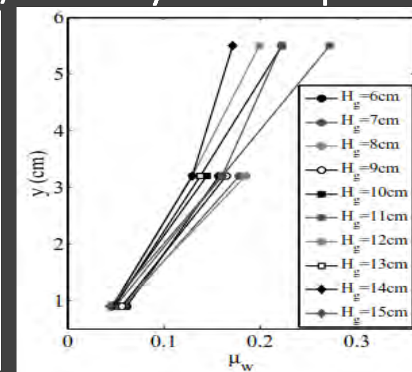
Contour of sp

Summary II

- Coulomb lateral wall friction requires a **non-constant developing μ_w**

  Depth-weakening simulated avalanche / steady flow experiments



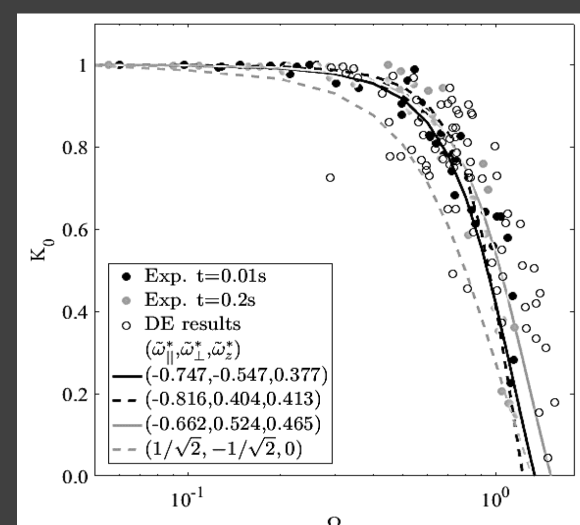


- Reveal concurrent **rotation as the friction reduction mechanism**

  Degradation function K(Ω) from grain-grain f

$$K_o(\Omega, \tilde{\omega}^*_\parallel, \tilde{\omega}^*_\perp) = \frac{1 + \Omega\tilde{\omega}^*_\perp}{[(1 + \Omega\tilde{\omega}^*_\perp)^2 + (\Omega\tilde{\omega}^*_\parallel)^2]^{1/2}}$$

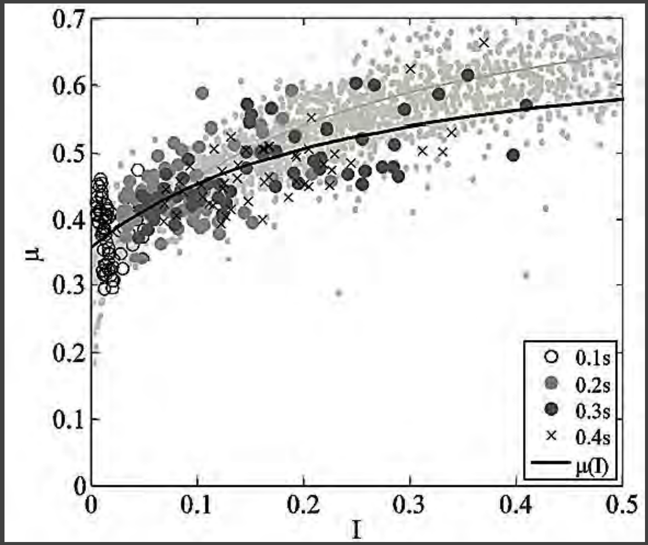
simulation / experiment



Conclusion

Bulk internal friction coefficient

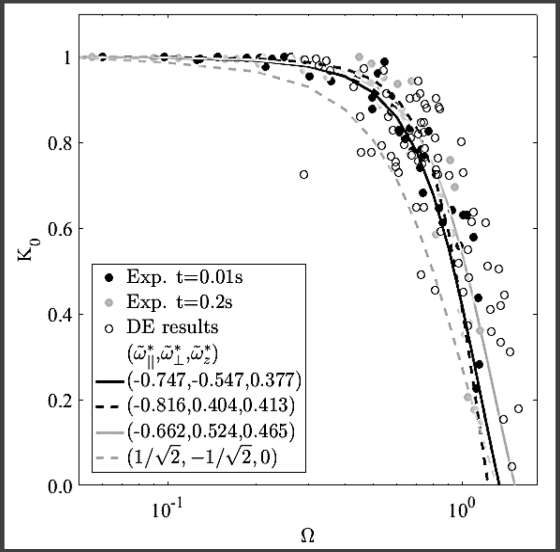
- Monotonic rise of μ –I above I_c~0.02
- Decay trend below I_c
- Non-monotonicity across I_c as a phase transition / bifurcation



Coulomb wall friction coefficient

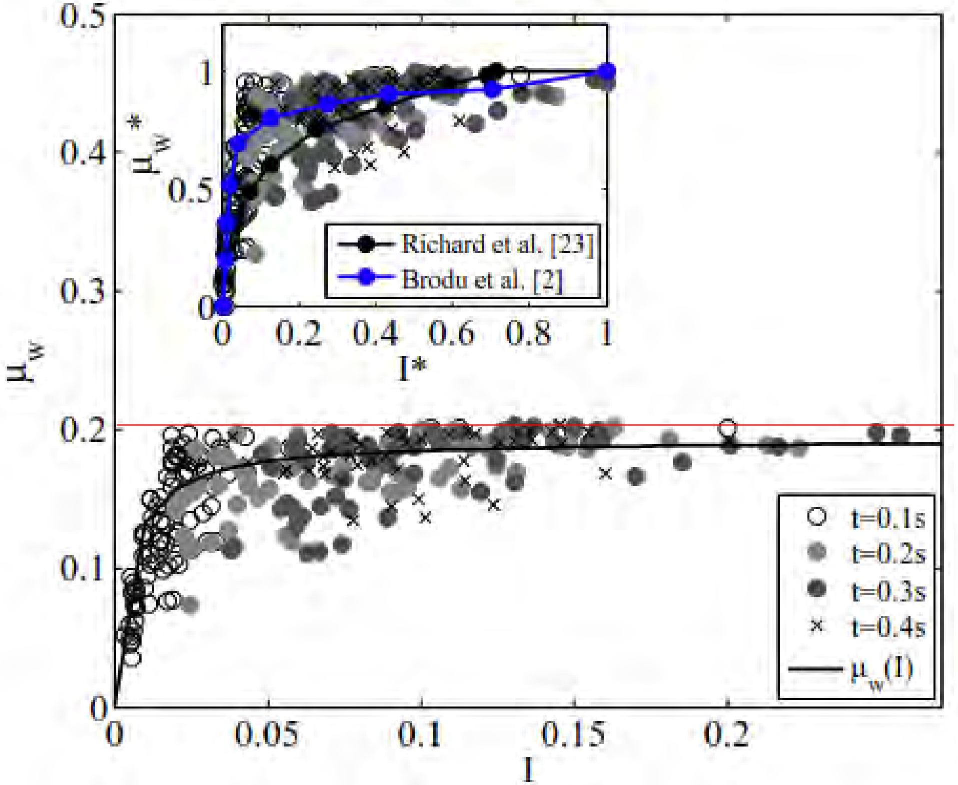
- Non-constant μ_w

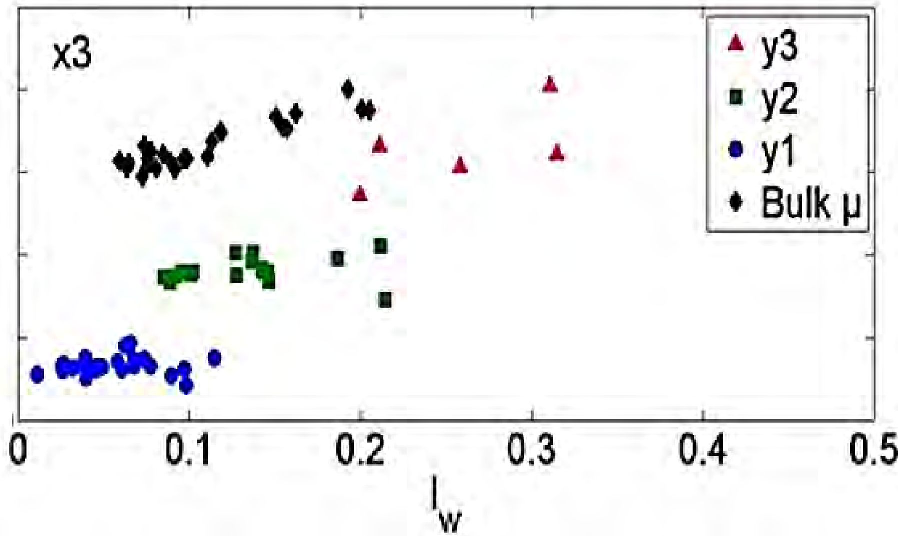
  Depth-weakening, developing
- Grain rotation to friction degradation
- Degradation function K(Ω) from micro f



Thank You

Credits to 黃永達、林正釧、邱廷彥、李庚霖



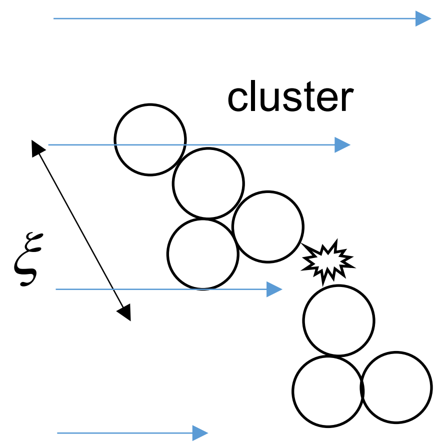


μ_w much smaller than μ

Flow dynamics away from jamming

- Small non-local effect $(l^2 \nabla^2 I << 1)$
- By Taylor expansion and scaling arguments, we have **β=2** and an eddy-viscosity-like stress scaling

$$\tau \sim \rho \xi^2 \dot{\gamma}^2$$



- Correlation length $\xi(\mu) = Ad\left[r(\mu) - r_c\right]^{-1/2}$
- Long-range momentum transmission due to collisions between clusters.

Ertas & Halsey 2002; Pouliquen 2004; Staron 2008; Mills et al 2013

Bistability-free approximation

- In the limit of $\chi=0$

$$t_0 \frac{DI}{Dt} = \zeta^2 \nabla^2 I + [r(\mu) - r_c] I - B I^3,$$

  - Single threshold μ_{stop}
  - Steady-state energy balance

$$\tau \dot{\gamma} = \rho l_e(\mu)^2 \dot{\gamma}^3 - \frac{A_0 l_e(\mu)^2}{T_r} P \nabla^2 I, \qquad l_e(\mu) = A_0^{-1} \sqrt{\mu}\, l(\mu)$$

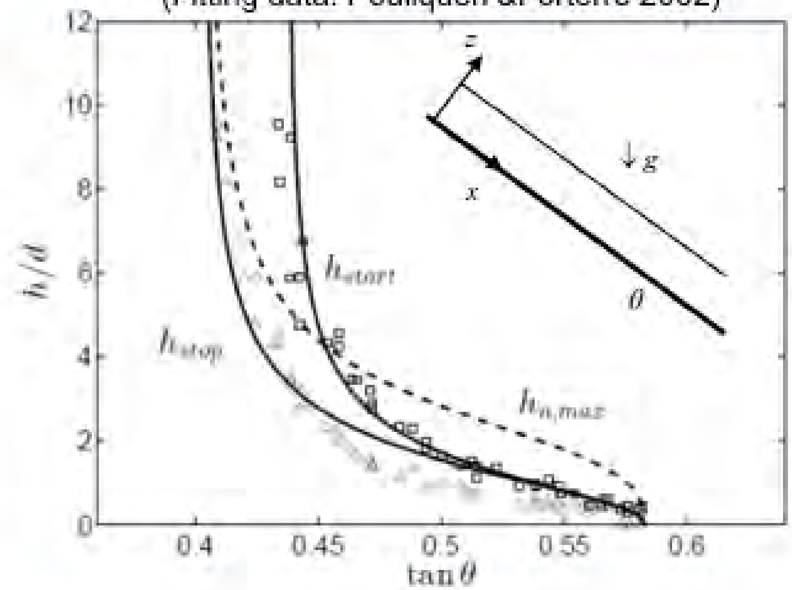
Shear work ($\tau \dot{\gamma}$)

Dissipation ($\rho l_e(\mu)^2 \dot{\gamma}^3$)

Diffusion of fluctuation during rearrangement via correlated motion ($\frac{A_0 l_e(\mu)^2}{T_r} P \nabla^2 I$)

Application to uniform incline flow

(Fitting data: Pouliquen &Forterre 2002)



- h_{start} and h_{stop} phenomenon

$$h_{stop}(\theta) \approx \frac{\pi}{2}\frac{\zeta}{\sqrt{r(\theta)-r(\mu^*)}}.$$

$$h_{start}(\theta) = \frac{\pi}{2}\frac{\zeta}{\sqrt{r(\theta)}}.$$

- In the **thick-layer limit,**

Predicted mean flow property	=	Pouliquen flow rule (1999)
$\bar{u} = (\pi\zeta/5d)\sqrt{\phi g h \cos\theta}(h/h_{stop}),$		$\bar{u}/\sqrt{gh} = \beta\, h/h_{stop}$

because of the following scaling equivalence:

$$\rho g h \sim \rho h_{stop}^{\ 2}(\bar{u}/h)^2 \longrightarrow \tau \sim \rho l^2 \dot{\gamma}^2$$

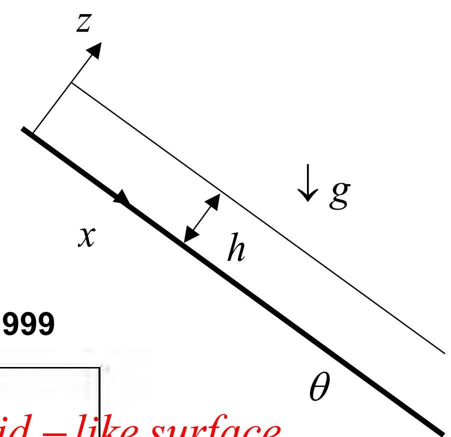
h_{stop} ~ Correlation length (Ertas & Halsey 2002; GDR midi 2004; Staron 2008; Baran et al. 2006)

Application to gravity-driven inclined flows

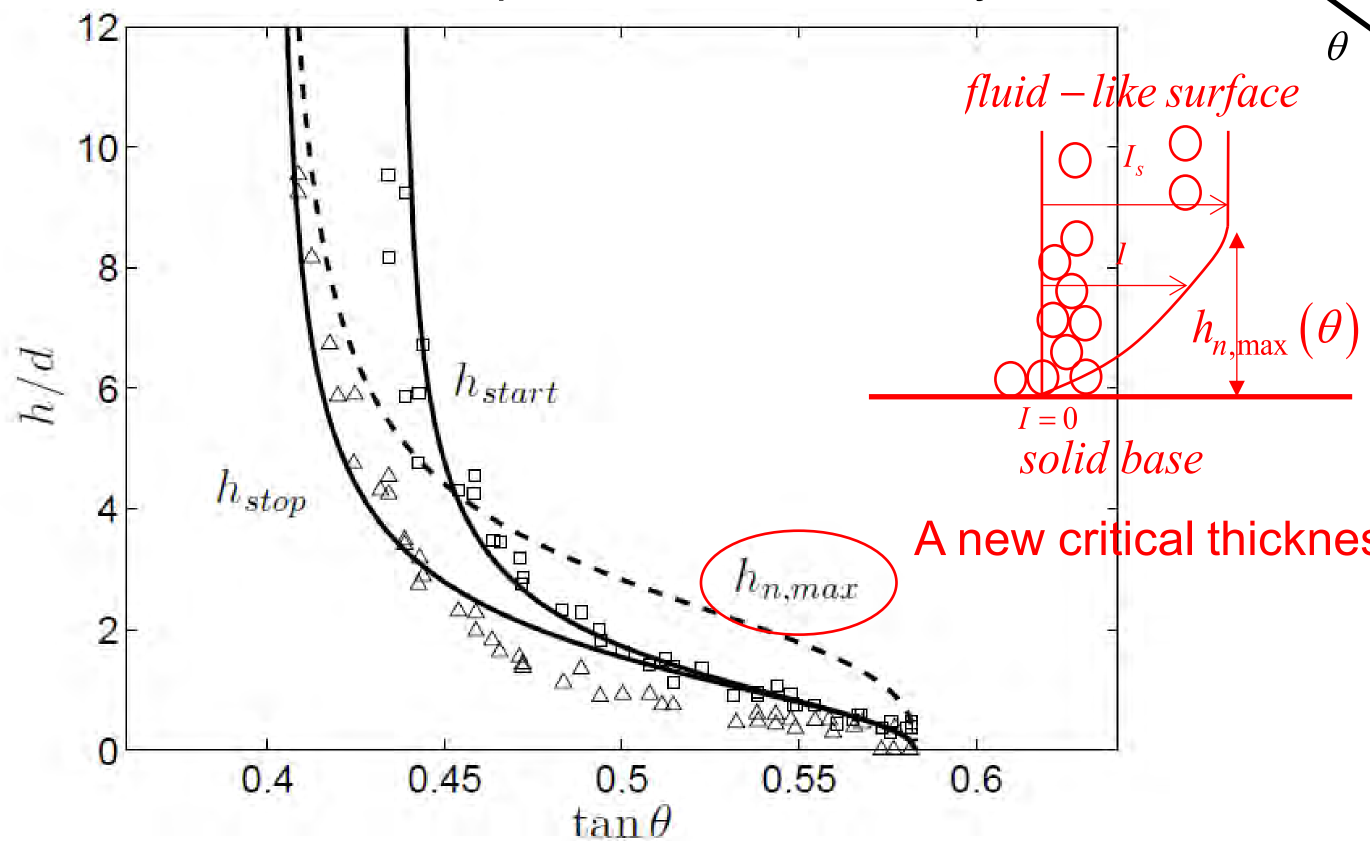
- **Hysteresis critical thickness.**

At a specific angle θ,

- Static pile ➔ Flow, when h> $\mathbf{h_{start}}$
- Flow ➔ Static pile, when h< $\mathbf{h_{stop}}$



Pouliquen 1996, 1999; Daerr & Douady 1999

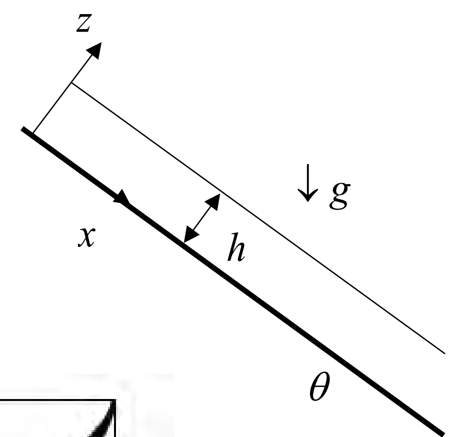


A new critical thickness!

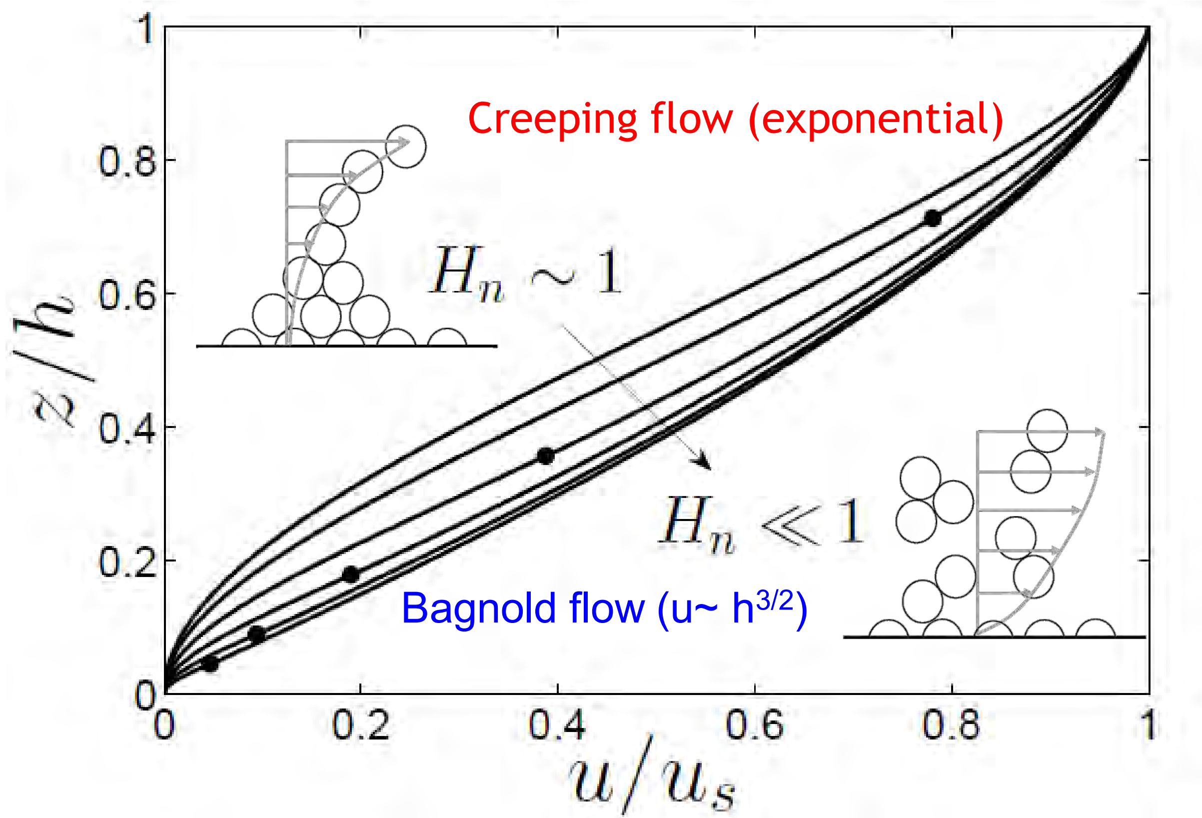
The data are extracted from Forterre & Pouliquen 2002

Application to gravity-driven inclined flows

- **Creeping and Bagnold flow velocity profiles**
  - Non-local parameter: $H_n \equiv h_{n,\max}(\theta)/h$
  - Observed in DEM (Silbert et al 2003)



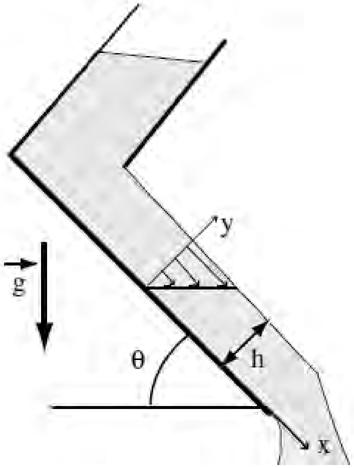
Normalized height

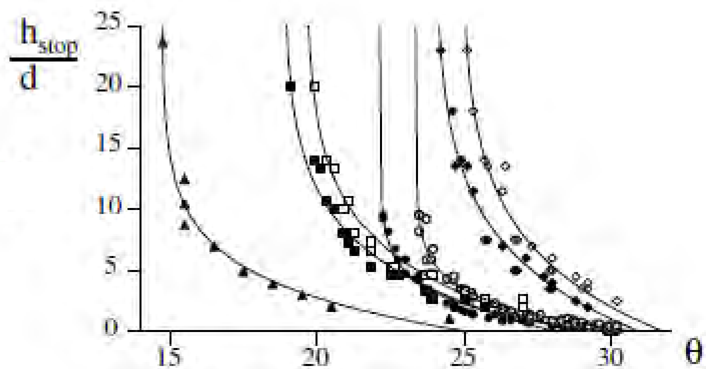


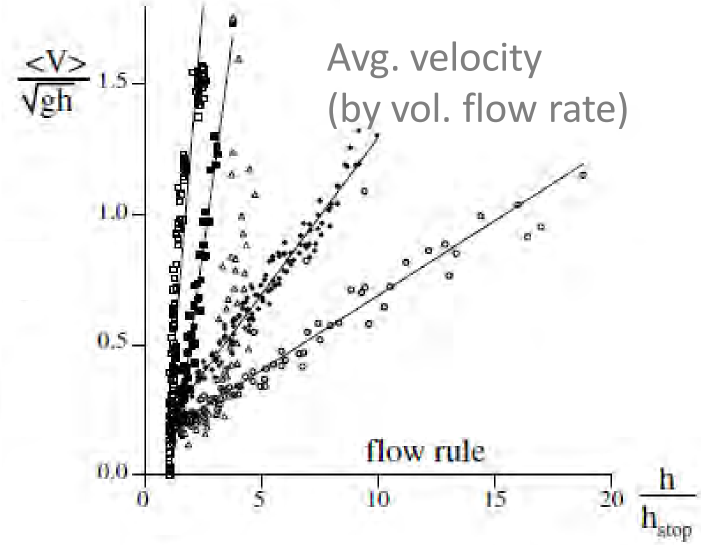
Normalized velocity

2D Inclined Free surface

Pouliquen et. al
(1999 onwards)







Empirical models:

$$\theta_{stop} \sim fn\left(\frac{h_{stop}}{d}\right)$$

$$\frac{\langle V \rangle}{\sqrt{gh}} = \alpha + \beta \frac{h}{h_{stop}(\theta)}$$

$$\Rightarrow \theta_{stop} = fn\left(\frac{d}{h_{stop}}\right) = \frac{\| U \|}{\sqrt{gh}}\frac{d}{h} - \alpha\frac{d}{h}$$

At equilibrium, $Tan(\theta_{stop}) = \mu_{eff}$