A COUPLED 1D AND 2D SPH-SWES MODEL FOR OPEN CHANNEL FLOWS IN COMPLEX CHANNEL GEOMETRIES

SPEAKER: DR. KAO-HUA CHANG (張高華)

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ACKNOWLEDGEMENT:

PROF. TONY WEN-HANN SHEU (許文翰 教授);

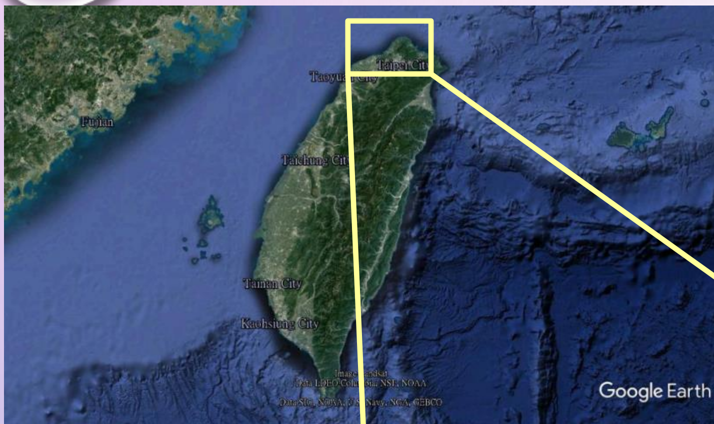
PROF. TSANG-JUNG CHANG (張倉榮 教授)

OUTLINE

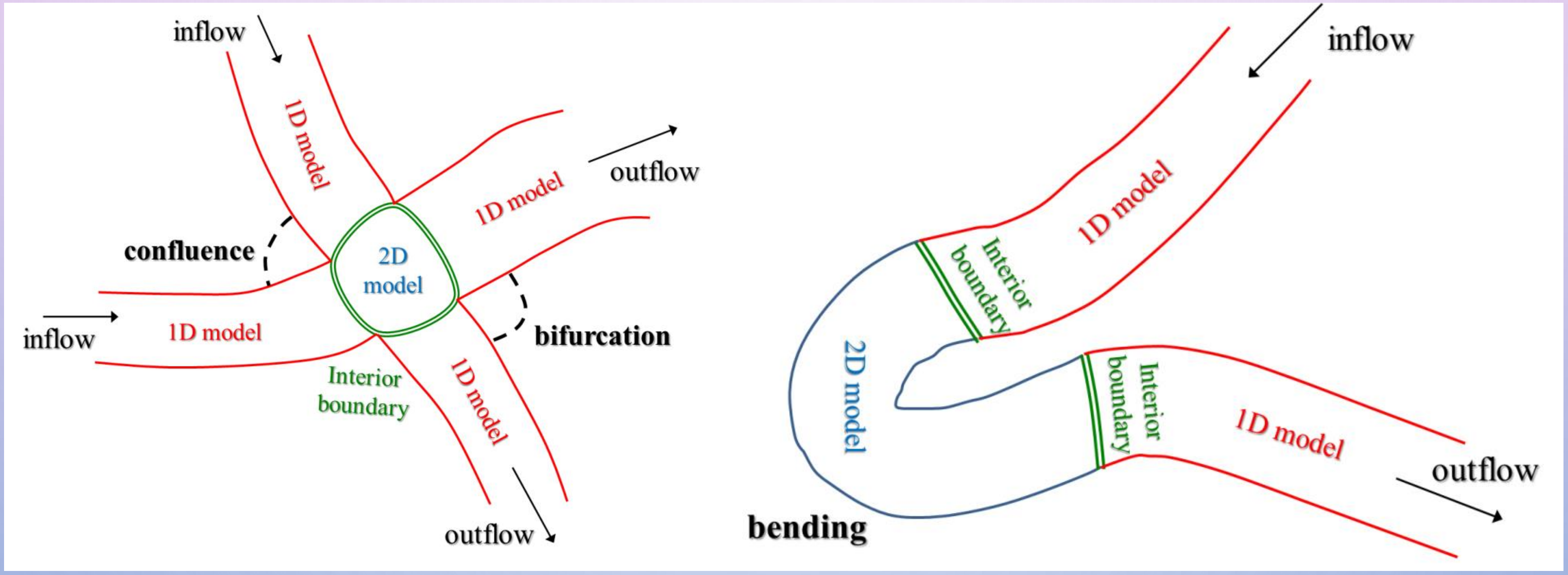
1. Motivation
2. SPH-SWEs Model
3. Boundary Treatments
4. Results and Discussion
5. Summary

MOTIVATION

- In hydraulic engineering, river flooding (*overbank flow* and *mud flow*) and river pollution are both concerned problems.
- How to predict discharge, depth and concentration is a key issue.



- For large river systems. 1D hydrodynamic models are extensively used due to their efficiency. Besides, 2D hydrodynamic models are also adopted to enhance the accuracy.

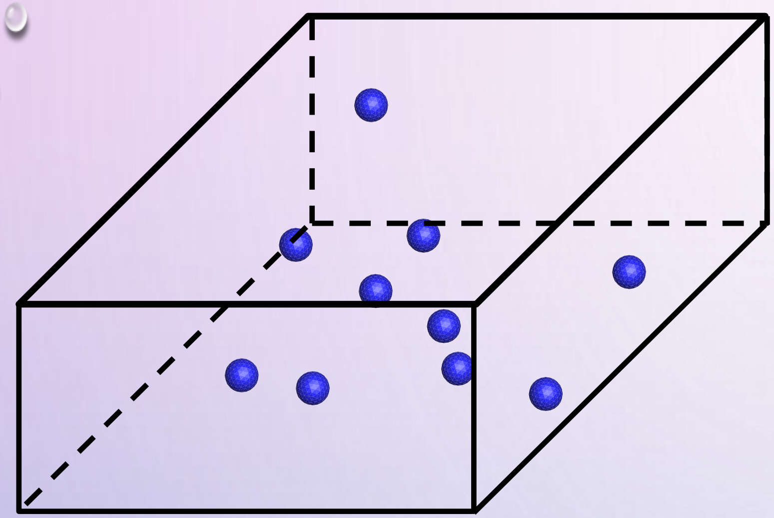


Goal

➢ Combining the efficiency of 1D model and the accuracy of 2D model.

SPH-SWEs MODEL

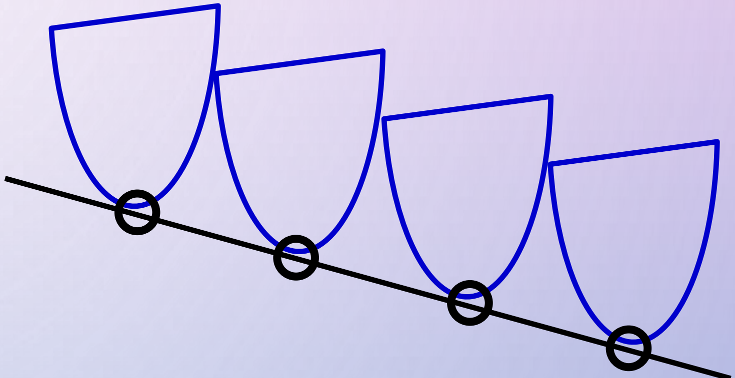
SHALLOW WATER EQUATIONS



3D Navier-Stokes equations

Density $\rho = \rho_w$

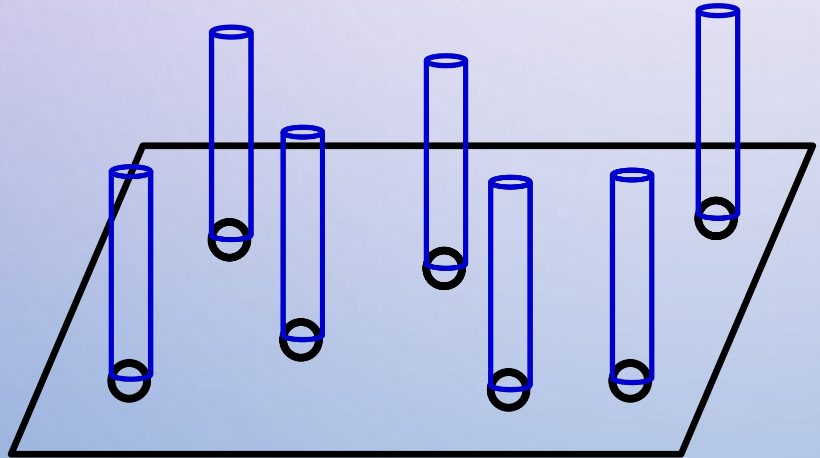
Mass $m = \rho V = \rho_w \left(\Delta x_0\right)^3$



1D shallow water equations

Density $\rho = \rho_w A$ **Area-averaged**

Mass $m = \rho V = \rho_w A \Delta x_0$



2D shallow water equations

Density $\rho = \rho_w d_w$ **Depth-averaged**

Mass $m = \rho V = \rho_w d_w \left(\Delta x_0\right)^2$

Pressure is assumed to be hydrostatic

$$P=\gamma d_w=\rho g d_w=\rho c^2,$$

where *c* is the celerity. $c=\sqrt{gd_w}$

1D shallow water equations

$$\frac{DA}{Dt}=-A\frac{\partial u}{\partial x}$$

$$\frac{DQ}{Dt}=-Q\frac{\partial u}{\partial x}-gA\frac{\partial\left(d_w+z_b\right)}{\partial x}-gAS_f$$

2D shallow water equations

$$\frac{Dd_w}{Dt}=-d_w\nabla\cdot\vec{U}$$

$$\frac{D\vec{U}}{Dt}=-g\nabla\left(d_w+z_b\right)-g\vec{S}_f$$

In the above,

$$\vec{U}=(u,v),\ \nabla=\frac{\partial}{\partial x}\vec{x}+\frac{\partial}{\partial y}\vec{y},\ \frac{D(\ )}{Dt}=\frac{\partial(\ )}{\partial t}+\vec{U}\cdot\nabla(\ )$$

A **is the wetted cross-section area,**
d_w is the water deoth,
Q **is the discharge,**
U **is the velocity,**
z_b is the ground elevation,
S_f is the friction slope and
g **is the gravity acceleration.**

SMOOTHED PARTICLE HYDRODYNAMICS

Kernel Function (ω)

(1) The integral of a kernel function within its compact domain should be equal to one.

$$\int_{\Omega} \omega\left(r_{ij}, h_i\right) \mathrm{d}V = 1$$

(2) As the smoothing length h approaching zero, the kernel function will become a Dirac delta function.

$$\lim_{h_i \to 0} \omega\left(r_{ij}, h_i\right) = \delta(r_{ij})$$

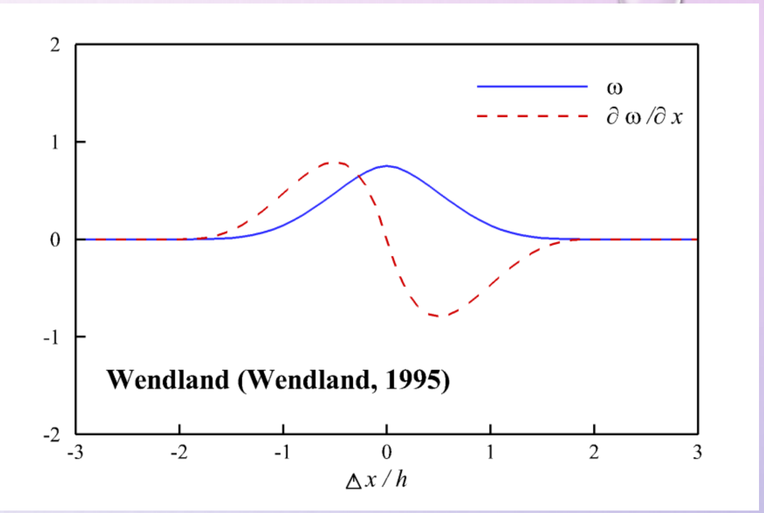
(3) The kernel function should be compactly supported.

$$\omega\left(r_{ij}, h_i\right) = 0 \quad \text{when} \quad r_{ij} > \kappa h_i$$

(4) The kernel function is assumed to be symmetric.

$$\omega\left(r_{ij}, h_i\right) = \omega\left(r_{ji}, h_i\right)$$

$$\nabla_i \omega\left(r_{ij}, h_i\right) = -\nabla_j \omega\left(r_{ij}, h_i\right)$$



$$\omega(s,h) = \frac{3}{4h} \begin{cases} (1+2s)(1-\frac{s}{2})^4 & 0 \le s \le 2 \\ 0 & s > 2 \end{cases}$$

where $s = \dfrac{\Delta x}{h}$

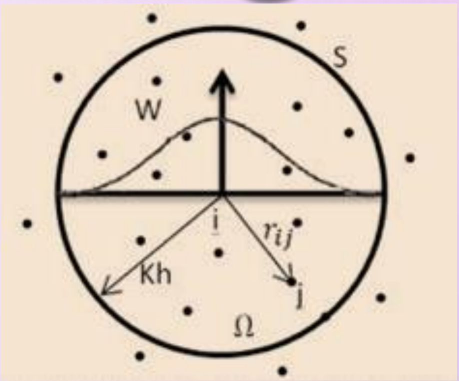
◆ *Summation operator*

$$\phi_i = \int_{\Omega} \phi(\vec{r})\,\delta(\vec{r}_i - \vec{r})\,\mathrm{d}V$$

$$\langle \phi \rangle_i = \int \phi(\vec{r})\,\omega(|\vec{r}_i - \vec{r}|, h)\,\mathrm{d}V$$

$$= \sum_j \frac{m_j}{\rho_j} \phi_j \omega(r_{ij}, h_i)$$ ⟵ **Discretization into particles**

where *m* is the mass of a particle,
***ρ* is the density of a particle,**
***ω* is the kernel function,**
***h* is the smoothing length and**
$\langle\phi\rangle_i$ denotes approximated ϕ_i.



◆ *Differential operator*

$$\langle \nabla \phi \rangle_i = \int_{\Omega} \nabla \phi(\vec{r})\,\omega(|\vec{r}_i - \vec{r}|, h)\,\mathrm{d}V$$

$$= \int_{\Omega} \nabla \left[\phi(\vec{r})\,\omega(|\vec{r}_i - \vec{r}|, h) \right] \mathrm{d}V - \int_{\Omega} \phi(\vec{r}) \nabla \omega(|\vec{r}_i - \vec{r}|, h)\,\mathrm{d}V$$

$$\nabla(\phi\omega) = \phi\nabla\omega + \omega\nabla\phi$$

$$= \int_{\partial\Omega} \phi(\vec{r})\,\omega(|\vec{r}_i - \vec{r}_i|, h)\,\vec{n}\,\mathrm{d}S - \int_{\Omega} \phi(\vec{r}) \nabla \omega(|\vec{r}_i - \vec{r}|, h)\,\mathrm{d}V$$

$$= -\int_{\Omega} \phi(\vec{r}) \nabla \omega(|\vec{r}_i - \vec{r}|, h)\,\mathrm{d}V$$

$$\nabla(\ ) = -\nabla_i(\ )$$

$$= \sum_j \frac{m_j}{\rho_j} \phi_j \nabla_i \omega(r_{ij}, h_i)$$ ⟵ **Discretization into particles**

◆Divergent operator in the asymmetric form

$$\nabla\cdot\vec{\phi}=\frac{1}{\rho}\left[\nabla\cdot\left(\rho\vec{\phi}\right)-\vec{\phi}\cdot\nabla\rho\right]$$

Using the differential operator $\langle\nabla\phi\rangle_i=\sum_j\frac{m_j}{\rho_j}\phi_j\nabla_i\omega_{ij}$

$$\left\langle\nabla\cdot\vec{\phi}\right\rangle_i=\frac{1}{\rho_i}\sum_j m_j\left(\vec{\phi}_i-\vec{\phi}_j\right)\cdot\nabla_i\omega_{ij}$$

◆Gradient operator in the symmetric form

$$\nabla\phi=\rho\left[\nabla\left(\frac{\phi}{\rho}\right)+\frac{\phi}{\rho^2}\nabla\rho\right]$$

Using the differential operator $\langle\nabla\phi\rangle_i=\sum_j\frac{m_j}{\rho_j}\phi_j\nabla_i\omega_{ij}$

$$\langle\nabla\phi\rangle_i=\rho_i\sum_j m_j\left(\frac{\phi_i}{\rho_i^2}+\frac{\phi_j}{\rho_j^2}\right)\nabla_i\omega_{ij}$$

Ref. 1) GR Liu, MB Liu, (2003). Smoothed particle hydrodynamics: A meshfree particle method, World Scientific, Singapore.

DIFFUSIVE TERMS

I. Artificial viscous term

$$\left(\frac{\mathrm{D}\vec{U}}{\mathrm{D}t}\right)_i = -g\sum_j m_j \frac{d_{w,i}+d_{w,j}}{d_{w,j}}\nabla_i\omega_{ij} - g\vec{S}_f + \sum_j \nu_{ij}V_j\left(\vec{U}_i-\vec{U}_j\right)\frac{\vec{r}_{ij}\cdot\nabla_i\omega_{ij}}{\left|\vec{r}_{ij}\right|^2}$$

Kinetic viscosity $\nu_{ij} = \alpha h_{ij} c_{ij}$, where $h_{ij} = 0.5\left(h_i + h_j\right)$, $c_{ij} = 0.5\left(c_i + c_j\right)$,

$\alpha = 0.2.$

2. Density diffusive term

$$\left(\frac{\mathrm{D}d_w}{\mathrm{D}t}\right)_i = \sum_j m_j\left(\vec{U}_i-\vec{U}_j\right)\cdot\nabla_i\omega_{ij} + \sum_j D_{ij}V_j\left(d_{w,i}-d_{w,j}\right)\frac{\vec{r}_{ij}\cdot\nabla_i\omega_{ij}}{\left|\vec{r}_{ij}\right|^2}$$

$D_{ij} = \beta h_{ij} c_{ij}$, where $h_{ij} = 0.5\left(h_i + h_j\right)$, $c_{ij} = 0.5\left(c_i + c_j\right)$, $\beta = 0.2.$

Ref. 1) JJ Monaghan, Rep Prog Phys, 68: 1703-1759, 2005.

2) M Antuono et al., Comput Phys Commun, 183: 2570-2580, 2012.

TIME MARCHING METHOD

Modified Verlet method

$$\vec{U}_i^{n+\frac{1}{2}} = \vec{U}_i^n + 0.5 \cdot \Delta t \left(\frac{D\vec{U}}{Dt} \right)_i^n$$

$$\rho_i^{n+\frac{1}{2}} = \rho_i^n + 0.5 \cdot \Delta t \left(\frac{D\rho}{Dt} \right)_i^n$$

$$\vec{r}_i^{n+\frac{1}{2}} = \vec{r}_i^n + 0.5 \cdot \Delta t \cdot \vec{U}_i^n$$

$\Rightarrow$

$$\vec{U}_i^{n+1} = \vec{U}_i^n + \Delta t \left(\frac{D\vec{U}}{Dt} \right)_i^{n+\frac{1}{2}}$$

$\Downarrow$

$$\vec{r}_i^{n+1} = \vec{r}_i^{n+\frac{1}{2}} + 0.5 \cdot \Delta t \cdot \vec{U}_i^{n+1}$$

$\Downarrow$

$$\rho_i^{n+1} = \rho_i^{n+\frac{1}{2}} + 0.5 \cdot \Delta t \left(\frac{D\rho}{Dt} \right)_i^{n+1}$$

$$\Delta t \le CFL \cdot \min_i \left(\frac{h_i}{\left| \vec{U}_i \right| + c_i} \right), \text{ where } CFL = 0.8.$$

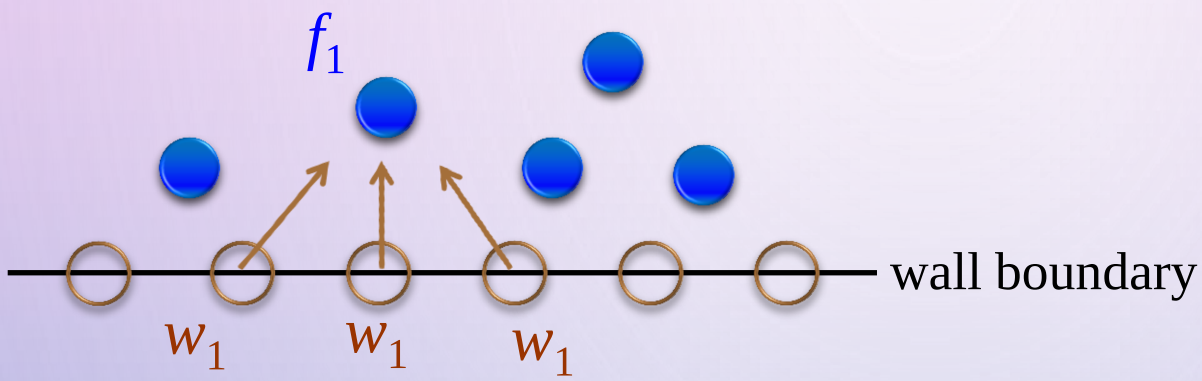
Ref. 1) D Molteni, A Colagrossi, Comput Phys Commun, 180: 861-872, 2009.

BOUNDARY TREATMENTS

WALL BOUNDARY

□Repellent particle

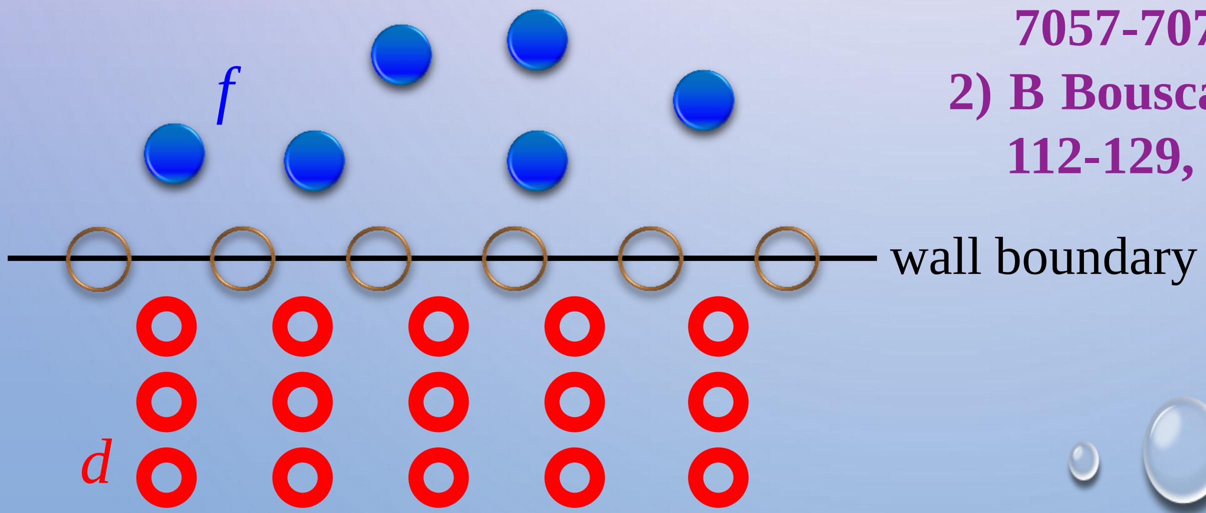
Ref. 1) JJ Monaghan, A Kos, J Waterw Port C-ASCE, 125: 145-154, 1999.



□Dummy particle

Ref. 1) S Adami et al., J Comput Phys, 231: 7057-7075, 2012.

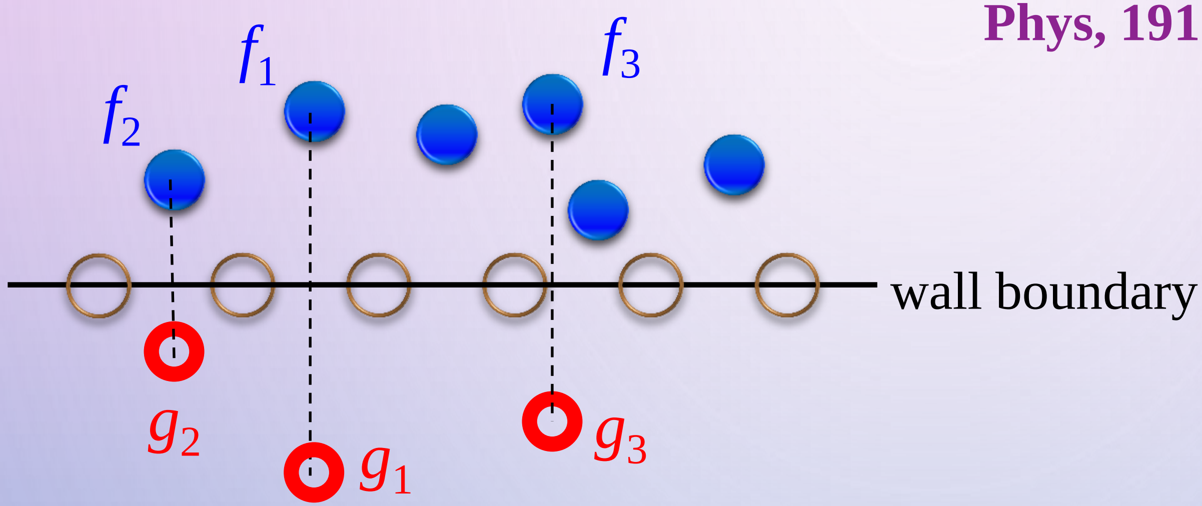
2) B Bouscasse et al., J Fluid Struct, 42: 112-129, 2013.



□ Ghost particle

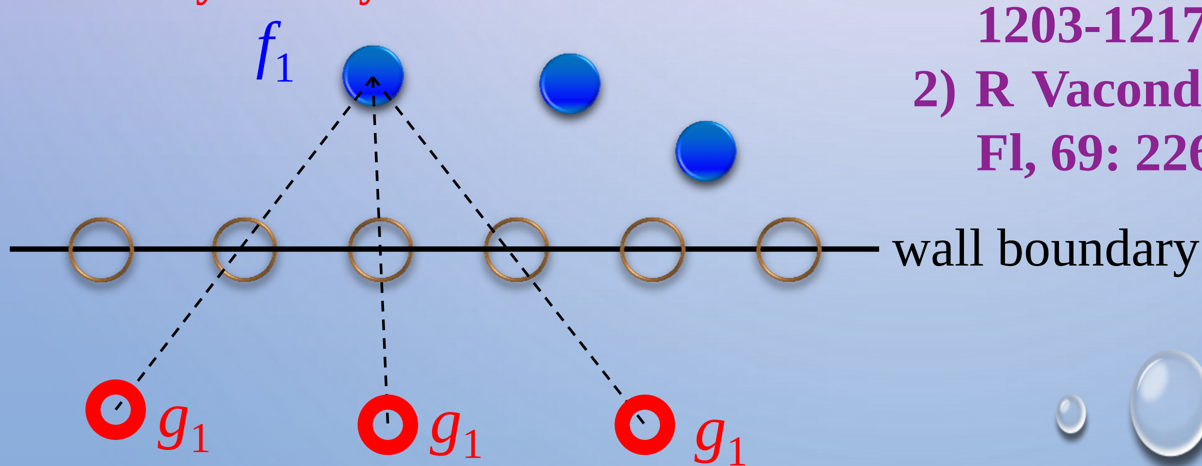
✓ **Plane-symmetry**

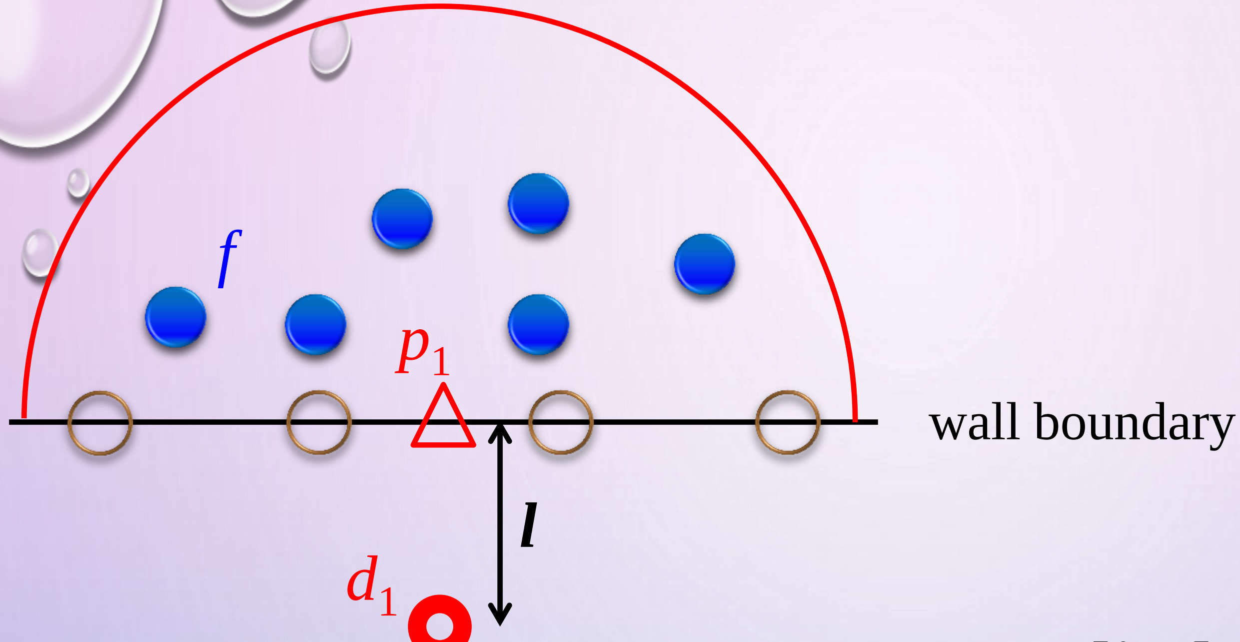
Ref. 1) A Colagrossi, M Landrini, J Comput Phys, 191: 448-475, 2003.

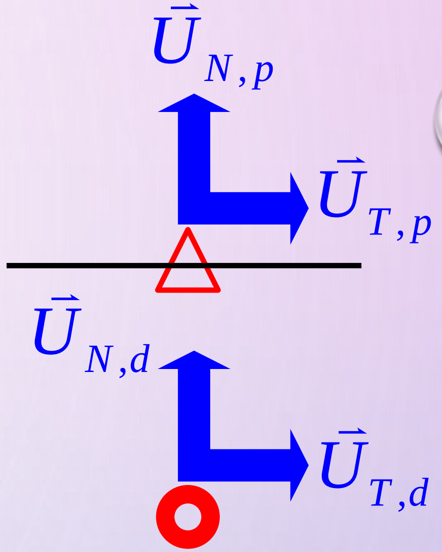


✓ **Point-symmetry**

Ref. 1) A Ferrari et al., Comput Fluids, 38: 1203-1217, 2009.
2) R Vacondio et al., Int J Numer Meth Fl, 69: 226-253, 2012.







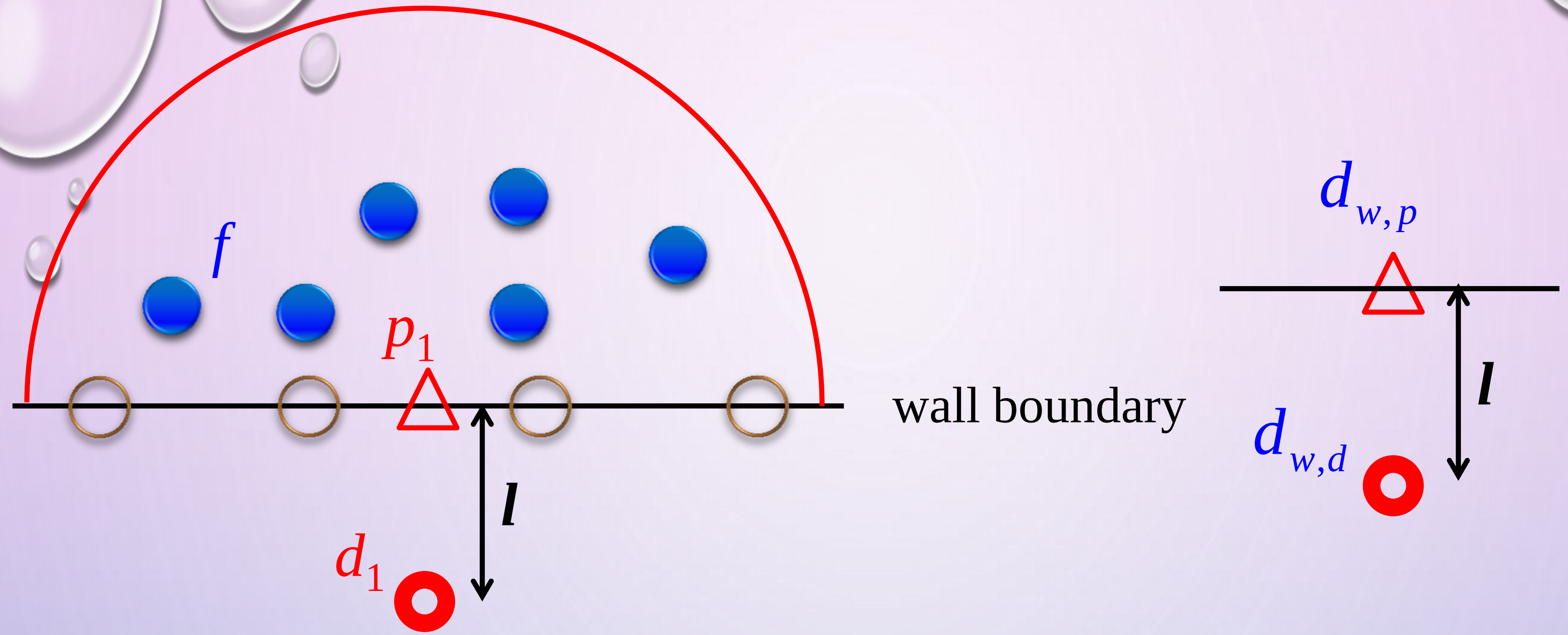
For no-slip boundary

$$\vec{U}_{i\in p} = \frac{\sum_{j\in f} V_j \vec{U}_j \omega_{ij}}{\sum_{j\in f} V_j \omega_{ij}} = \sum_{j\in f} V_j \vec{U}_j \tilde{\omega}_{ij} \Longrightarrow$$

$$\nabla\cdot\vec{U} \begin{cases} \vec{U}_{N,p} = 0 \\ \vec{U}_{T,p} = 0 \end{cases}$$

$$\nabla^2\vec{U} \begin{cases} \vec{U}_{N,d} = \vec{U}_{N,p} \\ \vec{U}_{T,d} = -\vec{U}_{T,p} \end{cases}$$

Ref. 1) S Adami et al., J Comput Phys, 231: 7057-7075, 2012.
2) B Bouscasse et al., J Fluid Struct, 42: 112-129, 2013.



Impermeable boundary condition

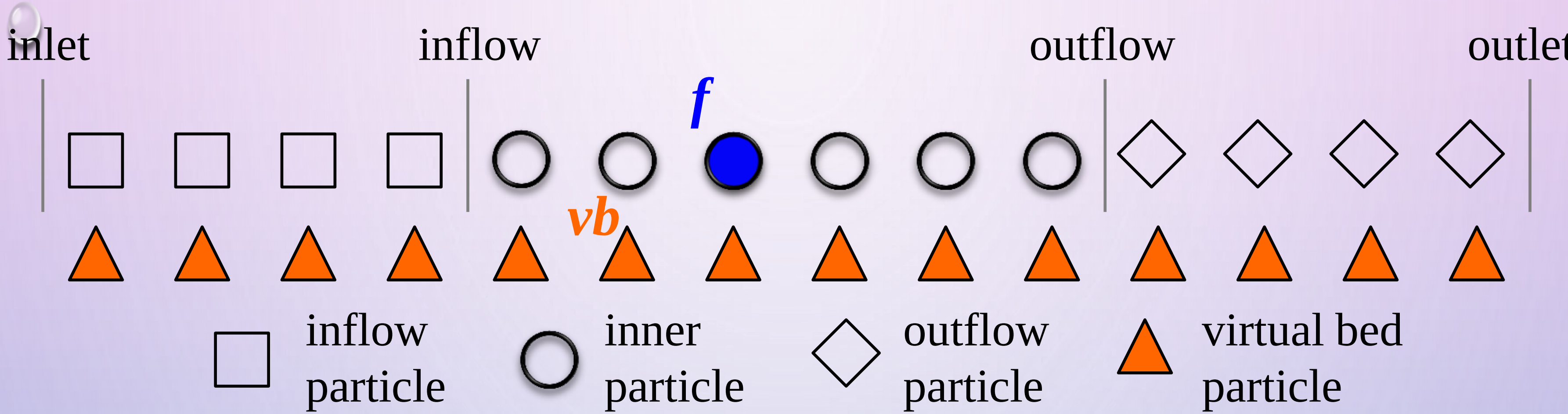
$$\frac{\mathrm{d}\vec{u}_d}{\mathrm{d}t}\cdot\vec{N} = \left[-g\nabla\left(d_w + z_b\right) - g\vec{S}_f\right]\cdot\vec{N} = 0$$

$$\Rightarrow \nabla d_w \cdot \vec{N} = \left(-\nabla z_b - \vec{S}_f\right)\cdot\vec{N} \Rightarrow \frac{d_{w,j\in p} - d_{w,i\in d}}{l} = \left[\sum_{j\in f} V_j\left(-\nabla z_{b,j} - \vec{S}_{f,j}\right)\tilde{\omega}_{ij}\right]\cdot\vec{N}_{ij}$$

$$\Rightarrow d_{w,i\in d} = \sum_{j\in f} V_j\left[d_{w,j} + \left(-\nabla z_{b,j} - \vec{S}_{f,j}\right)\cdot\vec{N}_{ij}\right]\tilde{\omega}_{ij}$$

Ref. 1) S Adami et al., J Comput Phys, 231: 7057-7075, 2012.
2) B Bouscasse et al., J Fluid Struct, 42: 112-129, 2013.

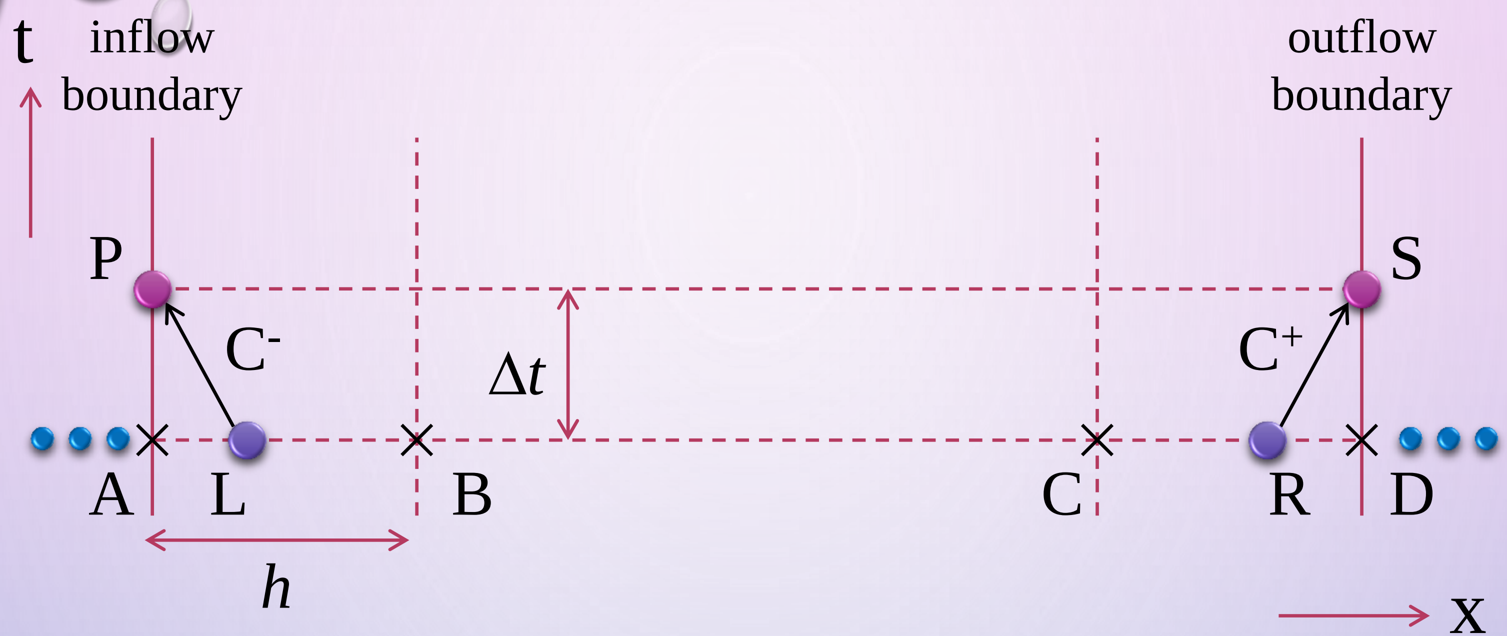
IN/OUT-FLOW BOUNDARY



$$\langle z_b \rangle_{i\in f} = \sum_{j\in vb} V_j z_{b,j} \tilde{\omega}_{ji},\ \langle n_a \rangle_{i\in f} = \sum_{j\in vb} V_j n_{a,j} \tilde{\omega}_{ji},$$

where $\tilde{\omega}_{ji} = \dfrac{\omega_{ji}}{\sum_{j\in vb} V_j z_{b,j} \omega_{ji}}$

Ref. 1) R Vacondio et al., Int J Numer Meth Fl, 69: 226-253, 2012.
2) TJ Chang, KH Chang, J Hydraul Eng-ASCE, 139:1142-1149, 2013.



Characteristics equations

$$C^+:\quad u_S = u_R - \frac{g}{c_R}\left(d_{w,S} - d_{w,R}\right) + g\left(S_{0,R} - S_{f,R}\right)\Delta t$$
$$\frac{x_S - x_R}{\Delta t} = u_R + c_R$$

$$C^-:\quad u_P = u_L + \frac{g}{c_L}\left(d_{w,P} - d_{w,L}\right) + g\left(S_{0,L} - S_{f,L}\right)\Delta t$$
$$\frac{x_P - x_L}{\Delta t} = u_L + c_L$$

- **Subcritical flows at outflow boundaries**

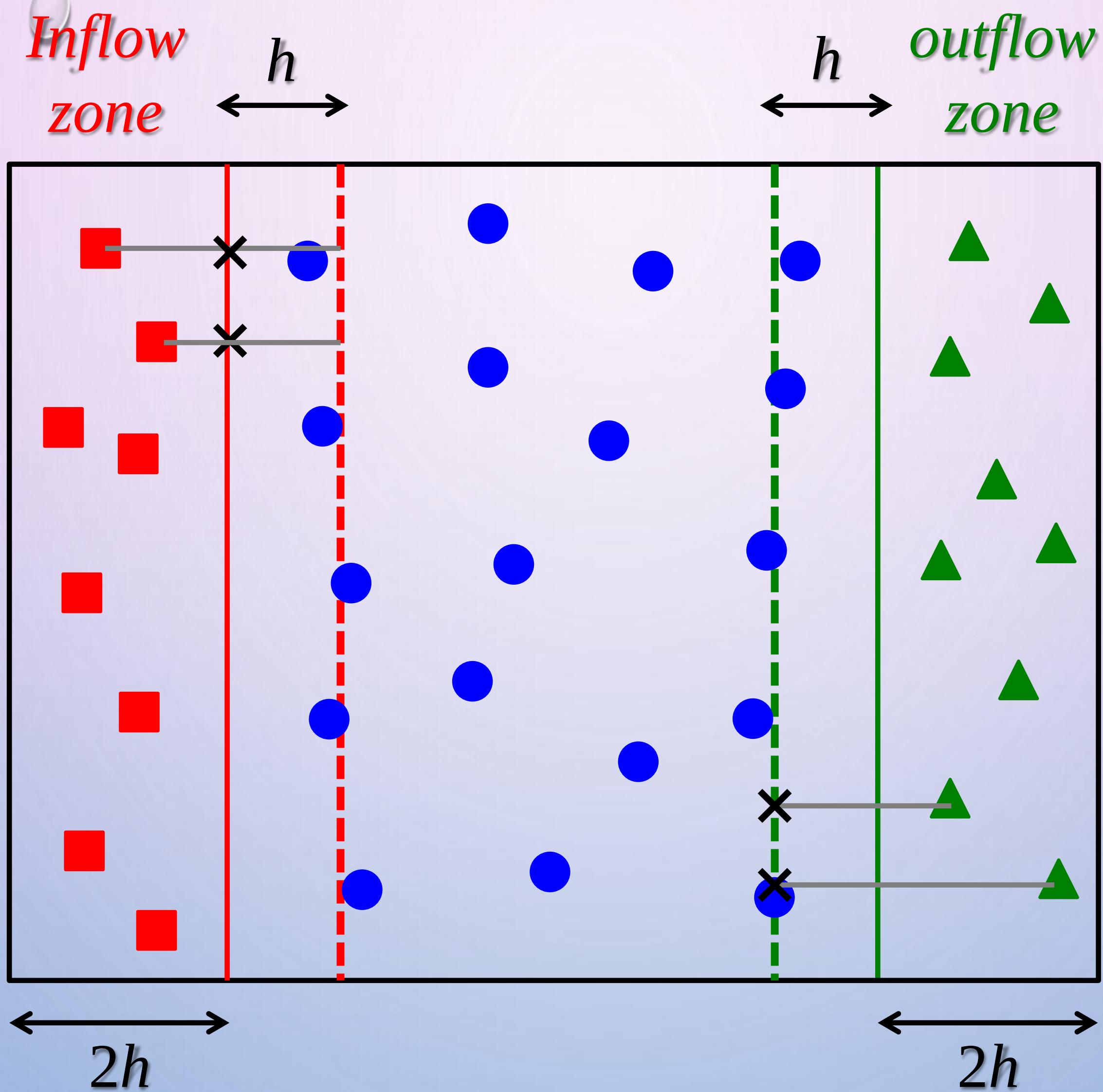
$$Q_S = A_S u_S = A_S\left[u_R - \frac{g}{c_R}\left(d_{w,S} - d_{w,R}\right) + g\left(S_{0,R} - S_{f,R}\right)\Delta t\right]$$

- **Subcritical flows at inflow boundaries**

$$Q_P = A_P u_P = A_P\left[u_L + \frac{g}{c_L}\left(d_{w,P} - d_{w,L}\right) + g\left(S_{0,L} - S_{f,L}\right)\Delta t\right]$$

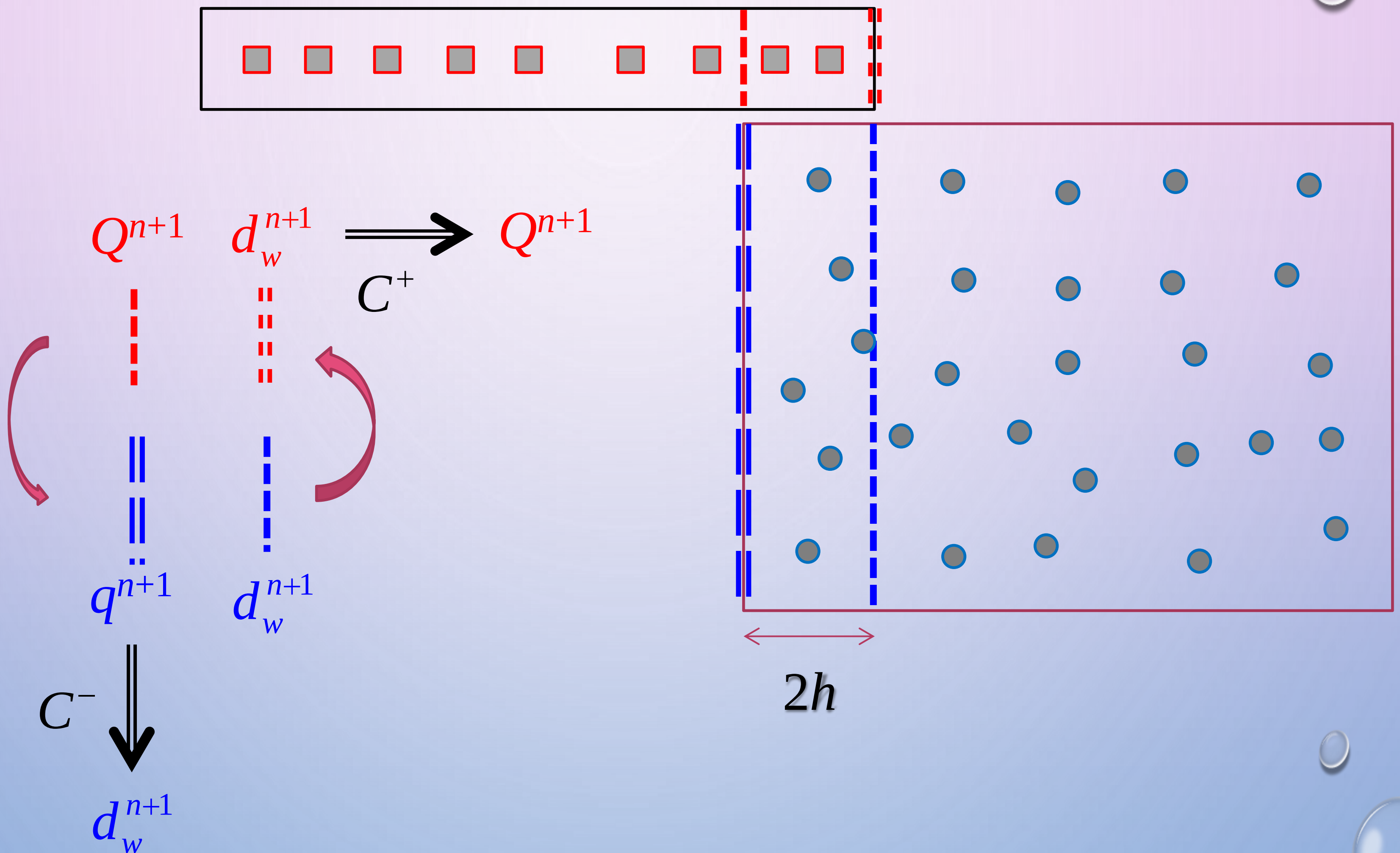
$$f(d_{w,P}) = A_P\left[u_L + \frac{g}{c_L}\left(d_{w,P} - d_{w,L}\right) + g\left(S_{0,L} - S_{f,L}\right)\Delta t\right] - Q_P = 0$$

Ref. 1) TJ Chang, KH Chang, J Hydraul Eng-ASCE, 139:1142-1149, 2013.



Ref. 1) R Vacondio et al., J Hydraul Eng-ASCE, 138: 530-541, 2012.
2) TJ Chang, KH Chang, J Hydraul Eng-ASCE, 139:1142-1149, 2013.

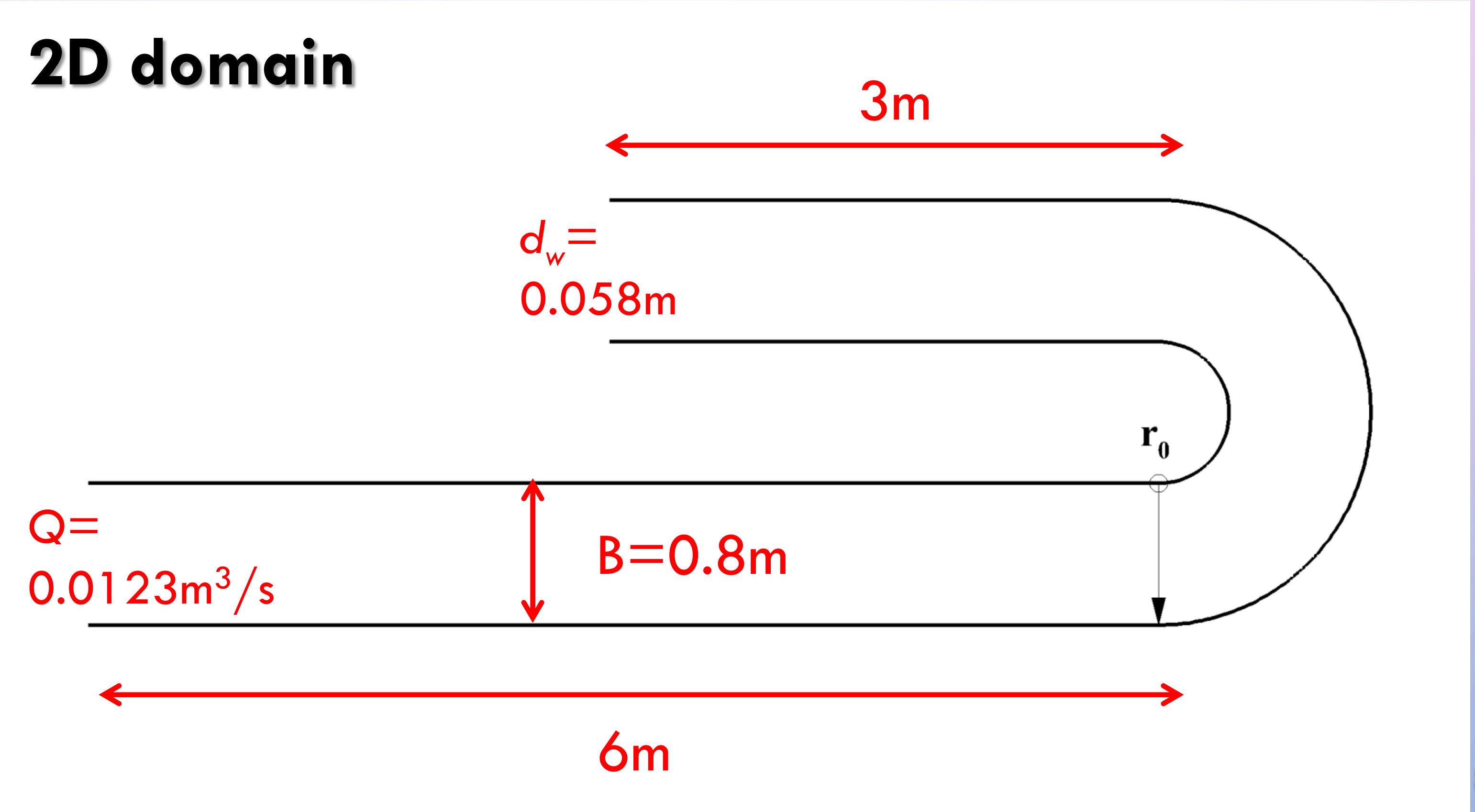
COUPLED BOUNDARY



Ref. 1) M Narayanaswamy et al., J Hydraul Res, 48:85-93, 2010.

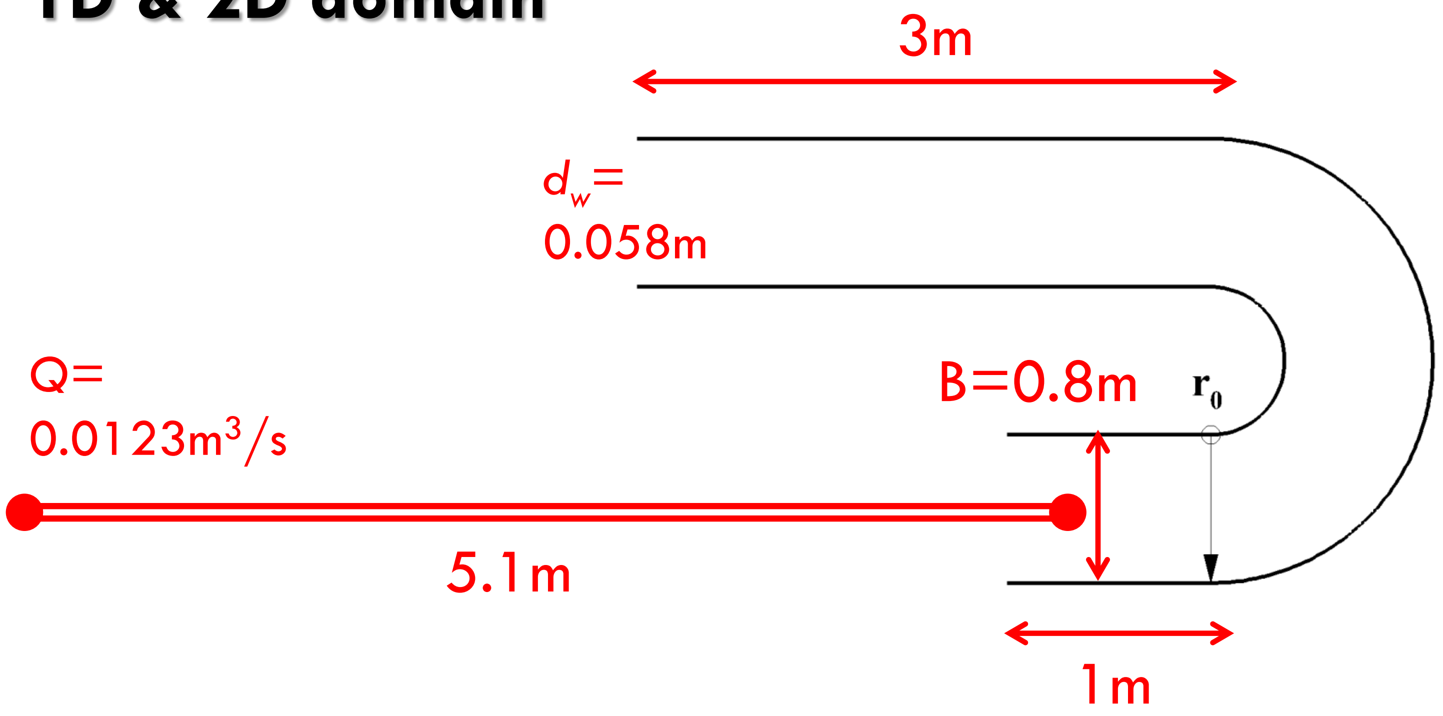
RESULTS & DISCUUSION

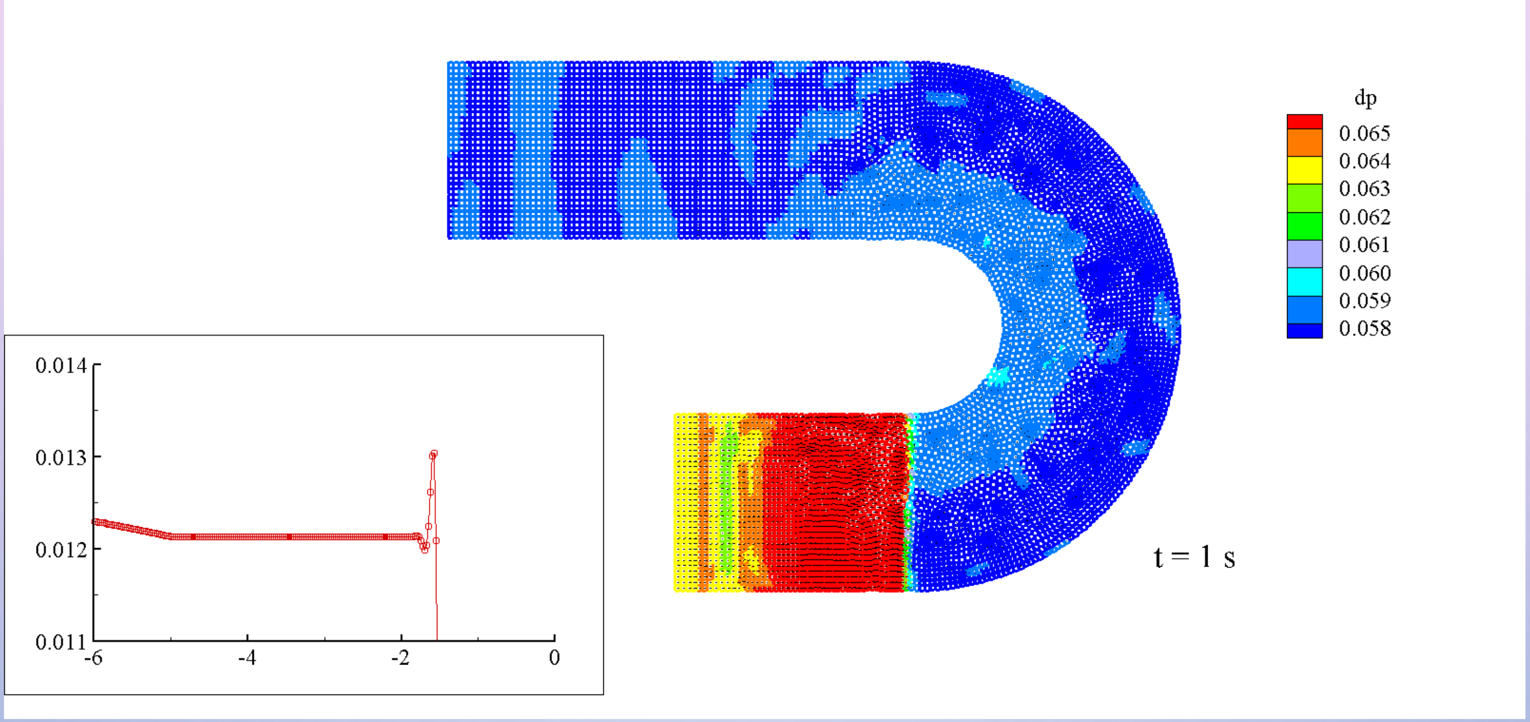
CASE 1 – 180° BENDING CHANNEL



1. $d_w/B<<1$ (=0.075 in this case)
2. No slip boundary condition will produce flow separation from the inner sidewall which was not observed in the laboratory.

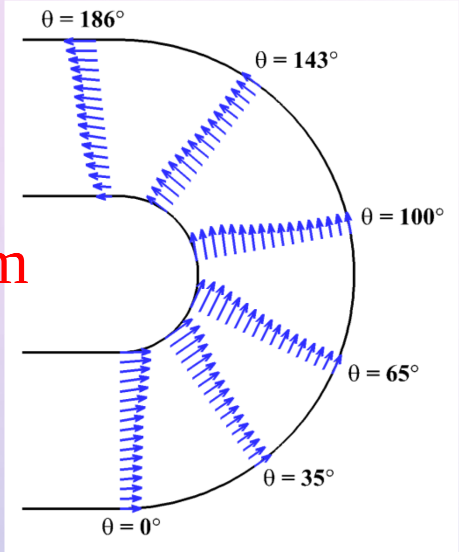
1D & 2D domain

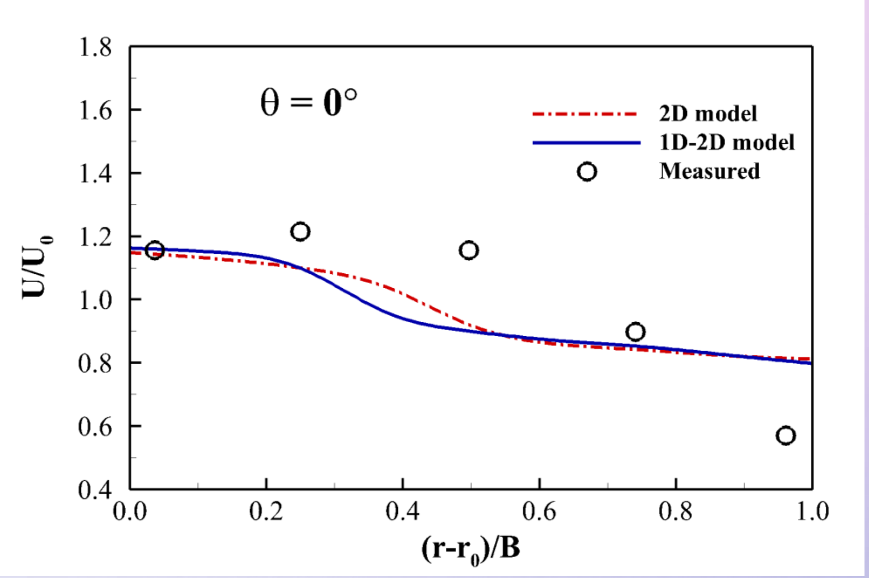


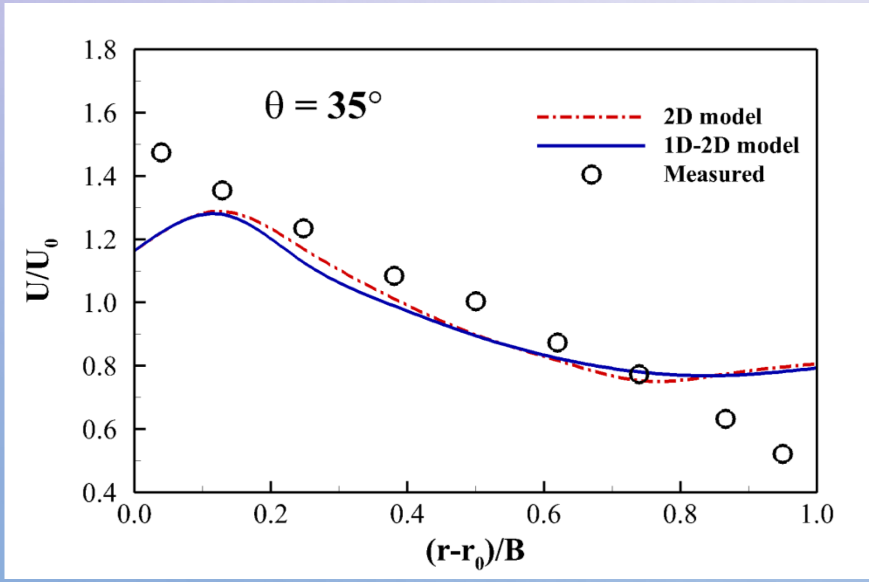

dp
0.065
0.064
0.063
0.062
0.061
0.060
0.059
0.058
t = 1 s
0.014
0.013
0.012
0.011
-6
-4
-2
0

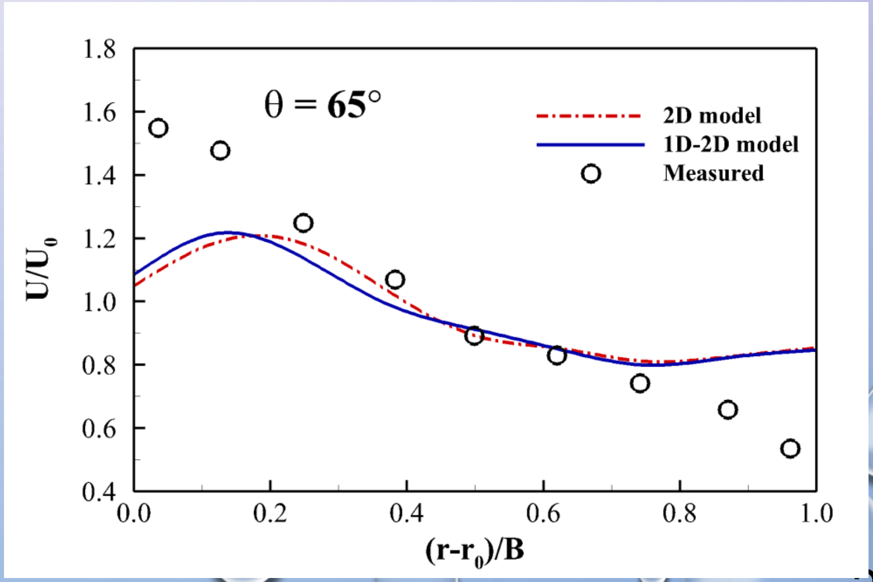
➢ Comparison between 2D and 1D-2D SPH-SWEs models

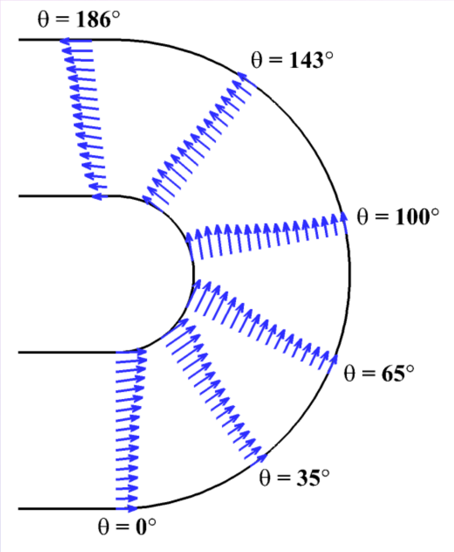
Δx_0=0.025m

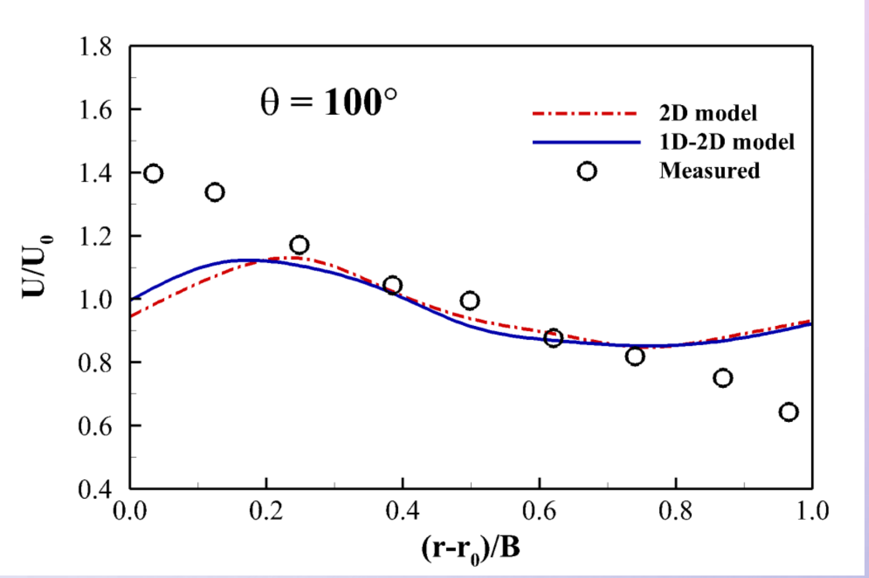


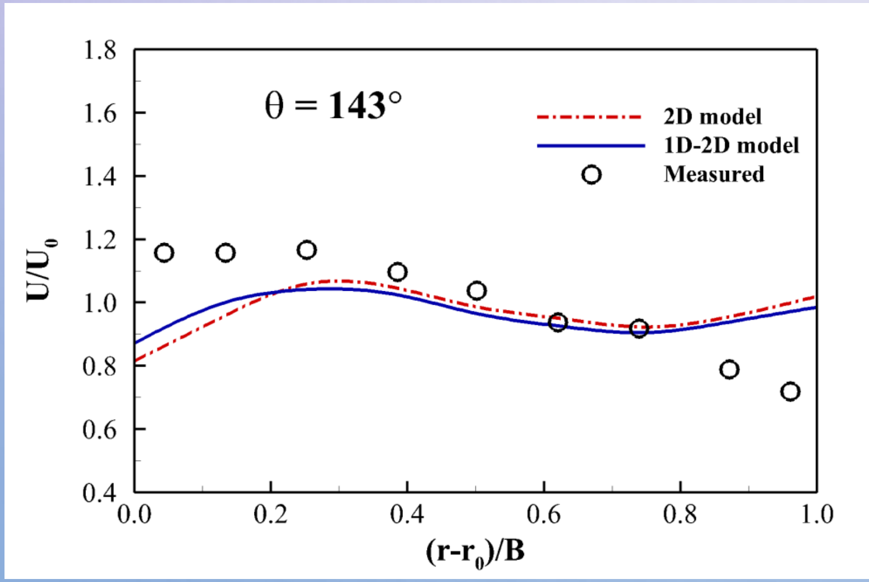


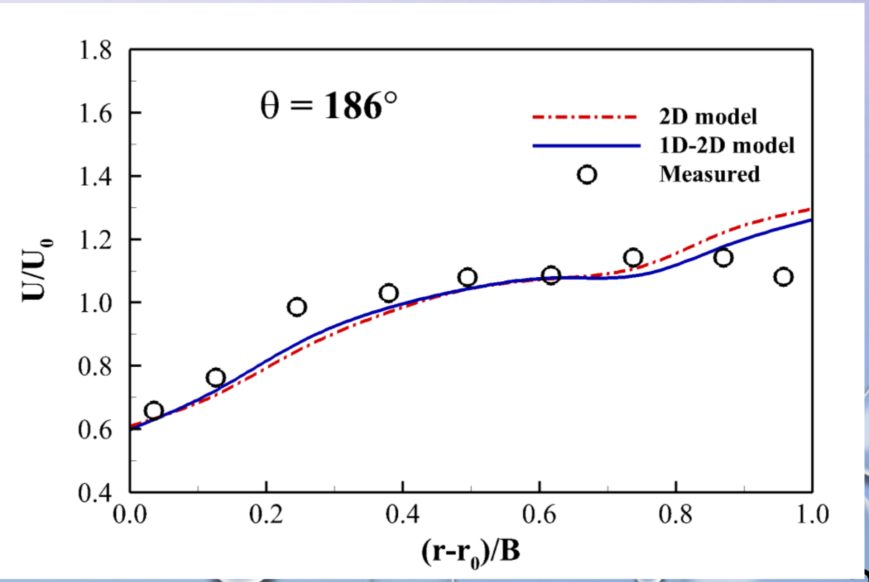





θ = 186°
θ = 143°
θ = 100°
θ = 65°
θ = 35°
θ = 0°

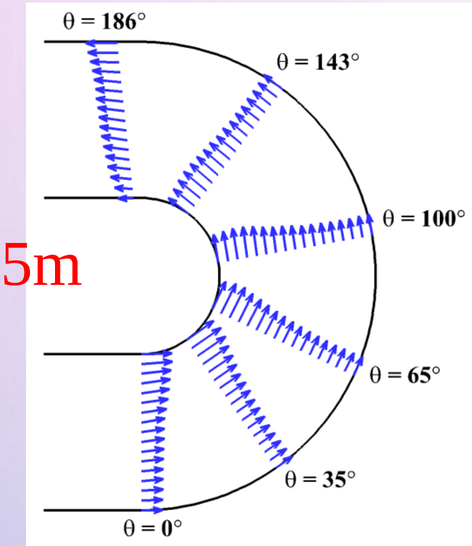

θ = 100°
2D model
1D-2D model
Measured
U/U0
(r-r0)/B

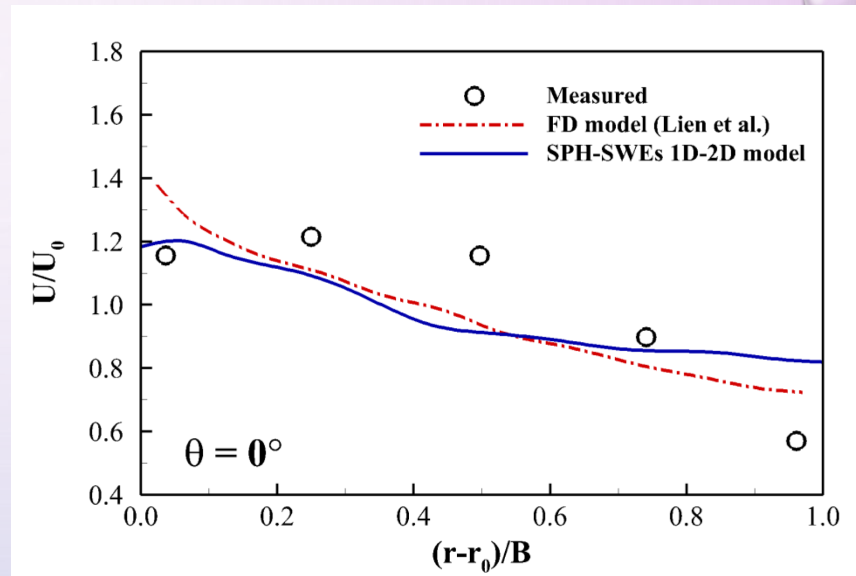

θ = 143°
2D model
1D-2D model
Measured
U/U0
(r-r0)/B

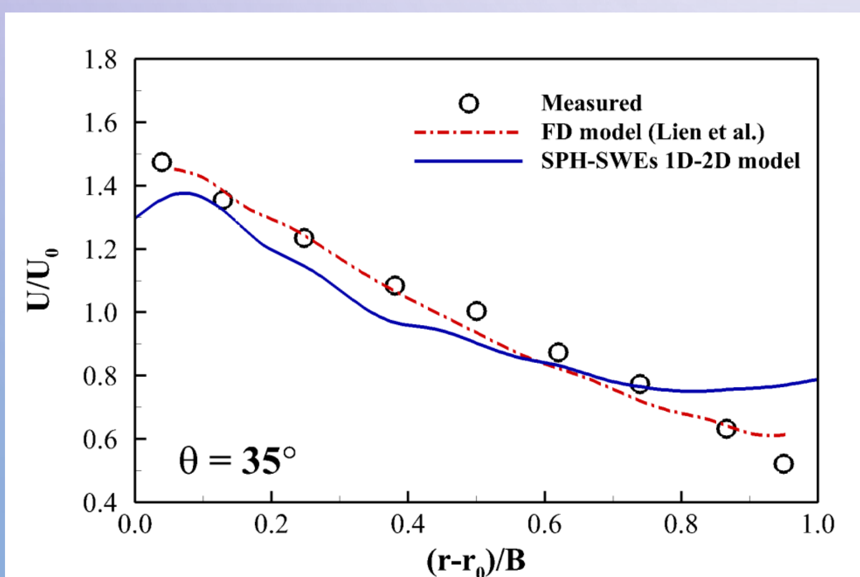

θ = 186°
2D model
1D-2D model
Measured
U/U0
(r-r0)/B

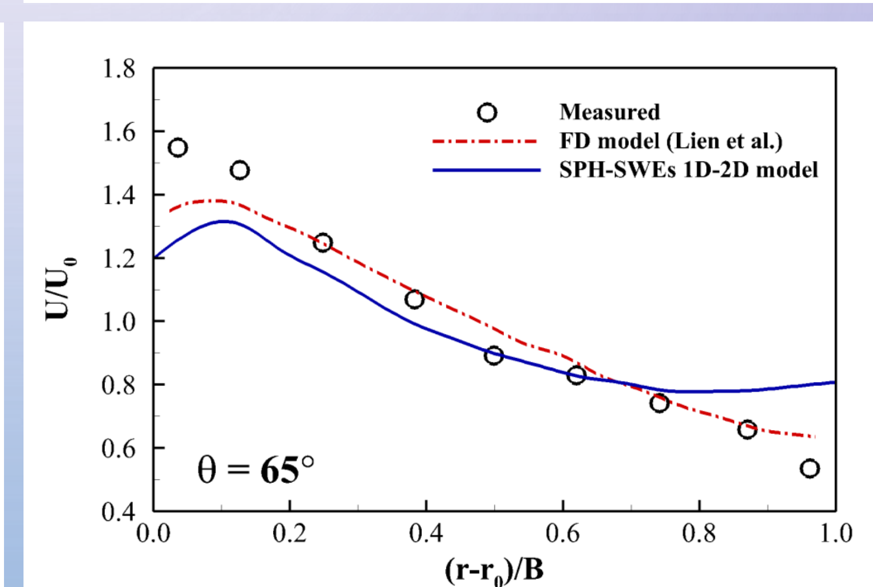
- **Comparison among measured data, FD model and 1D-2D SPH-SWEs model**

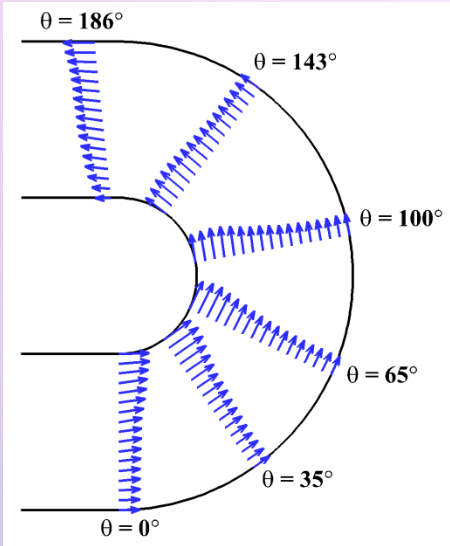
Δx_0=0.0125m

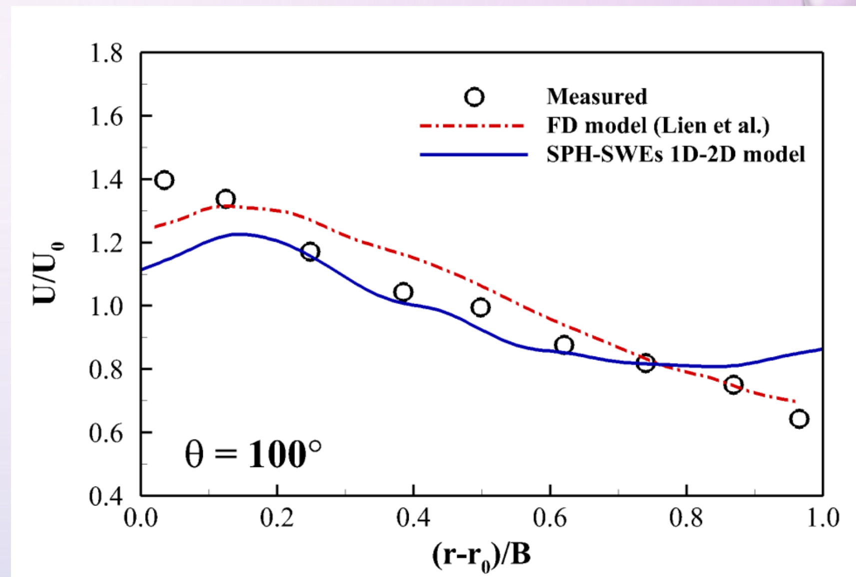


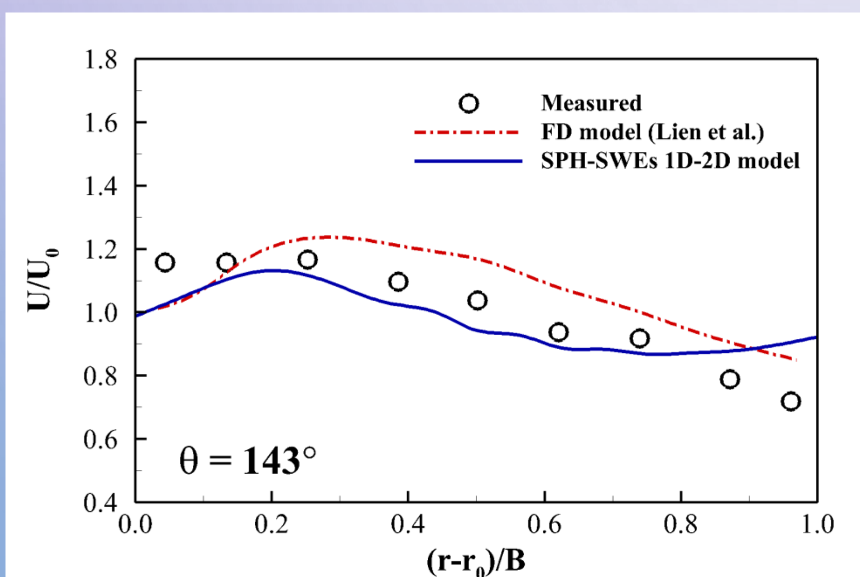


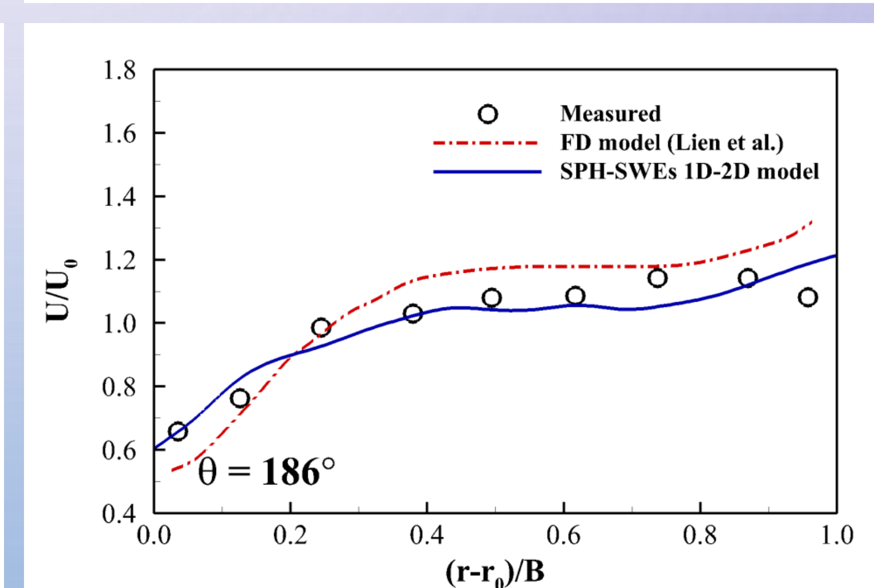




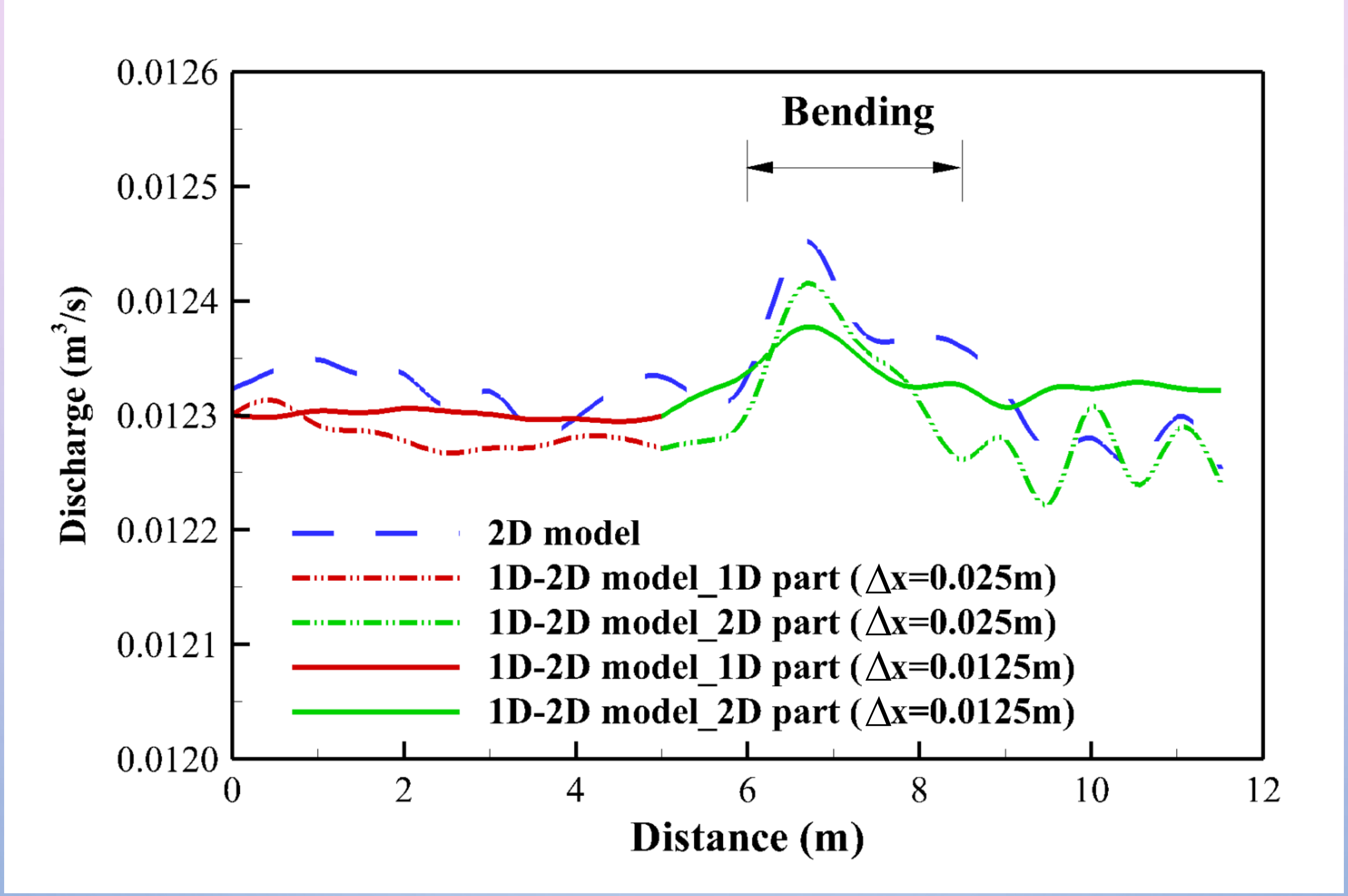

θ = 186°
θ = 143°
θ = 100°
θ = 65°
θ = 35°
θ = 0°


Measured
FD model (Lien et al.)
SPH-SWEs 1D-2D model
θ = 100°
U/U_0
$(r-r_0)/B$


Measured
FD model (Lien et al.)
SPH-SWEs 1D-2D model
θ = 143°
U/U_0
$(r-r_0)/B$

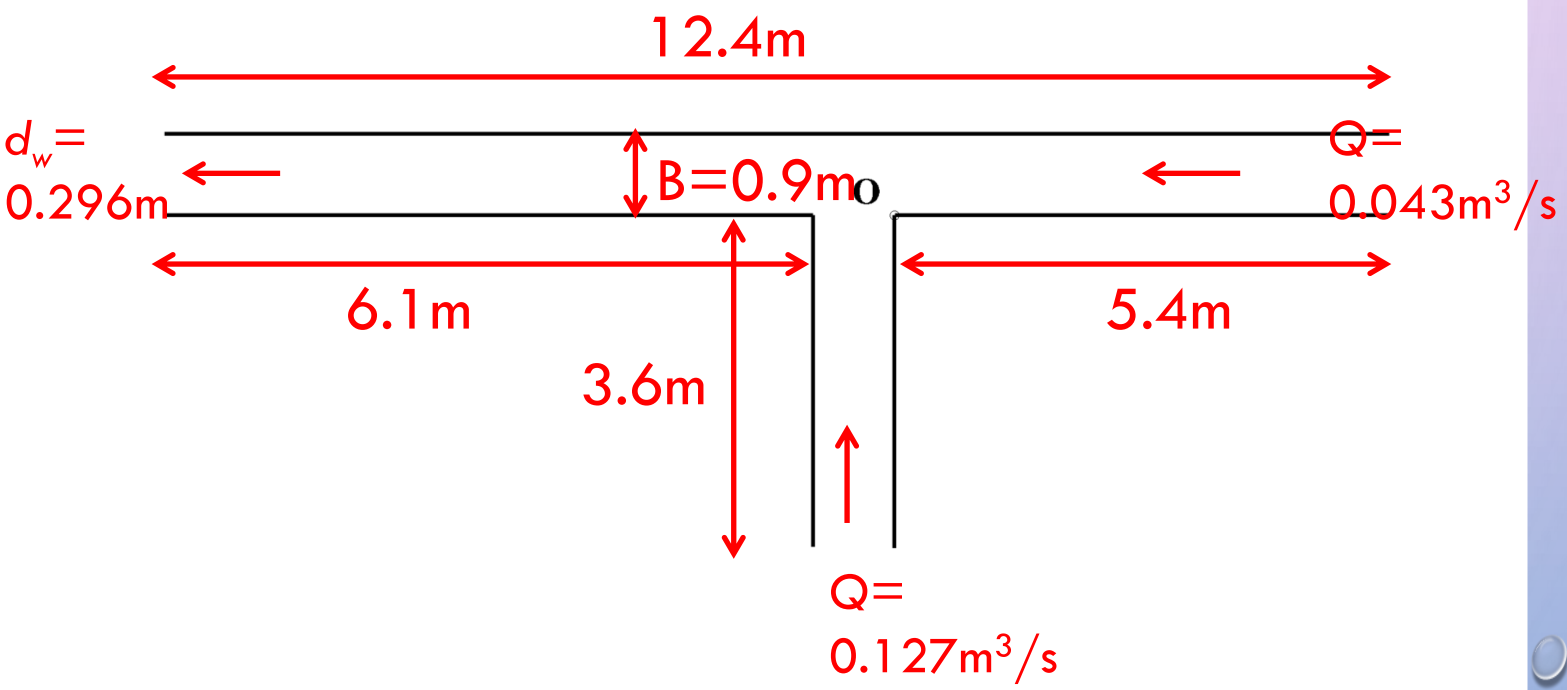

Measured
FD model (Lien et al.)
SPH-SWEs 1D-2D model
θ = 186°
U/U_0
$(r-r_0)/B$

- **Comparison of mass conservation between 2D and 1D-2D SPH-SWEs models**



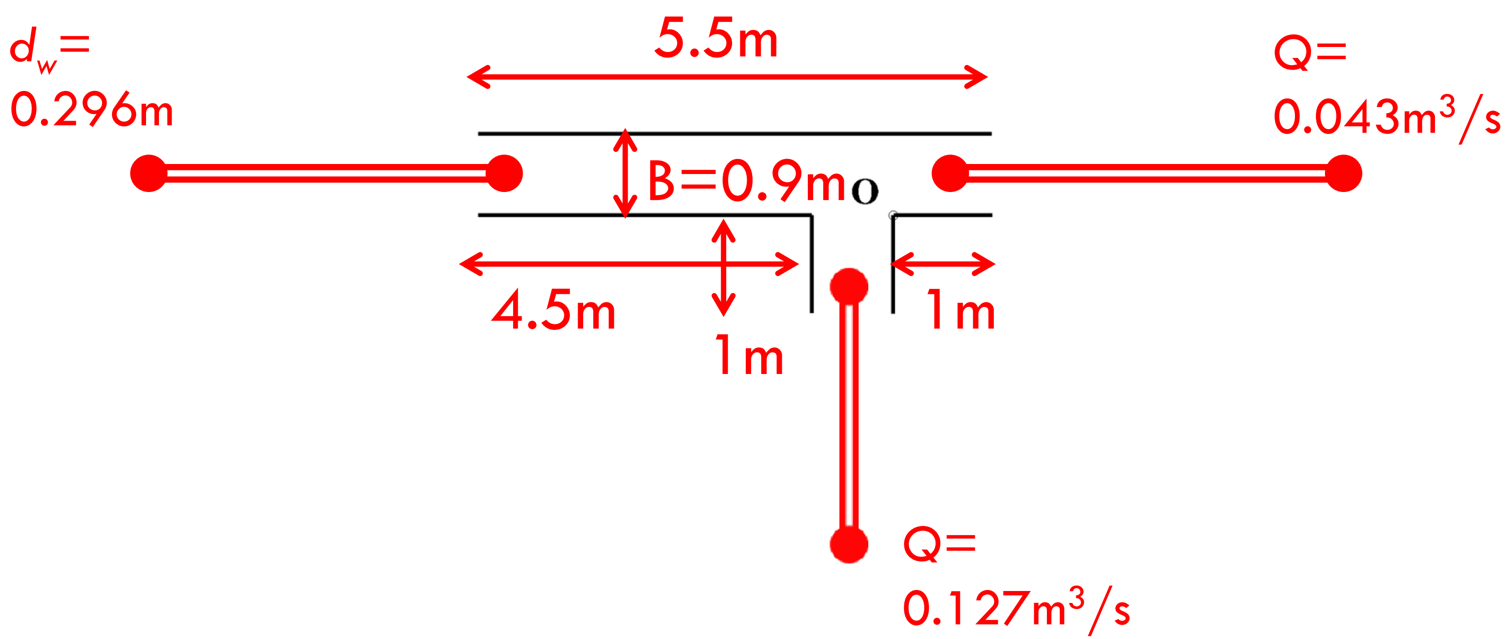
CASE II: 90 JUNCTION CHANNEL

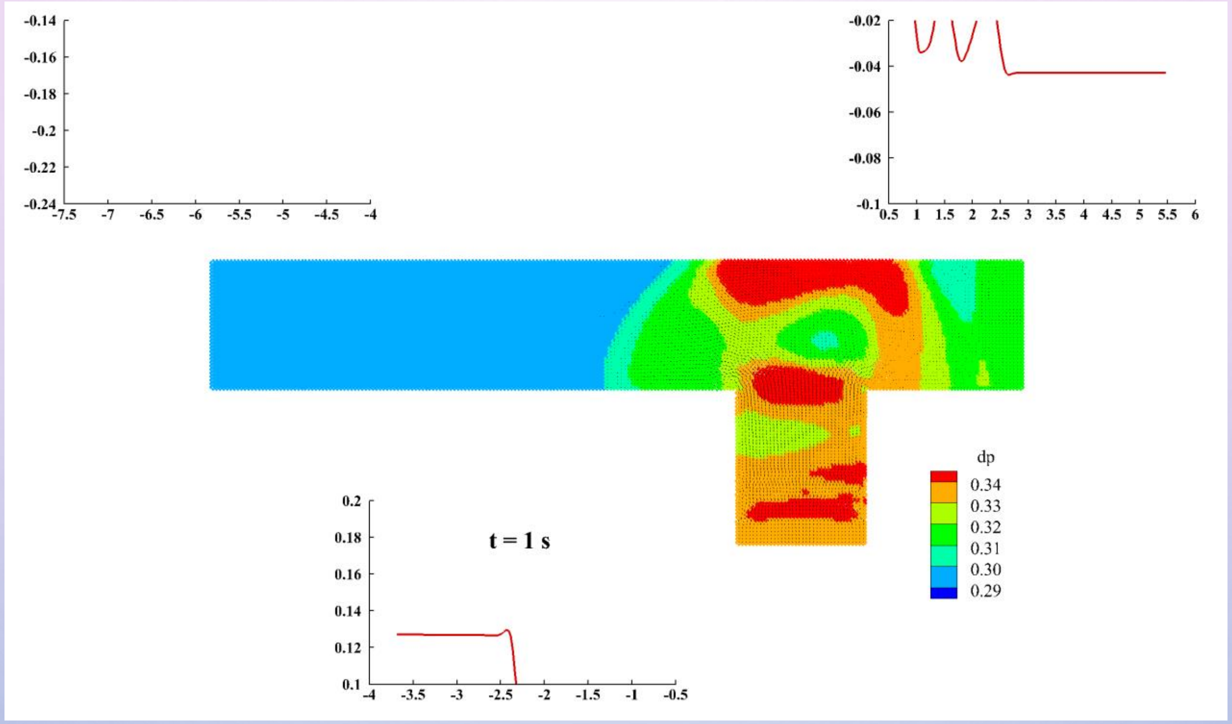
2D domain



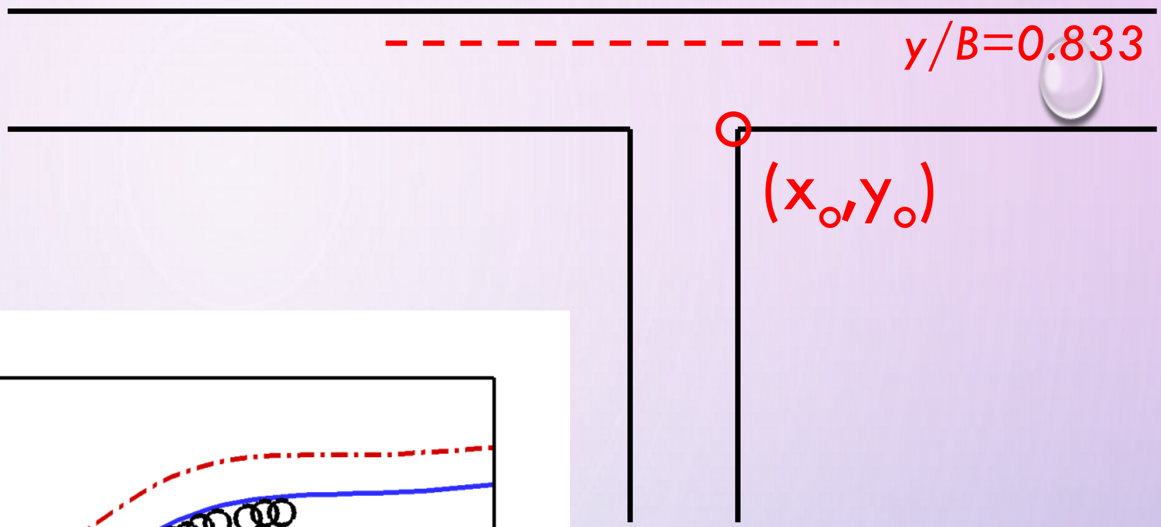
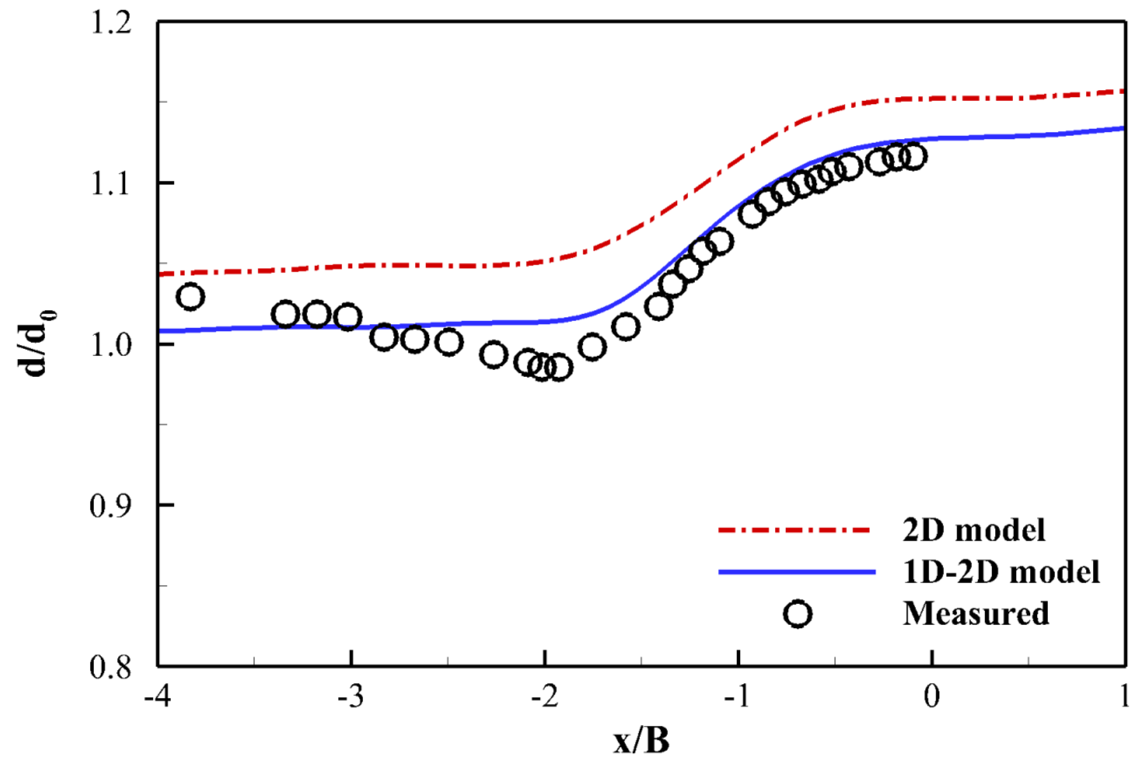
CASE II: 90 JUNCTION CHANNEL

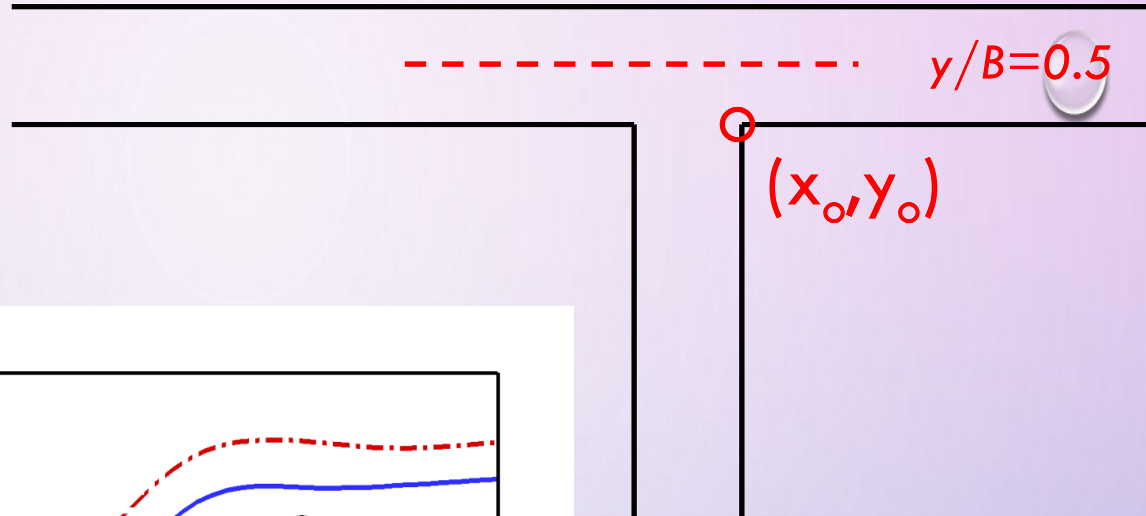
1D-2D domain

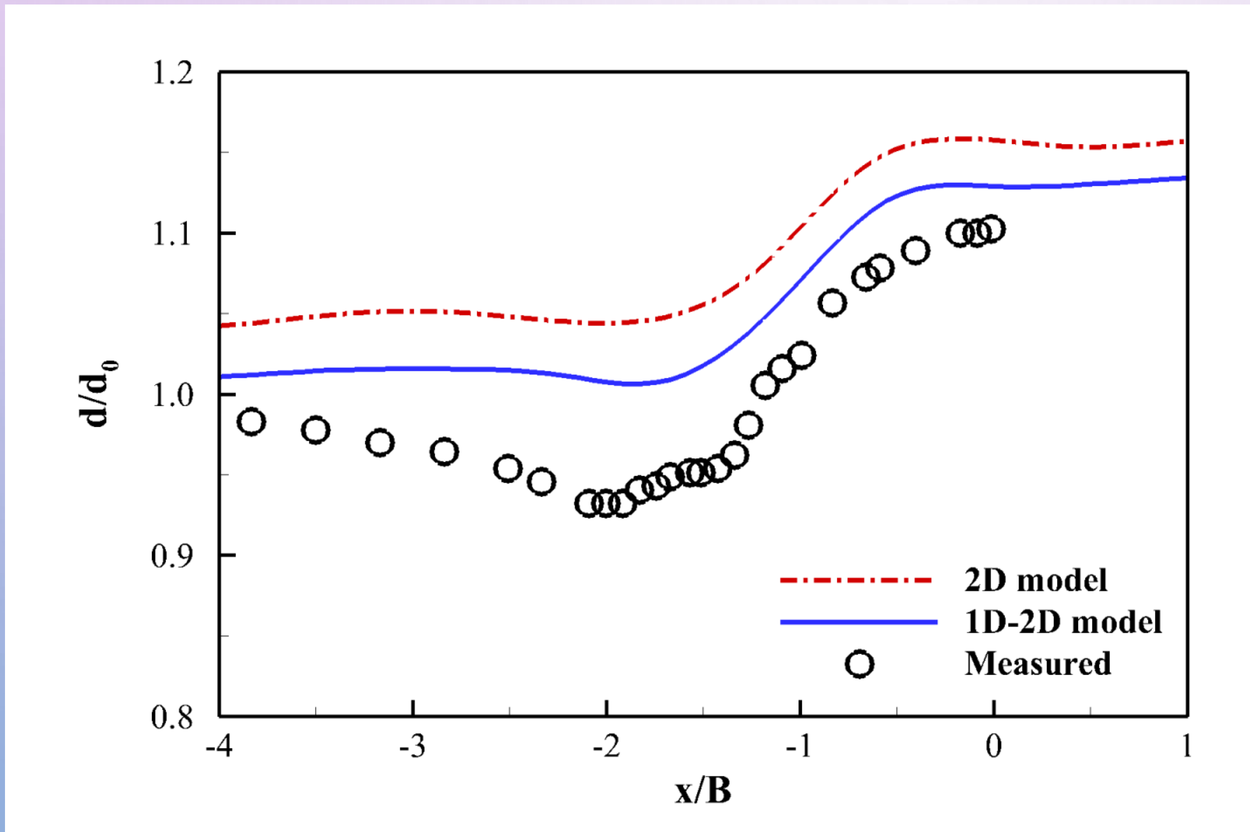


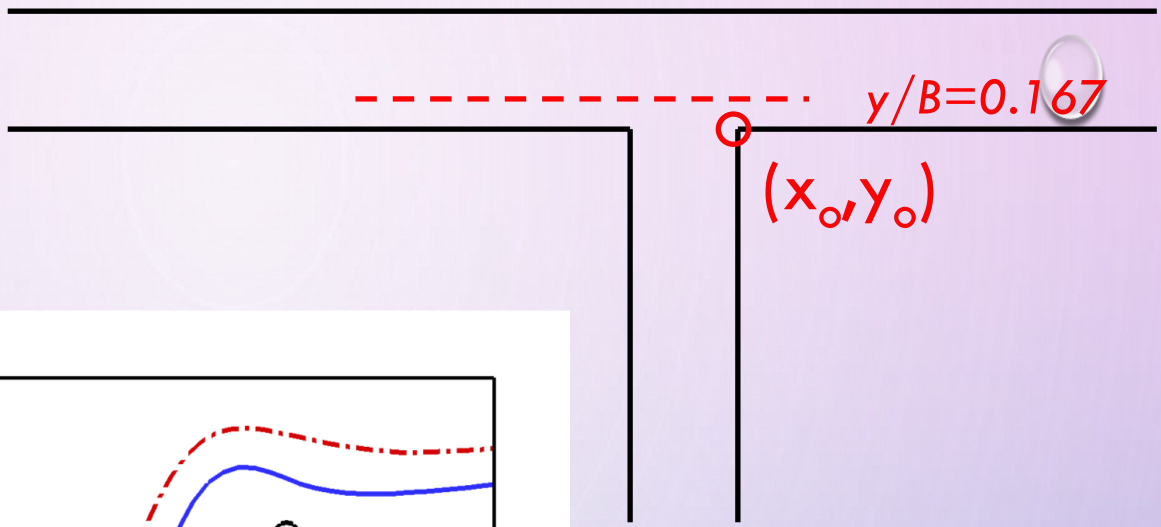


-0.14
-0.16
-0.18
-0.2
-0.22
-0.24
-7.5 -7 -6.5 -6 -5.5 -5 -4.5 -4
-0.02
-0.04
-0.06
-0.08
-0.1
0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6
dp
0.34
0.33
0.32
0.31
0.30
0.29
0.2
0.18
0.16
0.14
0.12
0.1
-4 -3.5 -3 -2.5 -2 -1.5 -1 -0.5
t = 1 s

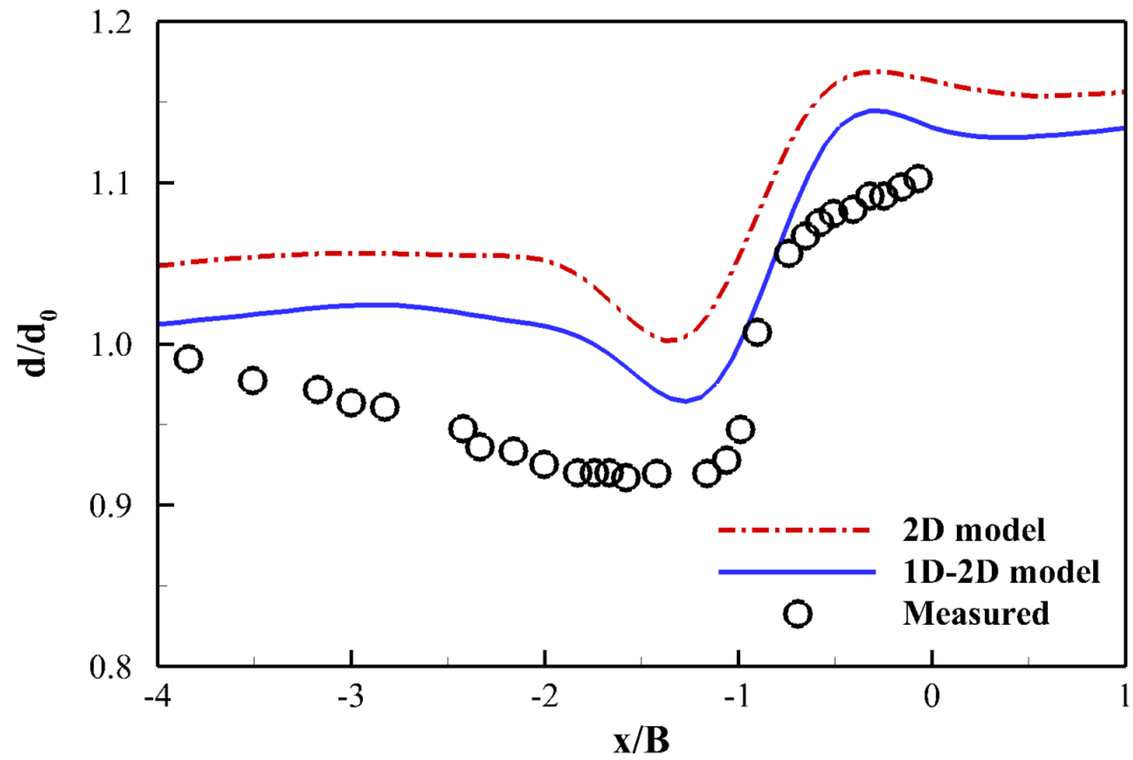

y/B=0.833
(xo,yo)

1.2
1.1
1.0
0.9
0.8
d/d0
-4
-3
-2
-1
0
1
x/B
2D model
1D-2D model
Measured


y/B=0.5
(xo,yo)

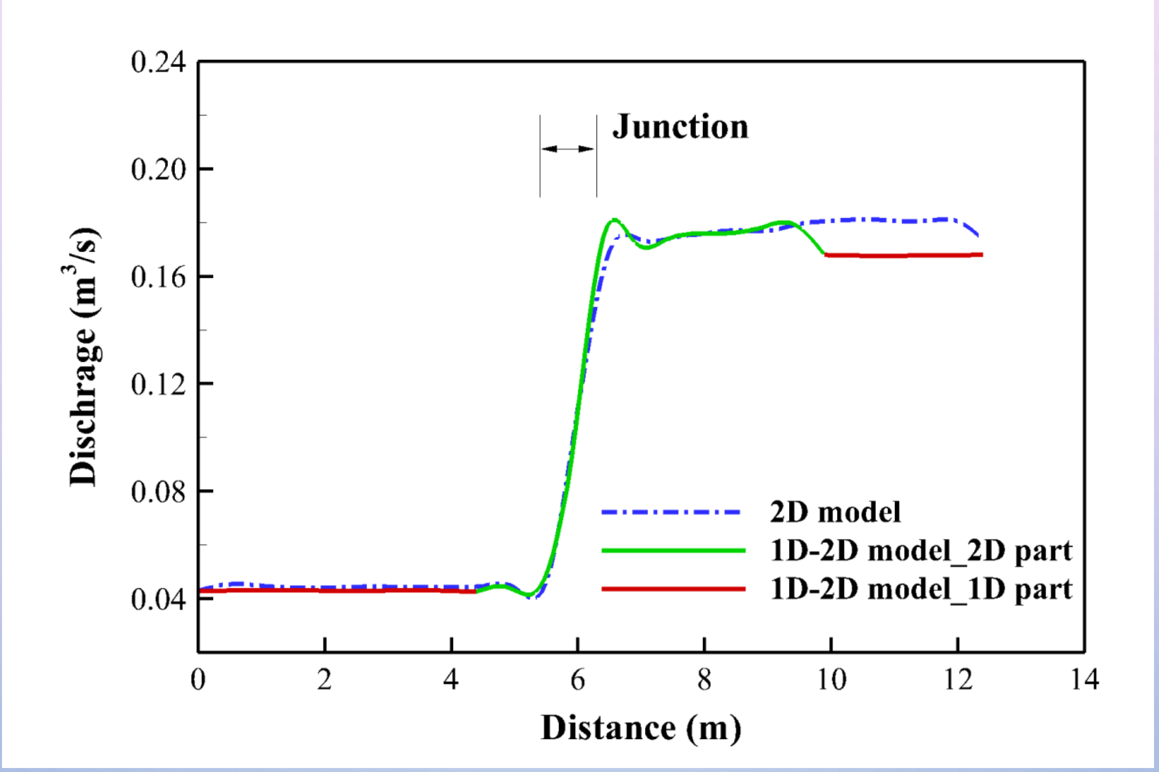

1.2
1.1
1.0
0.9
0.8
d/d0
-4
-3
-2
-1
0
1
x/B
2D model
1D-2D model
Measured



y/B=0.167
(xo,yo)



1.2
1.1
1.0
0.9
0.8
d/d0
-4
-3
-2
-1
0
1
x/B
2D model
1D-2D model
Measured


Junction
Discharge (m^3/s)
0.24
0.20
0.16
0.12
0.08
0.04
0
2
4
6
8
10
12
14
Distance (m)
2D model
1D-2D model_2D part
1D-2D model_1D part

SUMMARY

Case I (150s)			
	No. of particles	CPU time (sec)	Speed up
2D model	15238	1.13E+05	-
1D-2D model	2D: 8999 1D: 296	6.90E+04	1.63

Case II (130s)			
	No. of particles	CPU time (sec)	Speed up
2D model	23472	6.83E+04	-
1D-2D model	2D: 9792 1D: 416	2.34E+04	2.9

THANKS FOR LISTENING