Application of Optimized High Order Compact Schemes for Aeroacoustics and Turbulence Simulations

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Taiwan
Introduction

- **What is Sound?**
  Pressure perturbation that propagates to our ears

- **What is Aeroacoustics?**
  A branch of science that is concerned with the sound generated by aerodynamic forces or turbulence.
Visualization

- Sphere flying over a perforated plate: $M=3.0$

@ An Album of Fluid Motion, Van Dyke
Jet Aeо-acoustics

Periodic waves from a supersonic jet

- It radiates weak shock of frequency 85kHz, directed primarily along a cone 60° from the axis

@ An Album of Fluid Motion, Van Dyke
Jet Flow
List of Lucasian Professors (University of Cambridge)

<table>
<thead>
<tr>
<th>Year appointed</th>
<th>Name</th>
<th>Speciality</th>
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<tbody>
<tr>
<td>1664</td>
<td>Isaac Barrow</td>
<td>Classics and mathematics</td>
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<tr>
<td>1669</td>
<td>Isaac Newton</td>
<td>Mathematics and physics</td>
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<tr>
<td>1702</td>
<td>William Whiston</td>
<td>Mathematics</td>
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<tr>
<td>1711</td>
<td>Nicholas Saunderson</td>
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<tr>
<td>1739</td>
<td>John Colson</td>
<td>Mathematics</td>
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<tr>
<td>1760</td>
<td>Edward Waring</td>
<td>Mathematics</td>
</tr>
<tr>
<td>1798</td>
<td>Isaac Milner</td>
<td>Mathematics and chemistry</td>
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<tr>
<td>1820</td>
<td>Robert Woodhouse</td>
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<tr>
<td>1822</td>
<td>Thomas Turton</td>
<td>Mathematics</td>
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<tr>
<td>1826</td>
<td>George Biddell Airy</td>
<td>Astronomy</td>
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<tr>
<td>1828</td>
<td>Charles Babbage</td>
<td>Mathematics and computing</td>
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<tr>
<td>1839</td>
<td>Joshua King</td>
<td>Mathematics</td>
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<tr>
<td>1849</td>
<td>George Gabriel Stokes</td>
<td>Physics and fluid mechanics</td>
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<tr>
<td>1903</td>
<td>Joseph Larmor</td>
<td>Physics</td>
</tr>
<tr>
<td>1932</td>
<td>Paul Dirac</td>
<td>Physics</td>
</tr>
<tr>
<td>1969</td>
<td>James Lighthill</td>
<td>Fluid mechanics</td>
</tr>
<tr>
<td>1979</td>
<td>Stephen Hawking</td>
<td>Theoretical physics</td>
</tr>
<tr>
<td>2009</td>
<td>Michael Green</td>
<td>Theoretical physics</td>
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</tbody>
</table>

← Founder of Aeroacoustics
Flow Induced Noise; Aeroacoustics

- **Lighthill Equation**

  **Continuity:**
  \[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \]  
  \[ (1) \]

  **Momentum:**
  \[ \frac{\partial}{\partial t} (\rho \vec{V}) + \nabla \cdot (\rho \vec{V} \vec{V} - \vec{t} + p \vec{I}) = 0 \]  
  \[ (2) \]

  \[ \frac{\partial}{\partial t} (1) - \nabla \cdot (2) = 0 \]

  \[ \frac{\partial^2 \rho}{\partial t^2} - \frac{\partial^2 \rho}{\partial x_i \partial x_j} (\rho v_i v_j - \tau_{ij} + p \delta_{ij}) = 0 \]

  **Addition and Subtraction**
  \[ a_0^2 \frac{\partial^2 \rho}{\partial x_i \partial x_j} = \frac{\partial^2}{\partial x_i \partial x_j} (\rho v_i v_j - \tau_{ij} + p \delta_{ij}) \]

  \[ T_{ij} = \rho v_i v_j - \tau_{ij} + (p - a_0^2 \rho) \delta_{ij} \]

  **Lighthill’s stress tensor**

  **Assumptions**

  \[ \rho = \rho_0 + \rho' \]

  **Inviscid, Isentropic**
  \[ \tau_{ij} = 0 \quad \text{and} \quad p' = a_0^2 \rho' \]

  \[ \rho'(\vec{x},t) = \frac{1}{4\pi a_0^2} \frac{\partial^2}{\partial x_i \partial x_j} \int_{\vec{y}} \left| \frac{\rho v_i v_j}{\vec{x} - \vec{y}} \right| \quad \text{d} \vec{y} \]

  \[ \text{Lighthill’s Quadrupole source} \]
## Previous Works

<table>
<thead>
<tr>
<th>Era</th>
<th>Year</th>
<th>Description</th>
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<tbody>
<tr>
<td>Pre-Lighthill</td>
<td>1937</td>
<td>Demming : monopole</td>
</tr>
<tr>
<td></td>
<td>1948</td>
<td>Gutin : dipole</td>
</tr>
<tr>
<td>Aeroacoustic Formulation</td>
<td>1952</td>
<td>Lighthill : quadrupole</td>
</tr>
<tr>
<td></td>
<td>1970</td>
<td>Ffowcs Williams Hawkings : moving body</td>
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<tr>
<td>Jet Era</td>
<td>~</td>
<td>Experiment, Flight Test</td>
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<tr>
<td>Computational Aeroacoustics</td>
<td></td>
<td>Spectral-like (Lele, 1992)</td>
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<td></td>
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<td>DRP (Tam, 1993)</td>
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<td></td>
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<td>OHOC (Kim &amp; Lee, 1995)</td>
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<td>CAA Benchmark</td>
<td>1995</td>
<td>Aeroacoustic conference</td>
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</tbody>
</table>
Computational Aeroacoustics (CAA)

- CAA = Computational Fluid Dynamics + Aerodynamics + Acoustics
- High-Order & High-Resolution Numerical Algorithms

Visualization of acoustic wave generated by turbulence & moving body

- $E_a \sim 10^{-4}$
- $E_f \sim 1$
Generation and Radiation
Diffraction
Anechoic Wind Tunnel (KAIST)

- **Anechoic Wind Tunnel**
  - Test Section: 35cm X 35cm
  - Max. Velocity: 62.8m/s
Problems of Concern

- High-order, high-resolution scheme for spatial derivative in nonlinear convection term and for time integration
  - Dissipation error: Central Scheme or Upwind scheme
  - Dispersion error: high-resolution
  - Truncation error: 4\textsuperscript{th} order
    - DRP, spectral-like, OHOC, MP, ENO

- Boundary conditions
  - Non-reflection condition
  - Inflow condition

- Artificial dissipation
Special Topics

- Low speed incompressible radiation
  - Hydrodynamic density + dipole: Hardin, 1992
  - Non-linear acoustic + quadrupole vortex: Lee & Koo, 1995
  - Splitting method: Moon, 2003

- Acoustic-flow feedback mechanism
  - Cavity tone; Compressible feedback; Heo & Lee
  - Screech tone; Compressible feedback; Lee & Lee
  - Incompressible acoustic-flow feedback; Kim & Lee

- Challenges to CAA
  - 3 dimensional simulation of rotor
Cylinder

- Pressure Contours

2-D Cylinder Flow with $Re = 400$ and $M = 0.3$
Aeolian Tone

M=0.2, Re=300

Karman vortex street
94. Kármán vortex street behind a circular cylinder at $R=140$. Water is flowing at 1.4 cm/s past a cylinder of diameter 1 cm. Integrated streaklines are shown by electrolytic precipitation of a white colloidal smoke, illuminated by a sheet of light. The vortex sheet is seen to grow in width downstream for some diameters. Photograph by Sadao Toneva.
Noise reduction by spiral line

- 나선 송전선에 의한 소음 감쇠 측정
Airframe Noise

- Chaotic wake flow behind a Blunt-Based Body
  - Airliner
  - Airplane spoiler
Cavity Tone
Cavity Tone

Rossiter
Colonius
Heo & Lee

Schlieren photographs of cavity noise (Krishnamurty Karamchetti, 1956)

Generation of cavity tone
Cavity Tone

Issues

- Flow frequency
- vs. acoustic resonance frequency
- Acoustic-vortex interaction
Cavity Tone

- Upstream Propagation of Noise
- Linear (Noise)
- Disturbance in the Free Shear Layer
- Flow Impingement at Downstream Wall
- Amplification of Disturbance in the Shear Layer
- Non-linear (Flow)
Cavity Tone

- Mode change of rectangular cavity
  - Mode changes as $M$, $Re_\theta$, $L/D$ become larger

Steady mode ➔ Shear layer mode ➔ Wake mode

M=0.5, L/D=2, $Re_\theta=200, \theta/D=0.04$
- <Shear layer mode>

M=0.5, L/D=4, $Re_\theta=200, \theta/D=0.04$
- <Wake mode>
Cavity Tone

- Rossiter’s equation
  - 1962
  - Fitting with experiment
  - Resonance frequency

\[
\frac{L}{U_c} + \frac{L}{c_0} = \frac{n - \beta}{f}, \quad n = 1, 2, 3, \ldots
\]

\[
St_n = \frac{fL}{U} = \frac{n - \beta}{M + \frac{1}{k}}, \quad n = 1, 2, 3, \ldots
\]

Issues
- Frequency \( f \)
- Length \( L \)
- Phase lag \( \beta \)
- Amplitude
Cavity Tone

Cross correlation

\[ R(x_1, x_2, \tau) = E[q(x_1, t)q(x_2, t + \tau)] \]
### Cavity Tone

#### Modified Rossiter’s equation

- **Original Rossiter’s eq.:**
  \[
  \frac{L}{U_c} + \frac{L}{a_\infty} = n - \beta, \quad n = 1, 2, 3, \ldots \quad \beta = 0.25
  \]

- **Integral form:**
  \[
  \int_{V_G}^{V_C} \left( \frac{1}{u} + \frac{1}{a-u} \right) dl \text{(along vortex convection path)} = \frac{n - \tilde{\beta}}{f_n}, \quad n = 1, 2, 3, \ldots
  \]

- **Case n=2**
  - Integral form & \( \beta = 0.25 \) 0.47
  - Integral form & \( \beta = 0 \) 0.53
  - Original Rossiter’s Eq. 0.77
  - CAA 0.52

- **Case n=1**
  - Integral form \( \beta = 0.25 \) 0.29
  - Integral form \( \beta = 0 \) 0.38
  - Original Rossiter’s Eq. 0.33
  - CAA 0.30
Jet Screech Tone

Governing Equations
Optimized High Order Compact Numerical Techniques
Governing Equations

- **Unsteady Compressible Euler Equations**
  - Fully conservative form
  - 3 dimensional formulation

\[
\frac{\partial}{\partial t} \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho e_t \\ (\rho e_t + p)u \\ (\rho e_t + p)v \\ (\rho e_t + p)w \end{pmatrix} + \frac{\partial}{\partial x} \begin{pmatrix} \rho u \\ \rho u^2 \\ \rho vu \\ \rho wu \\ (\rho e_t + p)u \\ (\rho e_t + p)v \\ (\rho e_t + p)w \end{pmatrix} + \frac{\partial}{\partial y} \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 \\ \rho vw \\ (\rho e_t + p)v \\ (\rho e_t + p)w \end{pmatrix} + \frac{\partial}{\partial z} \begin{pmatrix} \rho w \\ \rho uw \\ \rho vw \\ \rho w^2 \\ (\rho e_t + p)w \end{pmatrix} = 0
\]

- Continuity equation in vector form

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0
\]

\[
\frac{D\rho}{D\tau} + \rho \nabla \cdot \vec{v} = 0 \quad \therefore \nabla \cdot \vec{v} = S \quad \text{where} \quad S = -\frac{1}{\rho} \frac{D\rho}{D\tau}
\]
Optimized High Order Compact

- DRP (Dispersion-Relation-Preserving)
  - Tam (1993)
  - Analytically optimized central scheme
- Spectral-like scheme
  - Lele (1992)
  - Numerically optimized compact scheme
- Optimized High Order Compact:
  - Kim & Lee (1996)
  - Analytically optimized compact scheme

\[
\beta f_{i-2} + \alpha f_{i-1} + f_i' + a f_{i+1} + \beta f_{i+2} = \\
\frac{a}{2\Delta x} \frac{f_{i+1} - f_{i-1}}{2 \Delta x} + \frac{b}{4 \Delta x} \frac{f_{i+2} - f_{i-2}}{4 \Delta x} + \frac{c}{6 \Delta x} \frac{f_{i+3} - f_{i-3}}{6 \Delta x}
\]
\[
\beta f_{i-2} + \alpha f_{i-1} + f_i' + a f_{i+1} + \beta f_{i+2} = a \frac{f_{i+1} - f_{i-1}}{2 \Delta x} + b \frac{f_{i+2} - f_{i-2}}{4 \Delta x} + c \frac{f_{i+3} - f_{i-3}}{6 \Delta x}
\]

<table>
<thead>
<tr>
<th>Scheme</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central scheme (2\textsuperscript{nd} order)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>DRP scheme</td>
<td>0</td>
<td>0</td>
<td>0.726325187522</td>
<td>-0.120619908868</td>
<td>0.003728657553</td>
</tr>
<tr>
<td>Original compact scheme (4\textsuperscript{th} order)</td>
<td>1/6</td>
<td>2/3</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Spectral-like scheme (4\textsuperscript{th} order)</td>
<td>0.5771439</td>
<td>0.0896406</td>
<td>1.3025166</td>
<td>0.9935500</td>
<td>0.03750245</td>
</tr>
<tr>
<td>OHOC (4\textsuperscript{th} order)</td>
<td>0.590010816707</td>
<td>0.097797917674</td>
<td>1.279672797796</td>
<td>1.051191982414</td>
<td>0.0044752688552</td>
</tr>
</tbody>
</table>
 ➢ Optimized compact difference scheme

\[
\beta f_{i-2} + \alpha f_{i-1} + f_i + \alpha f_{i+1} + \beta f_{i+2} \\
\approx a \frac{f_{i+1} - f_{i-1}}{2\Delta x} + b \frac{f_{i+2} - f_{i-2}}{4\Delta x} + c \frac{f_{i+3} - f_{i-3}}{6\Delta x}
\]

✓ Penta-diagonal formulation and 4th-order spatial accuracy
✓ Fourier analysis of dispersion errors in the wave-number domain

➤ Maximum resolution characteristics

\[
\kappa = k\Delta x = \frac{2\pi}{\lambda} \Delta x
\]

\[
\kappa = 2.5, \quad \frac{\lambda}{\Delta x} \approx 6.3
\]

a. 2nd-order central differences
b. 4th-order central differences
d. DRP scheme
3. OSOT scheme
6. OFOP scheme
e. exact differentiation
Numerical Techniques

- Optimized high-order compact (OHOC) : Kim & Lee (1996)
- Low dissipation and dispersion Runge-Kutta (LDDRK) : Hu et al. (1996)
- Adaptive nonlinear artificial dissipation (ANAD) : Kim & Lee (2001)
- Generalized characteristic boundary condition (GCBC) : Kim & Lee (2000)
Rocket Noise

- Ariane 4: Among 116 launch 3 fail (Noise)
- Ariane 5: Among 60 launch 4 fail (Electronic Equipment Operation)
- Conestoga Rocket: 6 Model (Electronic Equipment due to Noise)

Ariane 5 rocket explosion

Conestoga rocket explosion
Screech Tone

- Issues
  - Structure damage
  - Electronic Equipment
  - Generation mechanism: instability mode & shock cell reflection
  - Powell (1953)
    \[
    \frac{1}{U_c} + \frac{1}{a_\infty} = \frac{1}{sf}
    \]
    - $U_c$: convection velocity
    - $s$: shock cell length
  - Tam (1988)
    \[
    \frac{1}{U_c} + \frac{1}{a_\infty} = \frac{k_1}{2\pi f}
    \]
    - $U_c$: phase velocity
    - $k_1$: fundamental wave number of the shock cell structure
Supersonic Flow

<Noise from rocket launcher> <Supersonic jet noise>
Jet Noise

- Rocket Noise
  - M=2.1, perfectly expanded condition
  - No shock cell structures
  - Only Mach waves are shown
Jet Noise

➤ Screech Tone

✓ M=1.18 (A2 mode)

• Two perturbations are observed at x/D=2.52 and x/D=5
• A large perturbation is observed at the top of 3\textsuperscript{rd} shock cell
• Concentric circles show good agreement with wave fronts
Jet Noise

- Screech Tone
  - At the initial stage, the pressure signal is not so stable and some irregular variation is observed

A pressure signal of M=1.18 jet
Sound Source
Jet Noise

- Screech Tone
  - M=1.18 (A2 mode): the components of both A1 and A2 modes exist at the beginning and A2 mode becomes dominant.

Graphs showing SPL (dB) versus non-dimensional frequency for long period (N=16,384) and short period (N=512), with time intervals t=t₁, t=t₂, t=t₃, t=t₄.
Screech Tone

- Axisymmetric mode change: A1 mode
  
  - $M = 1.08$
Screech Tone

- Axisymmetric mode change: A2 mode
  - $M = 1.18$
Jet Noise

- Screech Tone
  - Instantaneous density contour
3 D Jet Screech

Screech Tone

- Two modes (A1 & A2) are observed
- Experimental results show rapid decrease of amplitude after Mach number 1.15 in 3D calculation
Supersonic Flow

- At the initial stage, the pressure signal is not so stable and some irregular variation is observed.

3D Mode: flapping, spiral mode

A pressure signal of $M=1.18$ jet

Vorticity contour

Density contour
Screech Tone Noise

- 3D mode ($M_j=1.43$, B & C):
  - Vorticity & Density Contours
    - Flapping mode (B)
    - Spiral mode (C)

Vorticity Contour

Density Contour

Instantaneous Density Contour
Screech Tone Noise

- 3D mode ($M_j = 1.43$, B & C): Density Contour
  - Strong waves propagates up and down.

![Diagram showing sectional variation](image)
3D mode ($M_j=1.43$, B & C): Spectral Analysis

- Two modes are observed.
  - B mode
  - C mode
3D mode (M_j=1.43, B & C): Directivity

- The directivities of B mode and C mode looks like the directivity of dipole and they are perpendicular each other.
Screech Tone

- 3 dimensional supersonic jet : M=1.15

>Density contour 1 (0.6<\rho/\rho_0<1.7)

>Density contour 2 (0.95<\rho/\rho_0<1.05)
Screech Tone

- Downstream propagation

- Upstream propagation

Overall range: \(0.6 < \rho / \rho_0 < 1.7\)

Narrow range: \(0.95 < \rho / \rho_0 < 1.05\)
Screech Tone

- 3 dimensional effect

![Graphs showing Mach numbers at different x/D values (x/D=1, x/D=5, x/D=10)]

- Experiment (T.R. Troutt et al.)
- 3D compact: Euler (Lee & Lee)
- 2D compact: Euler (Lee & Lee)
- 2D ENO: N-S (Kim & Lee.)

<Radial Mach number> <Central Mach number>
Application
Naro Rocket

- Naro Rocket

- Mach wave

- Observer

- Sudden rise

- Gradual decrease

- 1.4 km
Jet Noise

- Naro Rocket Noise
BVI Noise

Governing Equations
Optimized High Order Compact Numerical Techniques
Blade Vortex Interaction

M=0.2

Inflow vortex

Euler, Mach=0.8, AcA=1.25deg, Gamma=-0.2, y_c=-0.26
Simulation methods and validations

Curle’s Acoustic Analogy for Stationary Airfoil in a Uniform Flow

\[ H(f)p'(x) = - \int_{f=0}^f F_i n_i \frac{\partial G(x; \xi)}{\partial \xi_i} dl - \int_{f>0} T_{ij} H(f) \frac{\partial^2 G(x; \xi)}{\partial \xi_i \partial \xi_j} d\xi \]

( Curle’s equation )

- **Dipole (loading) source:**
  \[ F_i = pn_i \]
  (surface pressure)

- **Quadrupole source:**
  \[ T_{ij} = \rho u_i' u_j' + (p - c_0^2 \rho) \delta_{ij} \]
  (area Lighthill stress)

\[ f = 0 \quad \text{and} \quad f > 0 \]

\[ G(x; \xi) = \frac{i}{4\beta} \exp \left( \frac{i k M (x - \xi)}{\beta^2} \right) H_0^{(2)} \left( \frac{k R^*}{\beta^2} \right) \]

: Green function for 2D convective wave equation.

\[ \triangleright \text{From FVM Simulation} \]

\[ \triangleright \text{Artificial area truncation } \sim 0.5 \text{ c} \]

\[ \triangleright \text{Implementation [Lockard, JSV 2000]} \]
Validation for BVI noise

- Mach=0.5, \( y_v = -0.2 \)

Pressure contour

Airfoil
Whistle Noise

Governing Equations
Optimized High Order Compact Numerical Techniques
Feedback Mechanism
Acoustic Systems in Biology

- Sound production by vocal fold

![Diagram of vocal and respiratory systems](image)
Laminar Whistle noise

• Feedback Mechanism

Most amplified instability generated by initial flow

Flow fluctuate near the T.E by the most amplified frequency

Flow fluctuation propagate to upstream

Amplified instability propagate to T.E

Propagated fluctuation amplify the instability

Numerical investigation of the tone noise mechanism over laminar airfoil
G. Desquesnes et al.
Laminar Whistle noise

- Whistle noise on Laminar Airfoil

Sound spectrum for airfoil
(Nash et al.)

Pressure contour for airfoil
(Desquesnes et al.)
Laminar Whistle Noise: Airfoil

- Change of whistle noise as flow velocity change

Inflow velocity: 0 → 130km/h
Acoustic Systems in Biology

- Silent Flight of Owl
• Whistle noise

Real auto-vehicle side-mirror

Simplified side-mirror

Noise spectrum
How people in science see each other
Acoustic – Flow Feedback
Locking Flow

- Example of flow locking

@ http://www.mae.cornell.edu/IJFD/1997_vol1/paper1/Parker.Flow.html
Locking Flow

- Analysis of locking flow
  - Resonance mode: Parker, 1967
  - Simple model: Welsh, 1984
  - Acoustic mode + flow: Stoneman, 1988

- A body in a duct

\[ S_f = \frac{fL}{U} \quad S_a = \frac{fL'}{a} \]

- Flow Strouhal No.  
  - U: flow velocity
  - a: acoustic speed
  - L: flow characteristic length

- Acoustic Strouhal No.  
  - L': acoustic characteristic length
Locking Flow

- Locking flow vs. unlocking flow
  - Pressure contour

<Flow acoustic locking>  <Flow unlocking>
Resonance frequency 686 Hz

- Beat tones from the natural vortex shedding and acoustic resonant frequencies before the resonance flow
- Single vortex shedding frequency in the region of resonance flow
- Recover to natural incompressible flow after the resonance flow
Two resonance regions depending on the suction velocities ranges
Each of the resonant regions shows the different flow and vortex patterns
Increase of the resonance frequencies according to the suction velocity

- Aeroacoustic sources can generate the acoustic energy, as well as modify the resonance frequencies.
Incompressible Flow-Acoustic Feedback

- Special treatment
  - Governing equation
  - Numerical formulation (extension of Stoneman’s formulation (1988))

- Continuity equation \( \nabla \cdot \vec{v} = S \) where \( S = \frac{1}{\rho} \frac{D\rho}{Dt} \approx \frac{1}{\rho} \frac{\partial \rho}{\partial t} \)

\[ \rho(x, y, z, t) = A(t) \Phi(x, y, z) \]

- Acoustic mode \( k^2 \Phi + \nabla^2 \Phi = 0 \) where \( k = 2\pi f \)

\[ \rho - \rho_0 = \sum_m A_m \Phi_m \]

\[ \sum_m \dot{A}_m \Phi_m + \rho_0 \nabla \cdot \vec{u} = 0 \]

\[ \rho_0 \left[ \vec{u}_i + (\vec{u} \cdot \nabla) \vec{u} \right] + \nabla p = \mu \nabla^2 \vec{u} \]
Vortex Sound ; Howe

From the momentum equation,
\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + \frac{1}{\rho} \nabla P = \nu \left( \nabla^2 \mathbf{v} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{v}) \right) \quad - - - (1)
\]

\[
\frac{1}{\rho} \nabla P = \nabla \left( \int \frac{dP}{\rho} \right)
\]

\[
\nabla \times \mathbf{\omega} = \nabla \times \nabla \times \mathbf{v} = \nabla (\nabla \cdot \mathbf{v}) - \nabla^2 \mathbf{v}
\]

\[
(\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{\omega} \times \mathbf{v} + \nabla (\frac{1}{2} \mathbf{v}^2)
\]

Crocco’s equation

\[
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{\omega} \times \mathbf{v} + \nabla B = -\nu \left( \nabla \times \mathbf{\omega} - \frac{4}{3} \nabla (\nabla \cdot \mathbf{v}) \right) \quad - - - (2)
\]

where

\[
B = \int \frac{dP}{\rho} + \frac{1}{2} \mathbf{v}^2
\]

Incompressible
Finally the integral solution is given as:

$$B(x, t) = \oint_{S^+} \left( B \nabla G + G \frac{\partial \mathbf{v}}{\partial \tau} \right) \cdot dS - \int_V H (\mathbf{\omega} \times \mathbf{v}) \cdot \nabla G \, d^3y$$

$$+ \nu \oint_{S^+} (\mathbf{\omega} \times \nabla G) \cdot dS$$
Compressible effects are only come from acoustic resonance phenomena.

Acoustic resonance is defined by the geometries of system.

Unsteady incompressible flow analysis is faster and lighter than compressible flow analysis and can accurately capture the aeroacoustic sources.

The strong points of unsteady incompressible flow analysis and acoustic modes can be combined.
Previous studies for acoustic resonant flows

Using the results of mode analysis
- Fixed acoustic resonant frequency
- Neglecting the feedback effects from flows
- Parametric case studies according to acoustic velocity
- 1 way coupled method

Continuity Equation
\[ \rho_0 \nabla \cdot \vec{u} = 0 \]

Momentum Equation
\[ \rho_0 \left[ \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} \right] + \nabla p = \mu \nabla^2 \vec{u} \]

This study

The strong points of the developed numerical method

- The magnitude and frequency of acoustic fields are defined by aerodynamic acoustic sources.
- Computational time can be reduced by using the ordinary differential equations and incompressible flow solver.
- Wall vibrations conditions and acoustic radiations loss can be considered by the surface integral terms of ordinary differential equations

The Eigen value problem of Helmholtz equation
\[ k_m^2 \Phi_m + \nabla^2 \Phi_m = 0 \]
\[ \Phi_m \bigg|_{\partial \Omega} = 0 \quad \frac{\partial \Phi_m}{\partial n} \bigg|_{\partial \Omega} = 0 \]

Volume source

Acoustic radiation

Wall vibration
Acoustic Liner

Governing Equations
Optimized High Order Compact Numerical Techniques
Acoustic Liner
Perforated Liner (Resonator Array)

Noise of a typical aero-engine (turbofan)

Perforate Liner
(Helmholtz resonator type)

Acoustic Liner
installed in Turbofan Engine Nacelle

© COPYRIGHT THE BOEING COMPANY
Perforated Liner (Resonator Array)

- CAA simulation of Helmholtz Resonator
  - Experimental Data: Measurement of single Helmholtz resonator by Hersh-Walker

![Diagram showing self-convection (blowing and suction) and impedance of single Helmholtz resonator.]

**Impedance of Single Helmholtz Resonator (P=140dB)**

- Line: Exp. [Hersh]
- Symbol: Present

- Imaginary
- Real

**Frequency [Hz]**

- 120 to 260
Jet Noise

- Nonlinear Impedance at Acoustic Liner
  - Property: Acoustic Impedance
  \[ Z = p' / \nu_n = R + iX \]
Jet Noise

➢ Tubular Liner

✓ Single hole simulation of NASA CT-57 liner
  • Experimental data: NASA grazing impedance tube
  • Theory: Zwikker-Kosten narrow tube

![Diagram of tubular liner with real and imaginary terms](image)

![Experimental setup: A. Air source plenum, B. Top-mounted liner fixture, C. Termination](image)

![Graphs showing real and imaginary terms of normalized impedance](image)
Acoustic Liner

- Nonlinear Propagation

Sonic boom propagation [Shim, Lee AIAA2001]
## Acoustic Liner

### Impedance Boundary Condition for Locally Reacting Liner

- **TDIBC implementation methods**

\[ Z(\omega) = \frac{v(\omega)}{p(\omega)} \]

<table>
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<th>Frequency-domain impedance model</th>
<th>Time-domain conversion</th>
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<td>coupling of LEE and impedance equation</td>
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<tr>
<td>Ozyoruk et al. (JCP 1998)</td>
<td>broadband model for CT liner</td>
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<td>Fung et al. (AIAA 2000, 2001)</td>
<td>several broadband models</td>
<td>modified numerical convolution integral</td>
<td>characteristic approach</td>
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Acoustic Liner

Lined Duct Propagation
- NASA Langley Grazing Impedance Tube Configuration
- Liner Impedance: constant depth ceramic tubular (CT) liner
- Sound Source
  - SPL: 120 dB, 140 dB, 160 dB
  - Waveform: Sinusoidal, Sawtooth

Sound Source with plane wave mode

Exit: Non-Reflecting

Hard Wall 8 in.
Impedance BC 16 in.
Hard Wall 8 in.~30 in.

2 in.
Lined Duct Propagation

- Experimental Data [Jones, 2003]
  - NASA Langley Grazing Impedance Tube
  - CT57 liner, M=0 case, 140 dB source

- Simulation:
  - LEE equation
  - Educed impedances for 140 dB source by NASA Langley*
Acoustic Liner

- Lined Duct Propagation with Nonlinear Effect
  - sinusoidal wave, fundamental freq=1 kHz (CT73 liner)

120 dB
Linear regime

140 dB
Nonlinear regime

160 dB

SPL curve over liner region is not changed.

- SPL of 2\textsuperscript{nd} harmonic component dominates one of source frequency
Orifice

Direct Simulation of Resonator

- Simulation of nonlinear characteristics of thin orifice

Vorticity contour
Pressure contour

Thin Orifice

amplitude of $U = u_f$

amplitude of $P = p/f^2$

Jing-Sun simulation
Present
linear theory
nonlinear theory
Orifice

- Direct Simulation of Resonator
  - Simulation of nonlinear characteristics of thin orifice
Orifice

- Helmholtz Resonator
  - Hersh-Walker’s experiment
    - Case of thick neck Helmholtz resonator

Wavelength > 400R=25T

Surface Pressure 75 dB

Pressure contour

Vorticity contour

Surface Pressure 120 dB

Pressure contour

Vorticity contour
Orifice

- Helmholtz Resonator

Impedance for 75dB

120dB

130dB

140dB

x-axis: freq (Hz)
Single Orifice Simulation

- Impedance w.r.t. Pressure

Real (Resistance)

Imaginary (Reactance)

Non-dimensional Pressure
CFD Simulation

\[ \hat{P} = 1 \]

Vorticity contour
CFD Simulation

$\tilde{P} = 1$

Mass-fraction contour
CFD Simulation – Animation

$\hat{p} = 1$

Mass-fraction contour
Red: Left fluid
Blue: Right fluid

Azimuthal Vorticity
White: Clockwise
Black: Counter-clockwise
Nonlinear Impedance

**CFD Simulation – Wake**

\[ \hat{P} = \frac{p}{(\rho \omega^2 R_0^2)} = 0.0001 \]

\[ \hat{P} = \frac{p}{(\rho \omega^2 R_0^2)} = 0.1 \]

Starting Vortex:
It is moving very slowly

At infinitesimal amplitude, no wake obviously.

Oscillating small-amplitude wake cannot travel away.
Regular and stable wake pattern.
Nonlinear Impedance

CFD Simulation – Wake

\[ \hat{P} = \frac{p}{\rho \omega^2 R_0^2} = 100 \]

Turbulent
Impedance (Reactance) vs. Wake Pattern

Reactance

1.000E+00
1.100E+00
1.200E+00
1.300E+00
1.400E+00
1.500E+00
1.600E+00
1.700E+00
1.800E+00
1.900E+00
2.000E+00

Linear Regime

Regular Wake Regime (2)

Turbulent Regime (4)

Onset of Vortex Shedding (1)

Onset of K-H instability (3)
Duct acoustics

- 6.0 kHz higher Mode Propagation
- 9.8 kHz Cut Off
- Resonator for Silencer

2.9 kHz: Plane wave mode
7.8 kHz: (1,0) mode
11.8 kHz: Higher modes
Sound Propagation in Duct

- Helmholtz Resonator in a Duct

Graph showing frequency response and transmission loss with Propagation, Noise Source, Helmholtz Resonator, and Reflection indicated. The graph depicts modes (1,0) and (2,0) with cut-on frequencies at 6.0kHz and 10.9kHz, respectively, and a peak at 7.0kHz.

Graphed data includes Resonator 2 and No muffer transmission loss in dB over frequency range from 0 to 15,000 Hz.
Application

- 13-Points DRP scheme (based on central difference)

\[
\left( \frac{\partial f}{\partial x} \right)_i = \frac{1}{\Delta x} \left( a_1 f_{i-1} + a_2 f_i + \cdots + a_4 f_{i+1} \right)
\]

\[
a_1 = 0.91595792650492, \quad a_2 = -0.3492225163622, \quad a_3 = 0.14398145036906, \quad a_4 = -0.0512369917290, \quad a_5 = 0.01327318112590, \quad a_6 = -0.0018126562895
\]

Governing equation (turbulent and incompressible channel flow)

\[
\nabla \bar{U} = 0, \quad \frac{\partial U_i}{\partial t} + \nabla (U_i \bar{U}) = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 U_i}{\partial x_i^2}
\]

where \( U_i \) : the \( i \)-th velocity component

\( p \) : pressure

\[
\frac{\partial^2 p}{\partial x_j \partial x_i} = -2 \frac{\partial U_i}{\partial x_j} \frac{\partial U_j}{\partial x_i} - \left( \frac{\partial^2 U_i}{\partial x_j \partial x_i} + \frac{\partial^2 U_j}{\partial x_i \partial x_j} \right) = f(x_i, t)
\]

with solution

\[
p = -\frac{1}{4\pi} \int_{V} f(x_i, t) dV
\]

Table 4. Resolution and grid spacing for fine, coarse and Moser grids. The dimensions of computational domain are the same in the three cases (4h x 2h x 4h/3)
Application

✓ Results

Fig. 14. R.m.s profiles for velocity components and pressure on fine grid (120x120x120). 02 (--.--).
CS (dashed line), DRP (solid line) configurations and Moser [23] (■)

Fig. 16. R.m.s profiles for velocity components and pressure on coarse grid. 02 (--.--).
CS (dashed line), DRP (solid line) configurations and CS on fine grid (■)
Most general expansion of any function can be written as

\[ F(r, \theta) = \sum_{m=0}^{\infty} (f_m(r) \cos(m \theta) + g_m(r) \sin(m \theta)) \]

where \( f_m \) and \( g_m \) are polynomial in \( r \) that have mth-order zeroes at \( r=0 \).

If \( m \) is even, \( f_m \) and \( g_m \) are both symmetric around \( r=0 \).

\[ F(r, \theta) = \sum_{m=0}^{\infty} \left( f_m(r) e^{im\theta} \right) \]

\[ = \sum_{m=0}^{\infty} c_{m} (m) r^m e^{im\theta} = r^m e^{im\theta} \sum_{m=0}^{\infty} c_{m} (m) r^{-m} = c_{m} (m) \sum_{m=0}^{\infty} c_{m} (m) r^{-m} \]

\[ u_x = \alpha_{00}, \]
\[ \frac{\partial u_x}{\partial r} = \alpha_{10} \cos(\theta) + \beta_{10} \sin(\theta), \]
\[ \frac{\partial^2 u_x}{\partial r^2} = 2\alpha_{01} \cos(\theta) + 2\alpha_{00} \sin(\theta) + 2\beta_{01} \sin(\theta), \]
\[ u_r = A_{10} \cos(\theta) + B_{10} \sin(\theta), \]
\[ \frac{\partial u_r}{\partial r} = A_{11} \cos(\theta) + A_{20} \cos(\theta) + B_{20} \sin(\theta), \]
\[ \frac{\partial^2 u_r}{\partial r^2} = 2A_{11} \cos(\theta) + 2B_{11} \sin(\theta) + 2A_{30} \cos(3\theta) + 2B_{30} \sin(3\theta). \]
\[
\frac{\partial u_x}{\partial x} + \frac{\partial u_r}{\partial r} + \frac{1}{r} \left( u_r + \frac{\partial u_{\theta}}{\partial \theta} \right) = \frac{\partial \alpha^{(x)}}{\partial x} + 2 A_{01}^{(r)},
\]
\[
\frac{\partial \rho}{\partial t} = -\left( \frac{\partial u_x}{\partial x} + \frac{1}{r} \frac{\partial \rho u_r}{\partial r} + \frac{1}{r} \frac{\partial \rho u_{\theta}}{\partial \theta} \right) = -\left( \frac{\partial \alpha^{(\rho)}}{\partial x} + A_{10}^{(r)} \alpha^{(\rho)}_{10} + B_{10}^{(r)} \beta^{(\rho)}_{10} + 2 A_{01}^{(r)} \alpha^{(\rho)}_{00} \right)
\]
\[
\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial r^2} + \frac{1}{r} \frac{\partial u_x}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u_x}{\partial \theta^2} = \frac{\partial^2 \alpha^{(x)}}{\partial x^2} + 4 \alpha^{(x)}_{01}.
\]

**FIG. 1.** Radial velocity contours in a plan situated at \( x = 5R_0 \) from the inlet section of the inviscid forced jet at time \( t = 12R_0/U \). Radial velocity fields are shown with 15 equally spaced contours between \(-0.0006/\) and \(0.0006/\). (a) Theoretical solution via linear stability analysis; (b) fine-grid solution; (c) medium-grid solution; (d) coarse-grid solution.

**FIG. 2.** Variation of error with mesh size for an inviscid test case.

\[
e_\Delta^{(2)} = \sqrt{\frac{\sum_{1}^{N} (q_\Delta - q)^2}{N}}, \quad e_\Delta^{(1)} = \frac{\sum_{1}^{N} |q_\Delta - q|}{N}, \quad e_\Delta^{(\infty)} = \max |q_\Delta - q|
\]
FIG. 4. Dilatation contours in the forced jet. (a) Cartesian coordinates; (b) method of Mohseni and Colonius; (c) series expansions treatment.

FIG. 5. Dilatation contours in an area situated after the end of the potential core for the turbulent jet. Dilatation fields are shown with 18 equally spaced contours between −0.08$/U_0/R_0$ and 0.08$/R_0$. (a) Method of Mohseni and Colonius; (b) series expansions treatment.
**Application**

- LES (Large Eddy Simulation) of compressible turbulent jets

\[ f = \frac{\rho f}{\rho} \]

Where the bar denotes the standard the LES filtering and \( \rho \) the density

Governing equation (turbulent and incompressible channel flow)

\[
\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_i} (\bar{\rho} u_i) = 0
\]

\[
\frac{\partial \bar{p}u_i}{\partial t} + \frac{\partial}{\partial x_i} (\bar{p} u_i u_j) = - \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \sigma_{i,j} - \frac{\partial}{\partial x_j} (u_i u_j - u_i u_j) \right) \\
\sigma_{i,j} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{i,j} \frac{\partial u_k}{\partial x_k} \right)
\]

\[
\frac{\partial \bar{E}}{\partial t} + \frac{\partial}{\partial x_i} (u_i (\bar{E} + \bar{p})) = - \frac{\partial}{\partial x_i} q_i + \frac{\partial}{\partial x_j} \left( \sigma_{i,j} u_i \right) - \frac{\partial}{\partial x_i} (\bar{E} u_i - \bar{E} u_i + \bar{p} u_i - \bar{p} u_i) \\
q_i = -k \frac{\partial T}{\partial x_i}
\]

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Re )</td>
<td>36,000</td>
<td>100,000</td>
</tr>
<tr>
<td>( N_x \times N_r \times N_0 )</td>
<td>192 \times 128 \times 64</td>
<td>320 \times 150 \times 96</td>
</tr>
<tr>
<td>( L_x )</td>
<td>45R_0</td>
<td>47.5R_0</td>
</tr>
<tr>
<td>( L_r )</td>
<td>8R_0</td>
<td>11R_0</td>
</tr>
<tr>
<td>( \Delta x_{\text{max}} )</td>
<td>0.42R_0</td>
<td>0.15R_0</td>
</tr>
<tr>
<td>( \Delta x_{\text{min}} )</td>
<td>0.14R_0</td>
<td>0.15R_0</td>
</tr>
<tr>
<td>( \Delta r_{\text{max}} )</td>
<td>0.1R_0</td>
<td>0.09R_0</td>
</tr>
<tr>
<td>( \Delta r_{\text{min}} )</td>
<td>0.03R_0</td>
<td>0.03R_0</td>
</tr>
</tbody>
</table>
Figure 1. The mean centerline velocity as a function of the axial coordinate $x$, obtained from the LES, DNS (Freund 1999), and experiment (Stromberg et al. 1980). DNS 3600: ---; Stromberg et al.: +; LES 36000: ----.

Figure 2. The mean velocity as a function of the radial coordinate at various different downstream positions. LES: $x = 3.2R_0$, ---; $x = 13.0R_0$, ----; $x = 19.5R_0$, -----; $x = 25.9R_0$, ·······. DNS: $x = 19.5R_0$, smallplus; $x = 25.9R_0$, ×.

Figure 4. Left: Contour plot of the density; Right: Contour plot of the total vorticity multiplied by the axial coordinate $x|\omega|$.

Figure 5. A contour plot of the dilatation $(\partial u_1/\partial x_1)$.
大學 (University)

格物 致知 正心 誠意
修身 齊家 治國 平天下

Material Property, Life,
Righteous Mind, Sincere Attitude
My Self, Family,
Country, Peaceful world
Thank You