

WORKSHOP ON INTEGRABLE SYSTEMS AND GEOMETRIC PDES

WORKSHOPS



CASIS

理論科學研究中心

Center for Advanced Study in Theoretical Sciences

2012. 10. 20-22

Toda equations, meromorphic connections, and harmonic maps (2)

Prof. Martin A. Guest

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Moduli spaces of meromorphic connections and their symmetries (2)

Prof. Daisuke Yamakawa

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SYZ Conjecture and Affine Differential Geometry (2)

Prof. John Loftin

At the large complex structure limit in the moduli of Calabi-Yau manifolds, Strominger-Yau-Zaslow conjecture that the Calabi-Yau structure reduces to a fibration of special Lagrangian tori over an affine base. In particular, the complex Monge-Ampere equation on the Calabi-Yau manifold reduces to a real Monge-Ampere equation $\det u_{ij} = 1$ on the affine base. Mirror symmetry then corresponds to a Legendre transform for the potential function u on the base, coupled to a Fourier type transform on the torus fibers. The base in the SYZ fibration is expected to have singularities of codimension 2. Thus for a Calabi-Yau threefold, the base is real 3-dimensional, and thus the singular set is likely to be a graph. I will discuss some old joint work with Yau & Zaslow to construct an affine structure plus a solution to the Monge-Ampere equation on a 3-ball minus a trivalent vertex of a graph. The proof involves a dimension reduction due to Baues-Cortes to a semilinear elliptic PDE on S^2 minus 3 points. This elliptic PDE is a structure equation for a classical affine differential object, an elliptic affine sphere in R^3 . With a solution to this PDE in hand, one can use a developing map to construct both an affine structure and a solution to the Monge-Ampere equation. I will discuss some open questions about this construction, and in particular attempts to use integrable systems techniques to recover the affine holonomy (which should be integral in order for the torus fibration to exist).

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